Developments in the Theory of Turbulence Spreading with Self-Consistent Flows

P.H. Diamond

[1] WCI Center for Fusion Theory, NFRI, Korea
[2] CMTFO and CASS, UCSD, USA

Joint US-EU Transport Taskforce Workshop TTF 2011
San Diego, USA
6 – 9 April 2011

Collaborators

- Z.H. Wang, DLUT and PKU; China
- H.J. Sun, NFRI, Korea and SWIP, China
- X.G. Wang, PKU; China
- A. Ulvestad; UCSD
- Ackn: Ö.D. Gürcan, X. Garbet, T.S. Hahm,
  C. Hidalgo, X. Zou

Outline

- Motivation : Turbulence spreading as a mechanism for fast transients, profile resiliency and ITB's
- Fundamentals of Model
  - Spreading ↔ Profiles ↔ Flows : Feedback loops and need for self-consistency
  - Model equations
  - Essentials of front dynamics
- Results from model studies
  - off axis heating → ingoing pulses → edge-core connection → profile resiliency

Outline (cont'd)

- Results, cont'd
  - intensity pulses can, but need not, penetrate gaps in excitation profile
  - intensity and heat pulse propagation can decouple in barriers
  - initial modeling of cold pulse propagation experiments

- The Quandary: Do zonal flow help or hinder spreading?
  - physics of wave packet propagation in zonal flow
  - spreading and local/non-local interaction in *k*

- Conclusions and Discussion

Motivation

- Some unresolved puzzles:
  - Cold pulse propagation:
    - ✓cool edge – core heat on ~ 1 msec
    - ✓indications of 'ITB' at inversion radius
    - ✓quenched for $n > n_{crit}$
  - Profile resiliency (stiffness):
    - ✓why do temperature profiles tend to exhibit small response to large perturbation ?
    - ✓edge + center heating: peaked profiles?

Motivation (cont'd)

- Some unresolved puzzles, cont'd
  - ITB's:
    - ✓ physics of threshold ?
    - ✓ when can avalanches penetrate nascent barrier?

- A highly relevant player in all-of-above:
  - **Turbulence Spreading !**

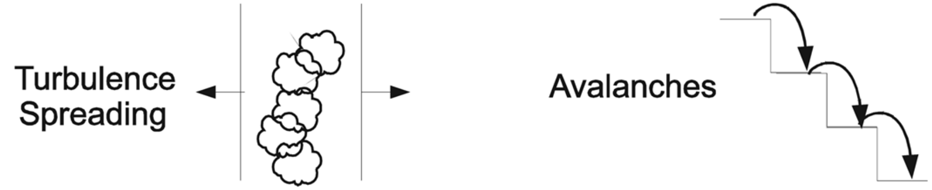
Fundaments of Model

- Spreading and Self-Consistency
  - "spreading" = tendency of turbulence to self-scatter (i.e. vortex mutual induction) and entrain stable regime
  - "spreading" closely linked to "avalanching", "avalanching" = tendency of excitation to propagate in space via local gradient change
  - Minimal model must:
    - ✓ treat intensity profiles, flows (ExB) self-consistently
    - ✓ be flux driven

Fundaments of Model (cont'd)

- Relation: Turbulence Spreading ↔ Avalanching

self-scattering **gradient feed-back**



entrainment of neighboring regions by localized turbulence;	sequential local mixing; over turning of adjacent cells
spatial coupling-nonlinear coupling accompanies avalanches;	spatial coupling-gradient evolution fluctuations and transport events;
nonlinear scattering diffusion $\frac{\partial I}{\partial t} \sim \frac{\partial}{\partial x}(D_0 I \frac{\partial I}{\partial x})$;	scale independent $Pdf \sim 1/\Delta x^{\alpha}$;
ubiquitous in $k-\epsilon$ models;	fractional kinetic transport;
based in spatial structure, wave interactions;	elements suggested in many experiments;
observed in gyro-kinetic simulations.	observed in simulations.

Close relationship of dynamics self-consistent profiles, flows, intensity a **MUST**

Fundaments of Model (cont'd)

- Model: Extended Fisher-Kolmogorov System

Fluctuation Intensity

$$\frac{\partial I}{\partial t}-\underbrace{\frac{\partial}{\partial x}\left(\chi_{turb}\frac{\partial I}{\partial x}\right)}_{\text{nonlinear diffusion, spreading}}=\underbrace{\gamma I}_{\text{linear growth}}-\boxed{\beta I^2}$$

nonlinear damping

Ion Temperature :

$$\frac{\partial\langle T\rangle}{\partial t}-\frac{\partial}{\partial x}\left[\left(\chi_{turb}+\chi_{neo}\right)\frac{\partial\langle T\rangle}{\partial x}\right]=S$$

neoclassical transport

ExB Shear :

$$V_E'^2=\frac{1}{e^2B_z^2}\left(\frac{\partial^2\langle T\rangle}{\partial x^2}+\frac{\partial\langle T\rangle}{\partial x}\frac{\partial\ln\langle n\rangle}{\partial x}-\langle T\rangle\left(\frac{\partial\ln\langle n\rangle}{\partial x}\right)^2+\frac{\langle T\rangle}{\langle n\rangle}\frac{\partial^2\langle n\rangle}{\partial x^2}\right)^2$$

(self-consistent)

$$\gamma=\gamma_0\frac{(R/L_T-R/L_T^{crit})}{\left(1+V_E'^2/V_c'^2\right)}\Theta(R/L_T-R/L_T^{crit})\qquad \chi_{turb}=\chi_0 I/(1+V_E'^2/V_c'^2)$$

excitation

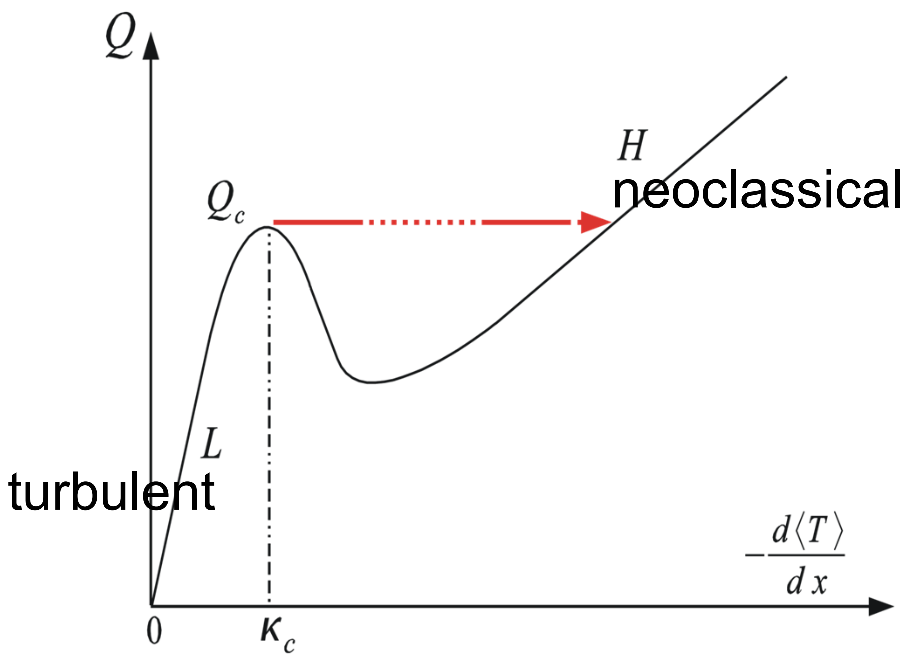
Heaviside function

transport, scattering

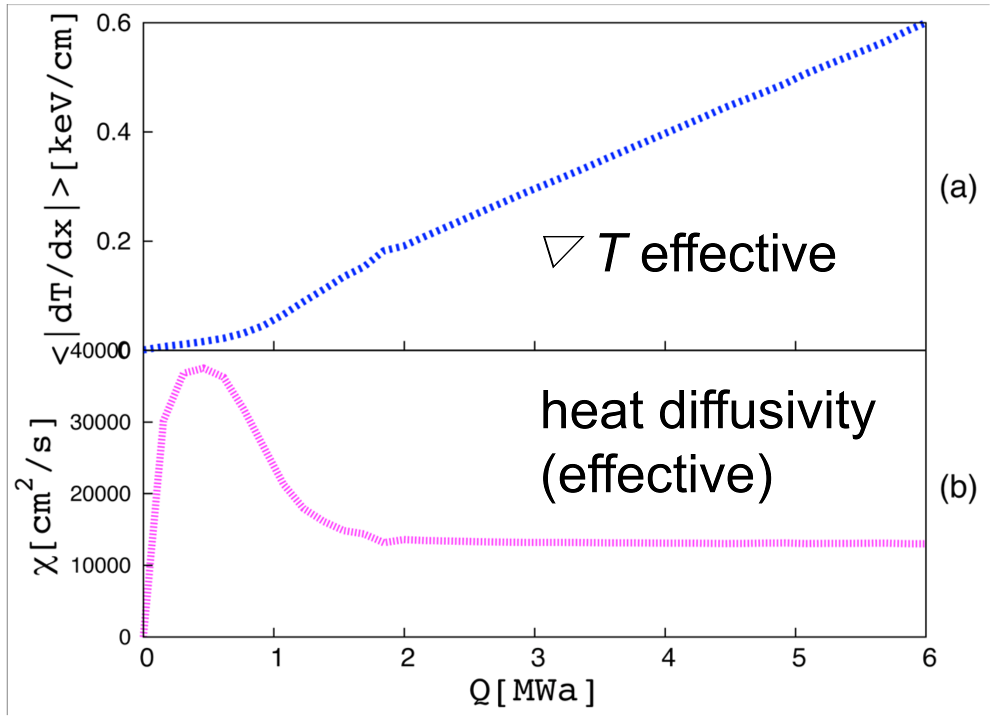
Fundaments of Model (cont'd)

- Model: Extended Fisher-Kolmogorov System

Bi-stability of Heat Flux due Shear Feedback



∇T and χ_{total} vs. Q



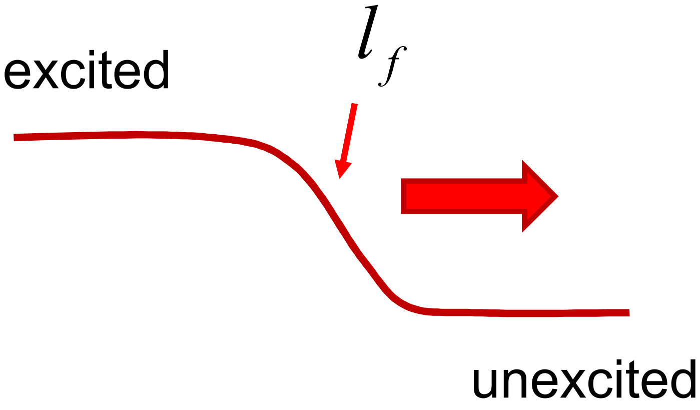
Fundaments of Model (cont'd)

- Fisher-Kolmogorov Fundamentals
  - Supercritical Reaction-Diffusion System
  - Leading edge – mesoscale

$$1/\tau_c > 1/\tau_f > 1/\tau_{transp}$$

fast transit

$$l_c < l_f < l_{system}$$



- $V_f \sim (\gamma D)^{1/2} \sim \rho_*^{\alpha} c_s$
  1. $\alpha = 1 \rightarrow$ Gyro Bohm D; $V_f \sim V_*$
  2. $\alpha = 1/2 \rightarrow$ Gyro Bohm D; $V_f \sim \rho_*^{1/2} c_s$

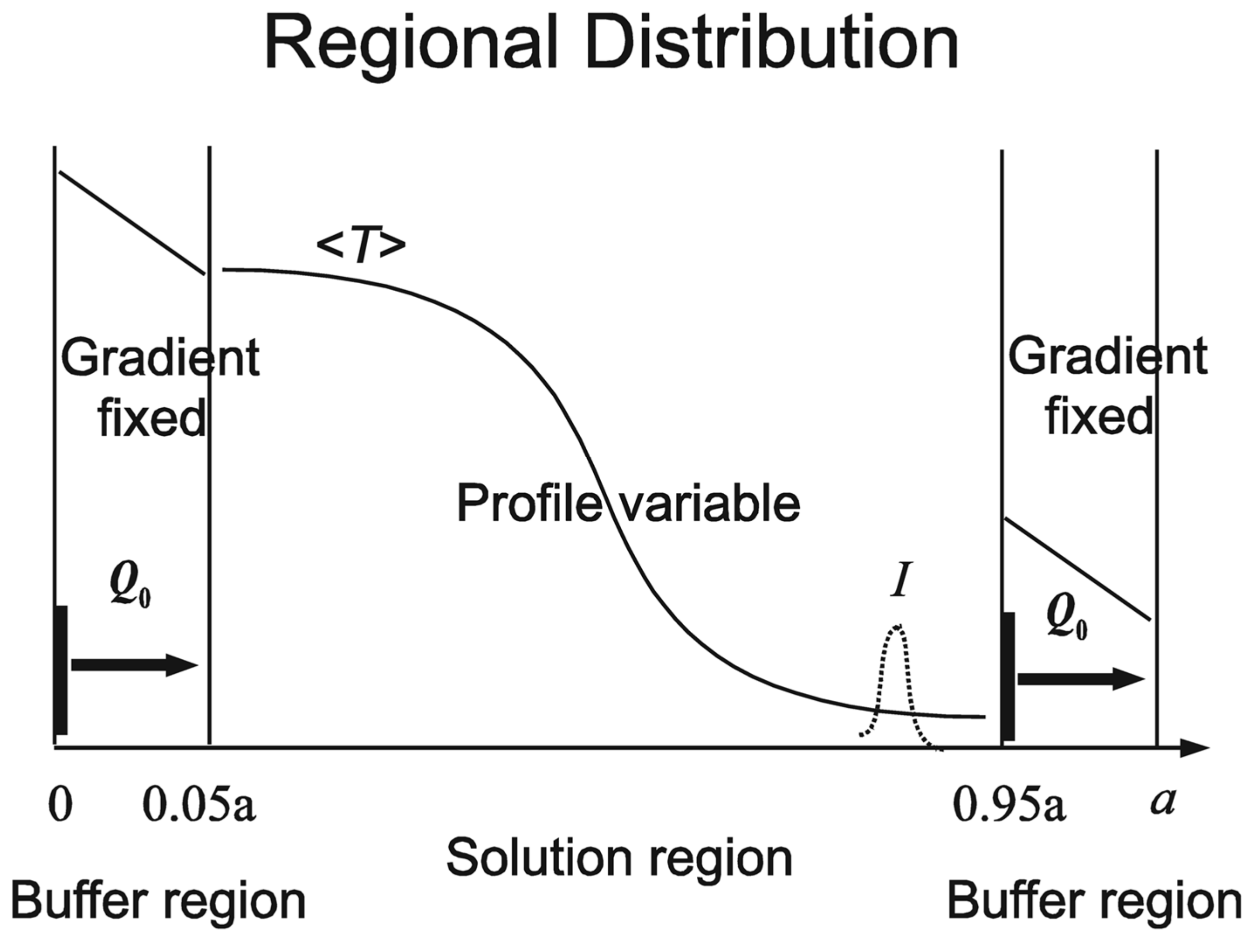
- $D = D_{GB}$, $V_f \sim \rho_* c_s / L_\perp \sim V_*$

$$\tau_f \sim \rho_* \tau_{transp}$$

$\tau_f \leq$ 1 msec (HL-2A; OH)

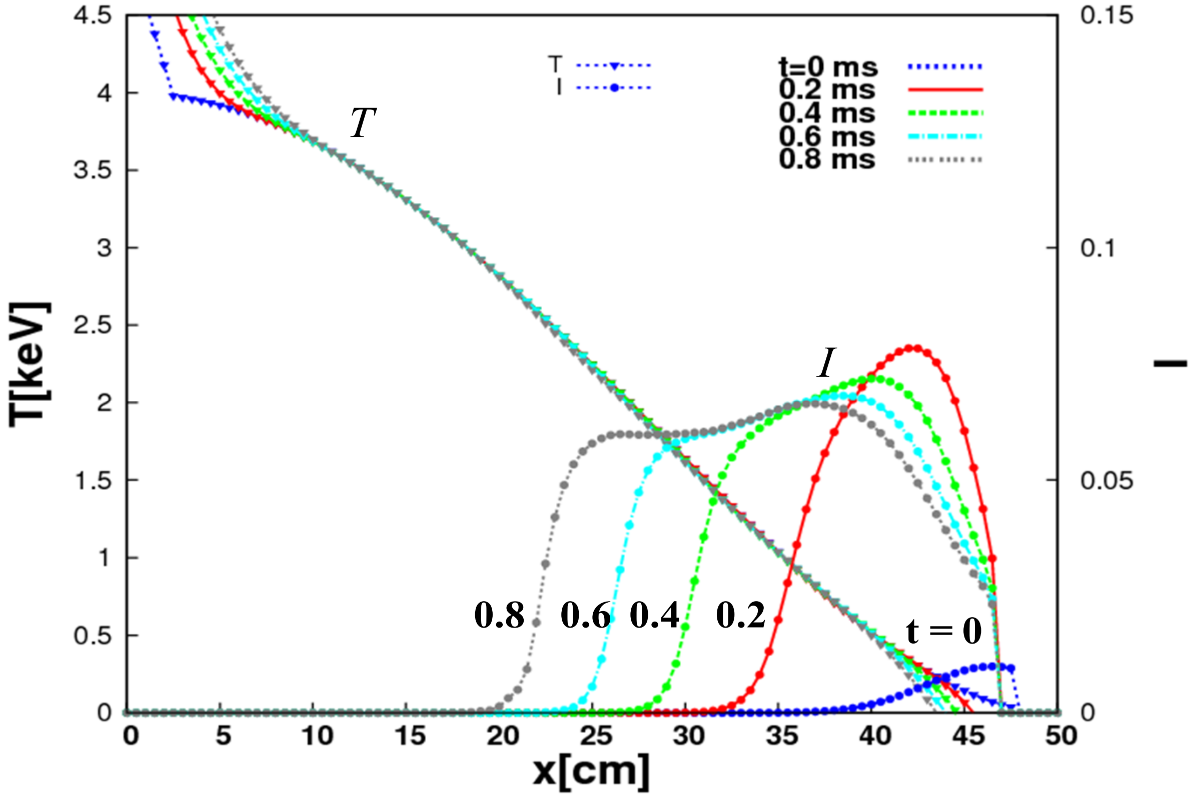
Results

- Computational Model Set-Up: Fixed Q Drive



Result (cont'd)

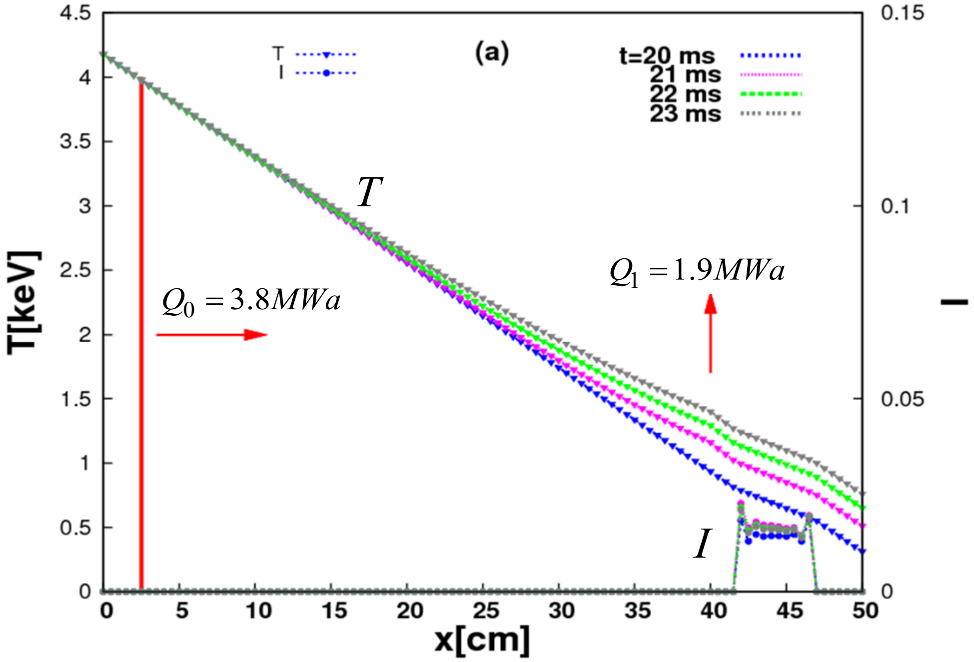
- Intensity and Heat Pulse Propagation
  - pulse initiated near edge
  - heat flux Q applied in center

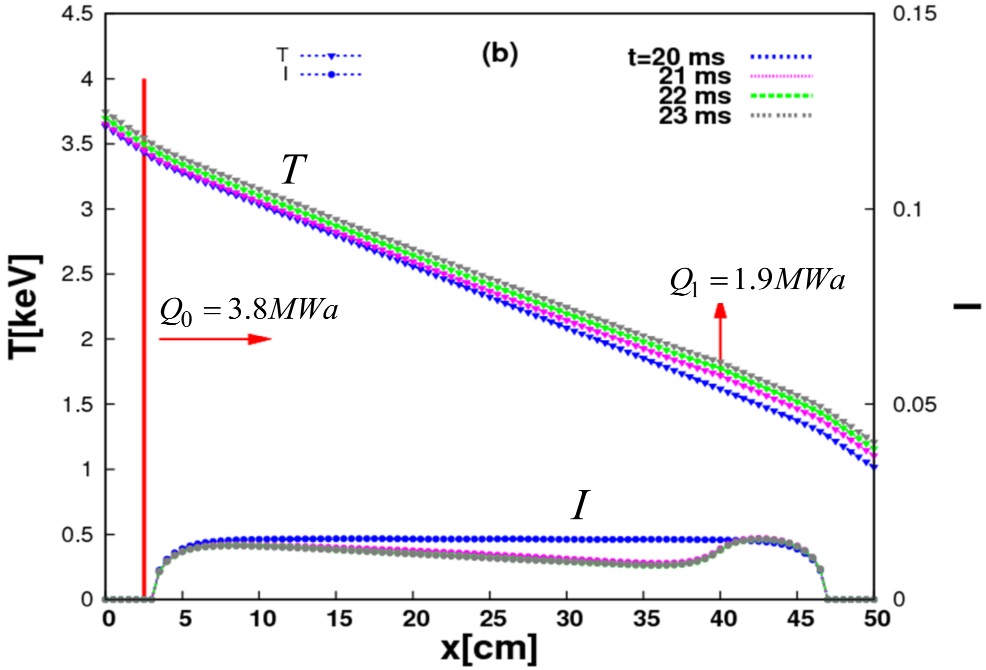


  - pulse maintains/ending edge during inward propagation

Result (cont'd)

- Spreading: possible explanation of profile resilience !?!





No spreading: $\chi_{turb} \to 0$ in *I*-equation

- local variability in ∇T at position edge source evident

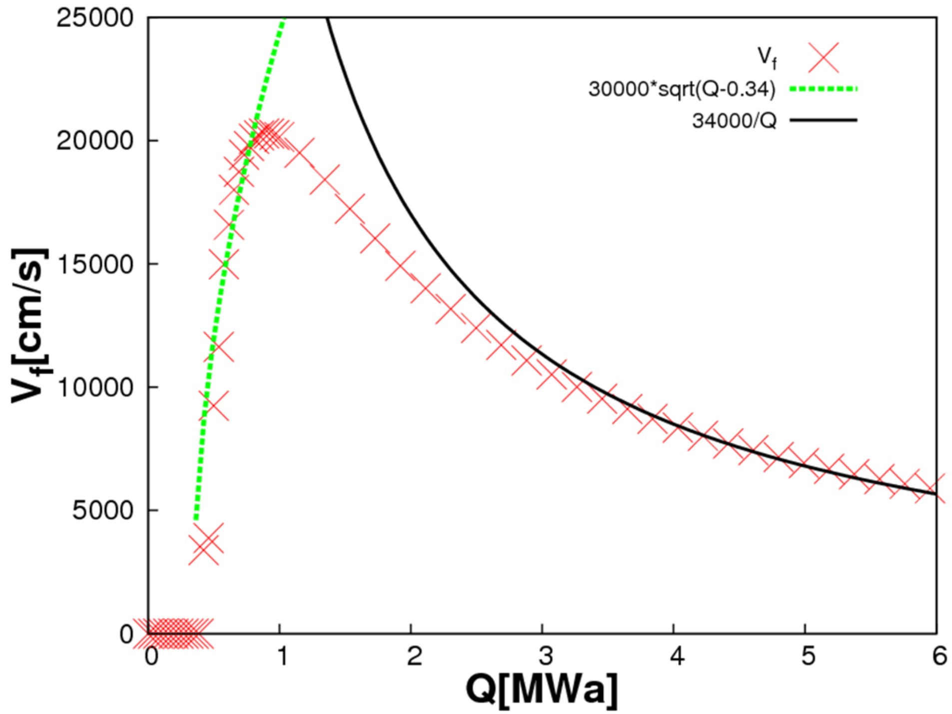
- *I*-dynamics localized

With spreading: χ_{turb} same for *I*, <*T*>

- profile very resilient
- rapid intensity pulse propagates inward. Modifies *I*-profile $(\tau_f << \tau_{transp})$,
- Leaving <T> ~ unchanged

Results (cont'd)

- Scaling: Intensity Pulse Speed vs. *Q*
  - $V_f(Q)$ bi-stable



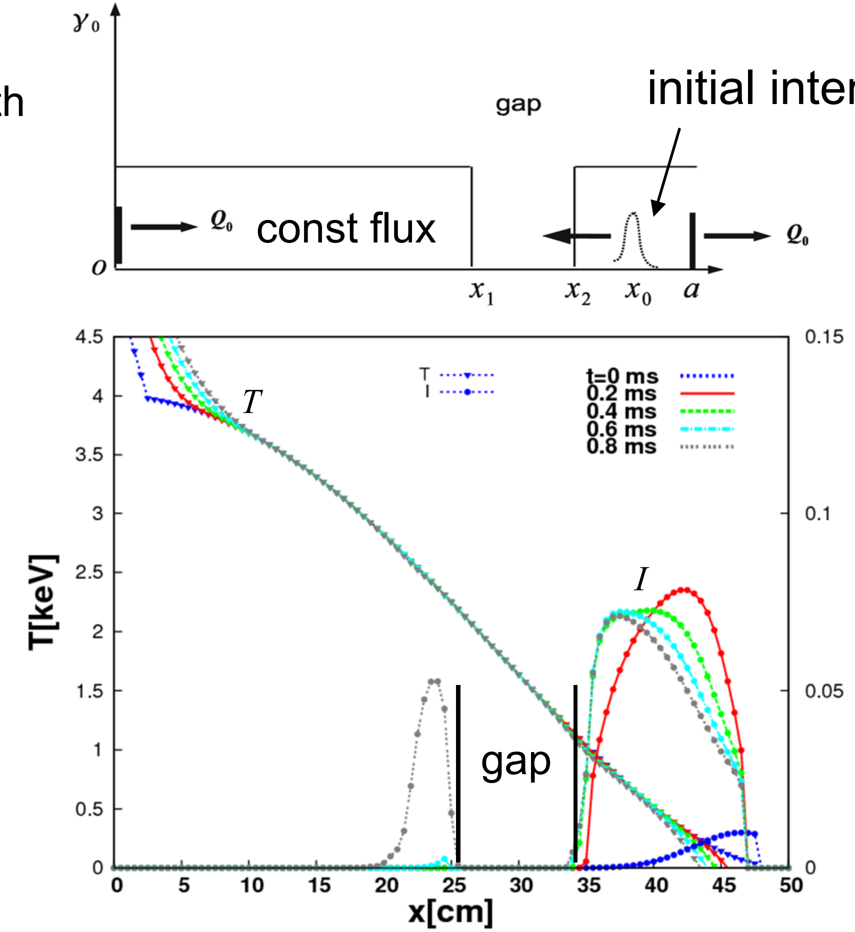
  - intensity pulse speed first ~ $Q^{1/2}$, then 1/Q
  - quantitatively consistent with analysis

Results (cont'd)

- Scattering experiments: Pulse Penetration of Gaps
  - Intensity pulse scattering from linear excitation gaps

growth

rate



Gap: - width

- location

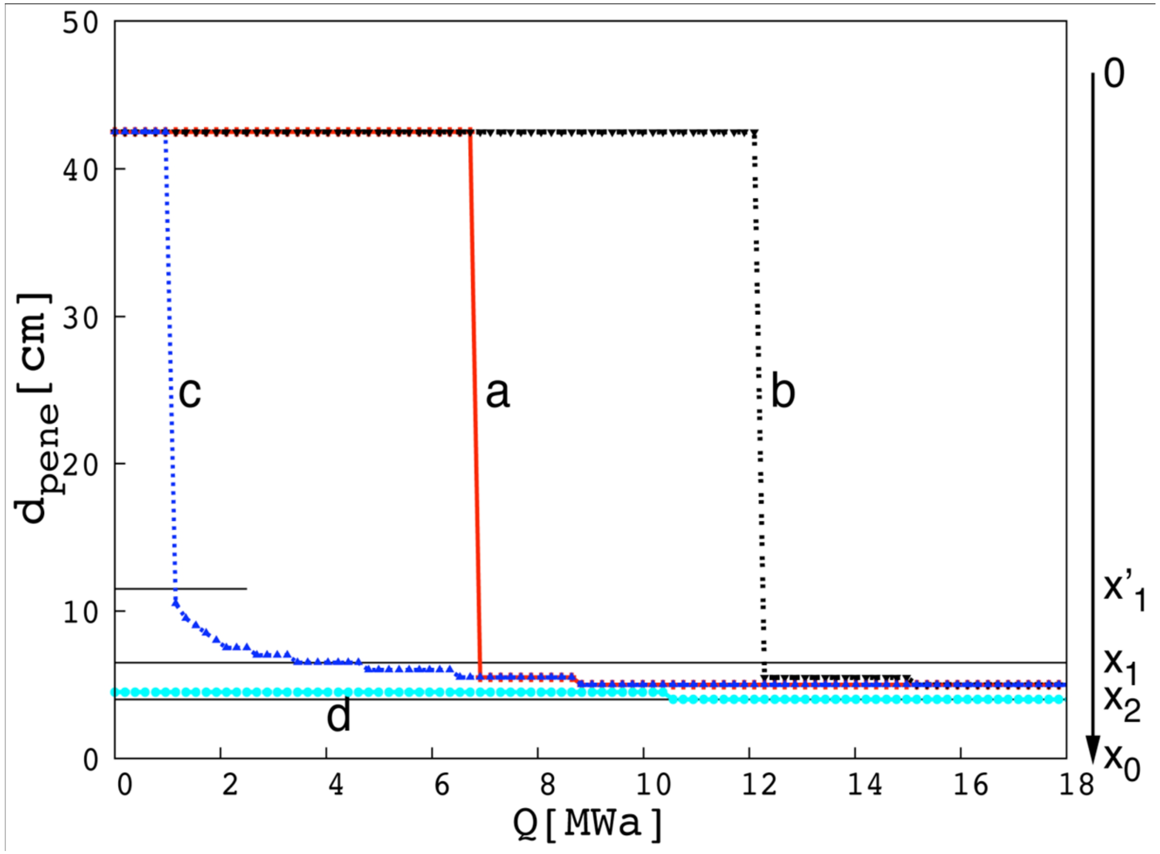
- marginal vs. damped

Pulse penetration of gap is variable

Results (cont'd)

- Intensity pulse penetration depth vs. Q
  - Q can block pulse penetration of excitation gap

a. narrow gap $Q > Q_{crit} \sim 7$ MW to block
b. narrow gap, decreased shearing → Q_{crit} increases
c. large gap, Q_{crit} decreases
d. damped gap, ~ no penetration for any Q



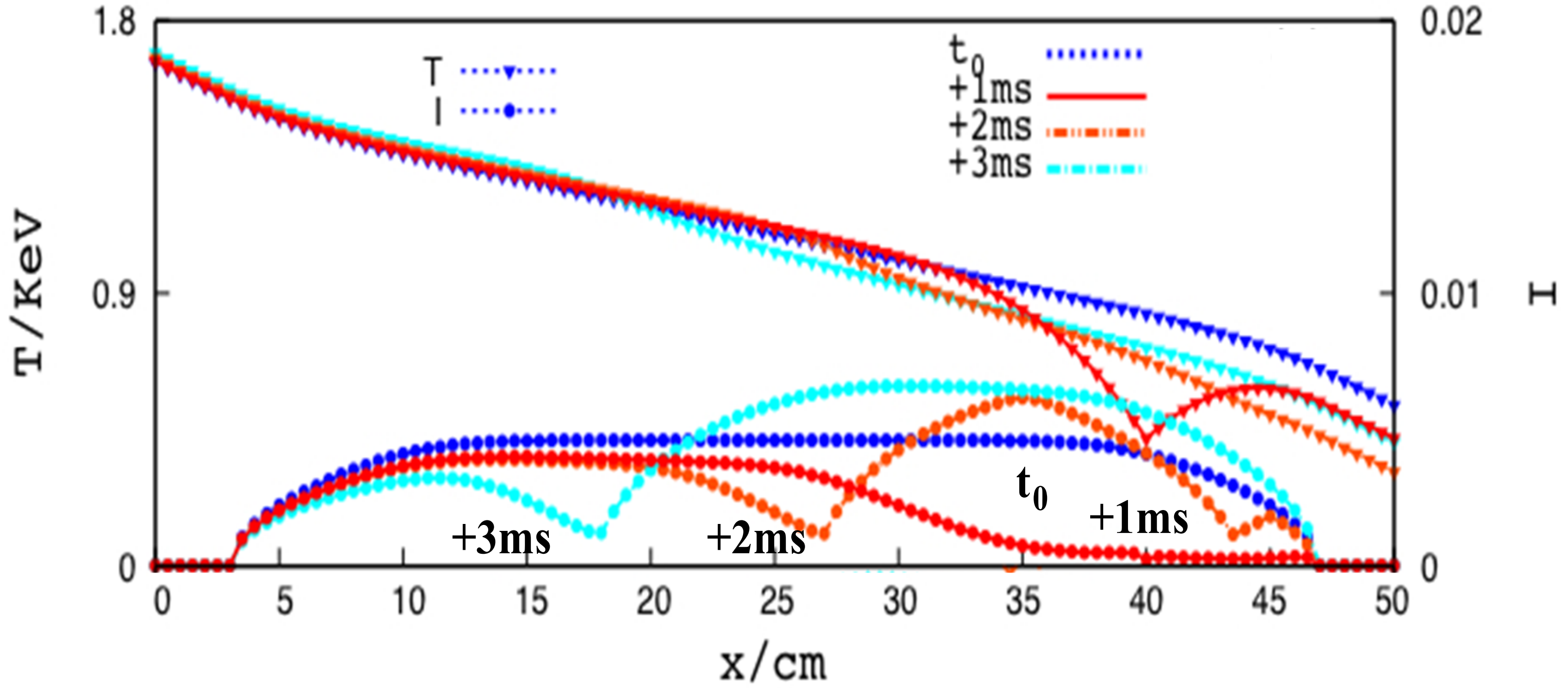
Results (cont'd)

- What here we learned so far?
  - self-consistent intensity, profiles, flows required
  - turbulence spreading can rapidly re-distribute excitation $\rightarrow$ fast intensity pulse as means for profile resiliency !?!
  - $V_f(Q)$ is bi-stable
  - pulse scattering experiments suggest that
    - gap penetration is variable
    - Q can block intensity pulse

$\therefore$ Is ITB formation related to keeping turbulence out, as well as heat in ?

Results (cont'd)

- To the Cold Pulse: Turbulence spreading and "Non-locality"



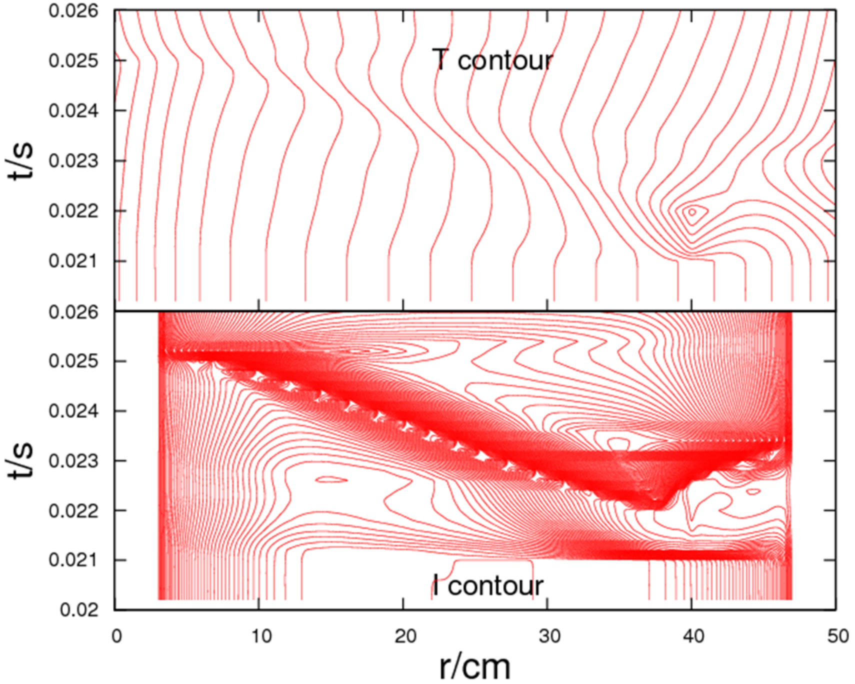
– cold pulse as edge + center, with negative edge source

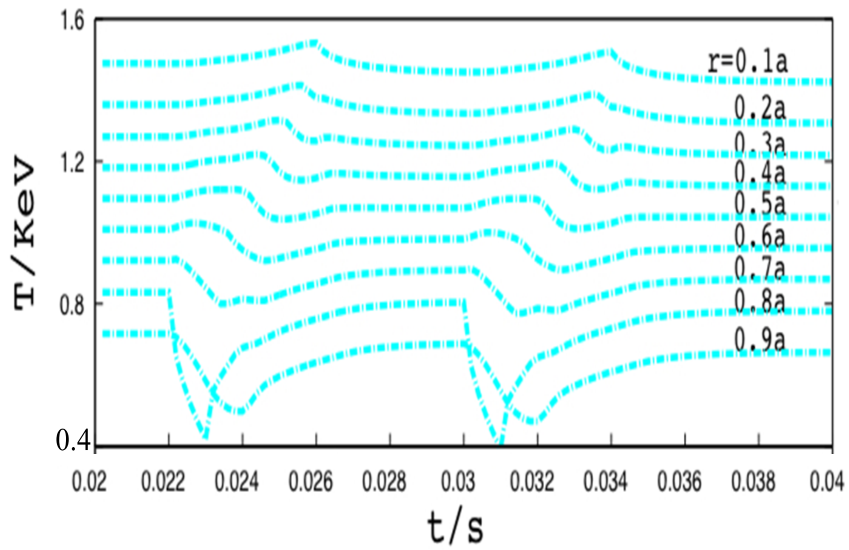
– fast intensity pulse to center; $\tau_f \sim 1$ msec

→ some T profile steepening → closer look ?!

Results (cont'd)

- Turbulence spreading and "Non-locality": Cold Pulse Propagation





- *t* vs *r* plots of heat and intensity pulse
- ~ constant V_f manifested
- more structure in *I*

- model manifests (weak) inversion !
- ∇T steepening due self-consistent shearing (Q bi-stability) is cause
- ITB? $\leftrightarrow$ sustain?

The Quandary of Zonal Flows

- Do zonal flows help or hinder the spreading? If promote, how effective?

- The conflict:
  - natural expectation re: shearing

  vs.

  - symmetry breaking effect on wave packet propagation

  and

  - purely non-local interaction (in scale)

  vs.

  - non-local + local interaction

The Quandary of Zonal Flows (cont'd)

- Zonal spreading
  - mechanism is linear group propagation
  - i.e. for Rossby waves

$$\omega = -\frac{\beta k_x}{\mathbf{k}^2}, \quad v_g = \frac{2\beta k_x k_y}{(\mathbf{k}^2)^2}$$

for symmetric spectrum $<k_x k_y> = 0 \rightarrow <v_{gy}> = 0$ no propagation

  - if zonal shear: $\frac{d}{dt} k_y = -\partial_y \left(k_x \langle v_x \rangle \right)$

$$k_y = k_{y0} - \int k_x \langle v_x \rangle' dt$$

$$\therefore v_{gy} = -2\beta k_x^2 \int \langle v_x \rangle' dt / \left(\mathbf{k}^2\right)^2$$

  - shear "correlates" k_y, k_x → no ambiguity in $<k_x k_y>$ but
  - inertia k^2 increase in time → efficiency?

The Quandary of Zonal Flows (cont'd)

- Zonal spreading, cont'd
  - n.b. not sufficient to establish propagation, need to establish/quantify:
    a. penetration, i.e. how far does turbulence penetrate into stable/damped region?
    b. efficiency, i.e. how much of initial source is radiated?

- analysis must include: growth/damping profiles and dissipation

- analysis should be non-perturbative, i.e. NLS models will miss enhanced inertia

The Quandary of Zonal Flows (cont'd)

Model and Analysis

- 1D, eikonal $\rightarrow$ **asymptotic, but non-perturbative**
- w =pseudomomentum $\rightarrow$ akin to wave momentum density

$$\underbrace{\partial_t w + \partial_y(v_{gr,y} w)}_{\text{group propagation}} = \Big(\underbrace{\gamma(y)}_{\text{growth}} - \underbrace{D_0(y)k_\perp^2}_{\text{damping}}\Big)\, w \qquad (1)$$

$$v_{gr,y} = \frac{2\beta k_x k_y}{(k_\perp^2)^2}$$

$$\partial_t\langle v_x\rangle = -\partial_y\langle v_y' v_x'\rangle - \overbrace{\nu\langle v_x\rangle}^{\text{drag, critical}} \quad \text{Reynolds stress} \qquad (2)$$

$$= \partial_y(v_{gr,y} w) - \nu\langle v_x\rangle \quad \text{pseudomomentum flux}$$

- n.b. $\partial_t(\langle v_x\rangle + w)$ =growth/damping $\rightarrow$ momentum conservation

The Quandary of Zonal Flows (cont'd)

Model and Analysis II

$$\frac{dk_y}{dt} = -k_{x0}\partial_y\langle v_x\rangle + D\nabla^2 k_y$$

- Eikonal equation $\rightarrow$ straining
- Model is non-perturbative
- Next:
  1. free, non-dissipative solution
  2. speed-amplitude relation
  3. numerical solution of dissipative system

Free solutions - Fronts and propagating nonlinear wave packets

- take: D_0, γ, ν, D,etc$\rightarrow 0$
- look for solutions of the form: $f(y - ct) \rightarrow$ nonlinear packets
- Then $k_y = k_{x0}\frac{v_x}{c} + k_{y0}$

$$v_x = -\frac{v_{gr,y}}{c}w + v_0$$

$$(v_{gr,y} - c)w = w_0$$

- Now, $w = -\epsilon k^2/\beta$ where $\epsilon =$ energy density

The Quandary of Zonal Flows (cont'd)

Model and Analysis III

- Final results:

$$k_y = \frac{2k_{x0}^2 k_y}{c^2} \frac{\epsilon}{k_{x0}^2 + k_{y0}^2} + k_{y0} + \frac{k_{x0}}{c} v_0$$

$$\left[\left(\frac{2\beta k_y k_{x0}}{k_{x0^2} + k_y^2} \right) - c \right] \epsilon k^2 = \epsilon_0 k_{x0}^2$$

- Suggests wave-packet bifurcations
- Simple, solvable limit: $k_{y0} + \frac{k_{x0}}{c} v_0 = 0, \quad \epsilon_0 = 0 \to$ choice
- $\to k_y = \pm \left(\frac{2k_{x0}^2 \epsilon}{c^2} - k_{x0}^2 \right)^{1/2}$
- and $\frac{c^2}{2k_{x0}^2} \frac{(2\epsilon - c^2)^{1/2} \beta}{\epsilon^2} = 1 \to$ **exact** speed-amplitude relation

The Quandary of Zonal Flows (cont'd)

Numerical Studies with Damping and Overshoot

- $c = c(\epsilon, \beta, k_{x0})$ is packet speed
- if $\epsilon \gg c^2 \rightarrow$

$$c = \left[\frac{\epsilon^3 (k_{x0})^2}{\beta^2} 2^{3/2} \right]^{1/4} \sim \epsilon^{3/4} \rightarrow v_{gr}$$

- Nonlinear packets happen, **if** free
- free solutions interesting, but of limited practical interest
- explore propagation with packet growth/damping profile, flow damping, etc.

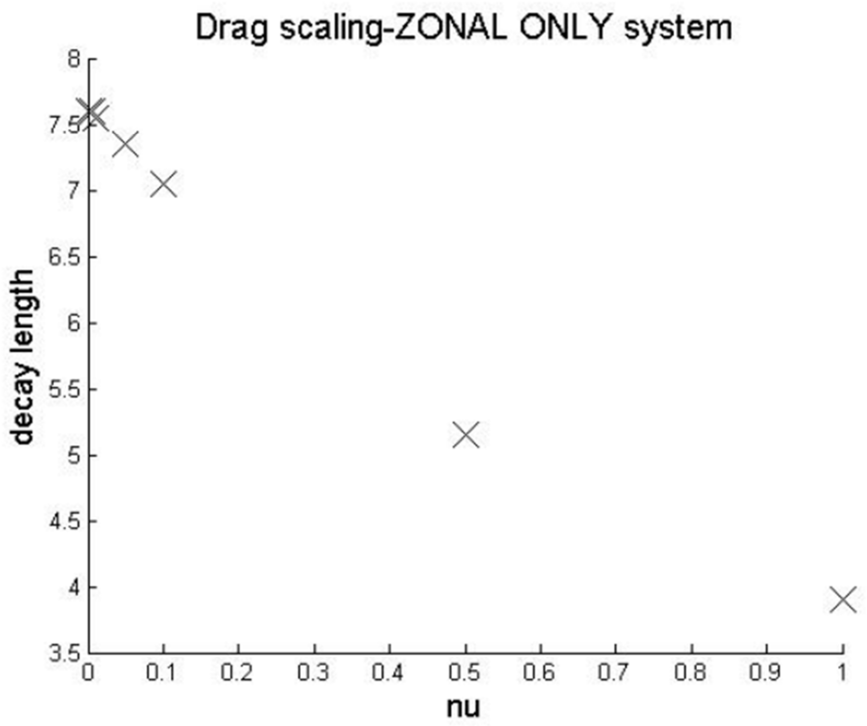
Issues:

- role of flow damping?
- efficiency of radiation packets?
- penetration depth

The Quandary of Zonal Flows (cont'd)

Wave Packet Decay Length Drops Rapidly with Increasing Flow Drag

Z.F. mediated spreading is inefficient



Decay length is defined as the length for the amplitude of the intensity pulse to decay to one half its initial value

The Quandary of Zonal Flows (cont'd)

Local and Zonal Evolution

Comparison Point: Local and Zonal Model

- Recall local scattering/mixing $\rightarrow$ propagating fronts

$$\partial_t \epsilon - \partial_x D_0 \epsilon \partial_x \epsilon = \gamma\epsilon - \alpha\epsilon^2$$

- Fisher equation with nonlinear diffusion
- resembles $k - \epsilon$ models
- derived via Fokker-Planck theory
- since $\epsilon = \frac{\omega_k}{k_x} w$, can combine local, zonal interactions in w equation

$$\partial_t w + \partial_y (v_{gr,y} w) - \partial_y \frac{D_0 \beta}{k^2} w \partial_y w + \alpha \frac{\beta}{k^2} ww = \left(\gamma - D_0 k^2\right) w$$

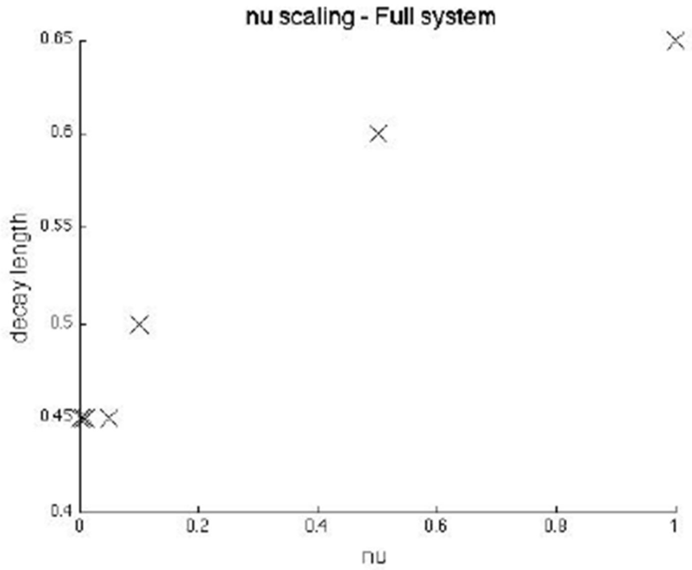
- $\langle v_x \rangle$,k_y equations as before

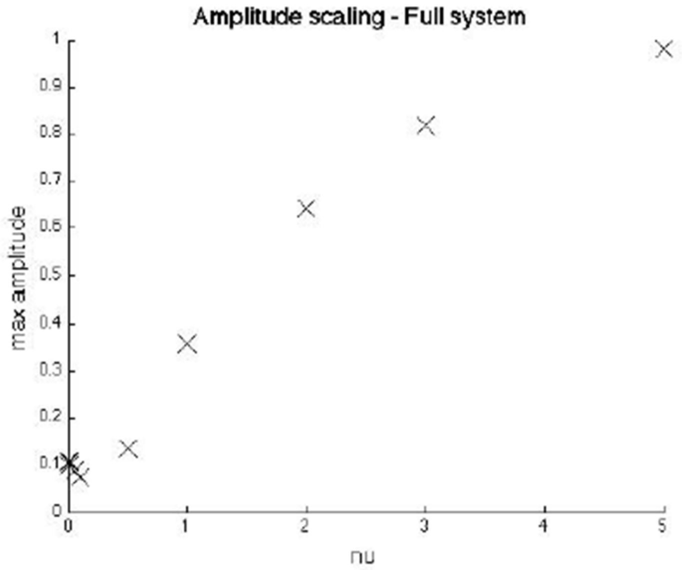
Note:

- in combined model, energy can propagate by:
  1. zonal coupling $\rightarrow v_{gr,y} w$
  2. local scattering $\rightarrow \partial_y \frac{Dw}{k^2} \partial_y w$
- but: local scattering robust, insensitive to zonal flow dissipation, phase relations
- naturally, explore synergy/complementarity

The Quandary of Zonal Flows (cont'd)

Scaling with Flow Drag in combined system





- In contrast to zonal-only system, decay length **increases** with ν. Maximum Envelope Amplitude **increases** with ν
- Local couplings robust to Z.F. damping

The Quandary of Zonal Flows (cont'd)

Bottom Line:

Zonal Flows may help spreading, but only a little…

Key Issues

- Theory
  - extend model to include $\langle V_{\varphi} \rangle$, $\langle V_{\theta} \rangle$ and $\langle n \rangle$ evolution
  - improve representation of scattering $\rightarrow$ i.e. beyond intensity diffusion (i.e. local + non-local interaction in *k*)
  - WKE + Zonal models and mean profiles
  - fractional kinetics formulation $\rightarrow$ how calibrate?

Key Issues

- Phenomenology
  - resilience: spreading and/or heat pinch (L. Wang, P.D. '11)
  - physics of inversions in cold pulse? shear flow or ? barrier evolution?
  - $n_{crit} \rightarrow$ OH power coupling?
  - spreading through
    - reversed shear
    - low order rationals

    ?
  - periodic excitation $\rightarrow$ SMBI