Lawrence Livermore National Laboratory

How micro-turbulence breaks magnetic surfaces

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Important collaborators …

- From IPP Garching:
  - Frank Jenko,
  - M.J. Pueschel,
  - F. Merz
- Formerly U of Wisconsin (and now IPP Garching):
  - David Hatch
- PPPL
  - Walter Guttenfelder

Equilibrium field structure and rational surfaces

- *q*-profile across flux-tube

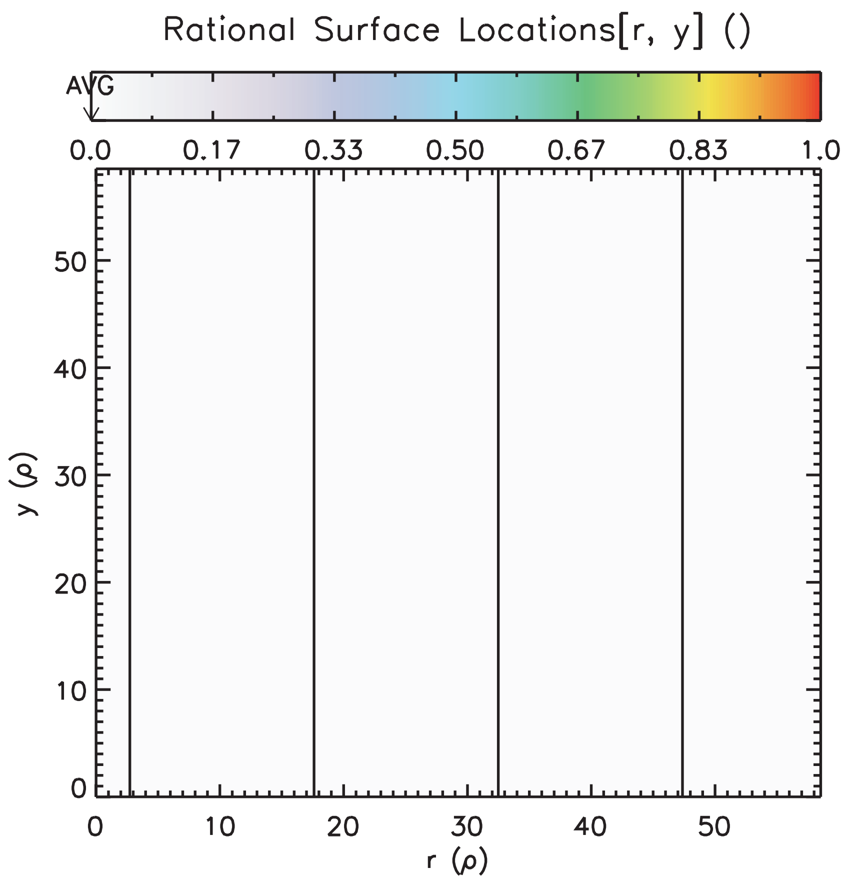
$$q(r) \approx q(r_0) + q'(r - r_0)$$
$$\approx q_0\left[1 + \left(\frac{\rho_i}{r_0}\right)\hat{s}\,\frac{(r - r_0)}{\rho_i}\right]$$

- At rational surface *q=m/n*

~1/n ~ρ*

$$\left(\frac{m}{n} - q_0\right) = \left(\frac{\rho_i}{r_0}\right)\hat{s}\,\frac{(r - r_0)}{\rho_i}$$

⇒ Flux-tubes have (high order) rational surfaces



Field-line trajectory in flux coordinates

- Equilibrium flux coordinates:
  - *(x,y)* are field-line labels
  - *s* measures distance along ***B***

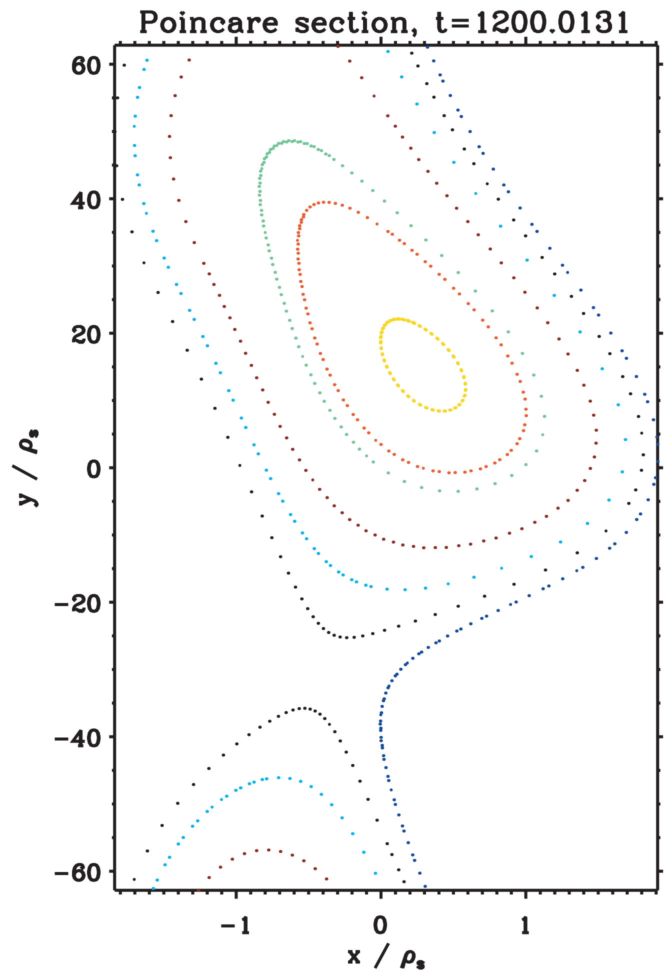
$$\frac{ds}{B} = \frac{dx}{\vec{B} \bullet \nabla x} = \frac{dy}{\vec{B} \bullet \nabla y}$$

- Naturally contravariant representation of $\delta B_\perp$

$$\delta \vec{B}_\perp = \nabla \times \delta A_\| \hat{b} \approx \nabla \delta A_\| \times \hat{b}$$

- Field-lines trajectories:

$$\frac{\partial x}{\partial s} = \frac{1}{B} \frac{\partial \delta A_\|}{\partial y} \, , \quad \frac{\partial y}{\partial s} = -\frac{1}{B} \frac{\partial \delta A_\|}{\partial x}$$



Analysis by F. Merz using data from GENE

$\langle \delta A_{\parallel} \rangle_{\theta}$ generates the one-turn field-line mapping

- Integrate field-line trajectory along ***B*** for one poloidal cycle

$$x_n(\pi) = x_n(-\pi) + \int_{-\pi}^{\pi} \frac{ds}{B} \frac{\partial \delta A_{\parallel}}{\partial y}$$

$$y_n(\pi) = y_n(-\pi) - \int_{-\pi}^{\pi} \frac{ds}{B} \frac{\partial \delta A_{\parallel}}{\partial x}$$

- Flux-tube periodicity:

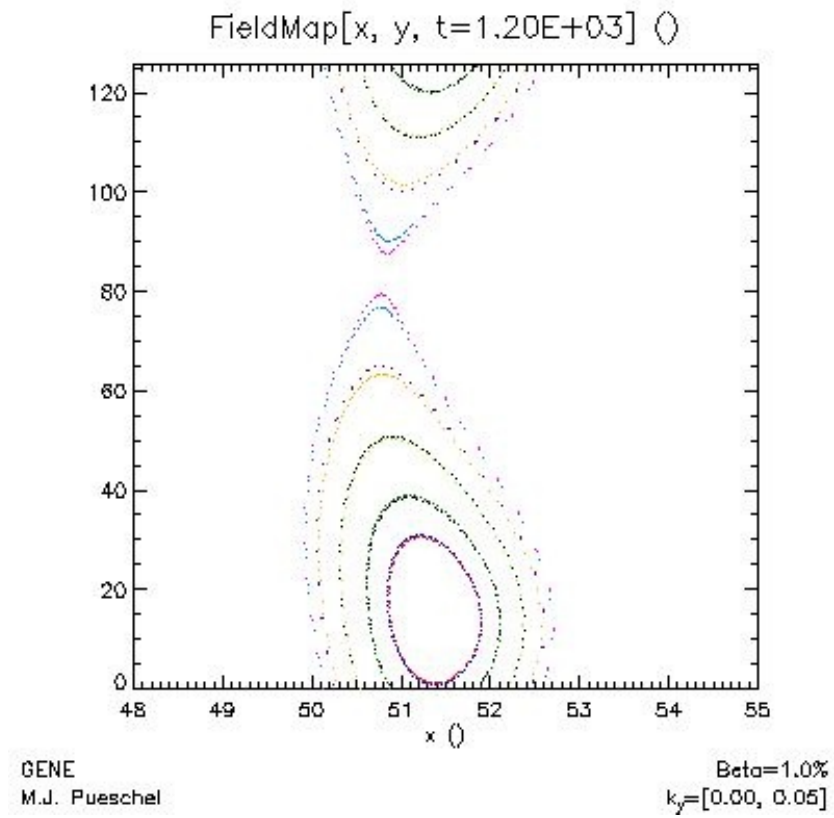
$$x_{n+1}(-\pi) = x_n(\pi)$$

$$y_{n+1}(-\pi) = y_n(\pi) + 2\pi\hat{s}x_{n+1}$$

- A one-turn map:

$$x_{n+1} = x_n + \frac{\partial}{\partial y} \langle \delta A_{\parallel} \rangle_{\theta}$$

$$y_{n+1} = y_n - \frac{\partial}{\partial x} \langle \delta A_{\parallel} \rangle_{\theta} + 2\pi\hat{s}x_{n+1}$$



Same data, Nevins iterating 1-turn map

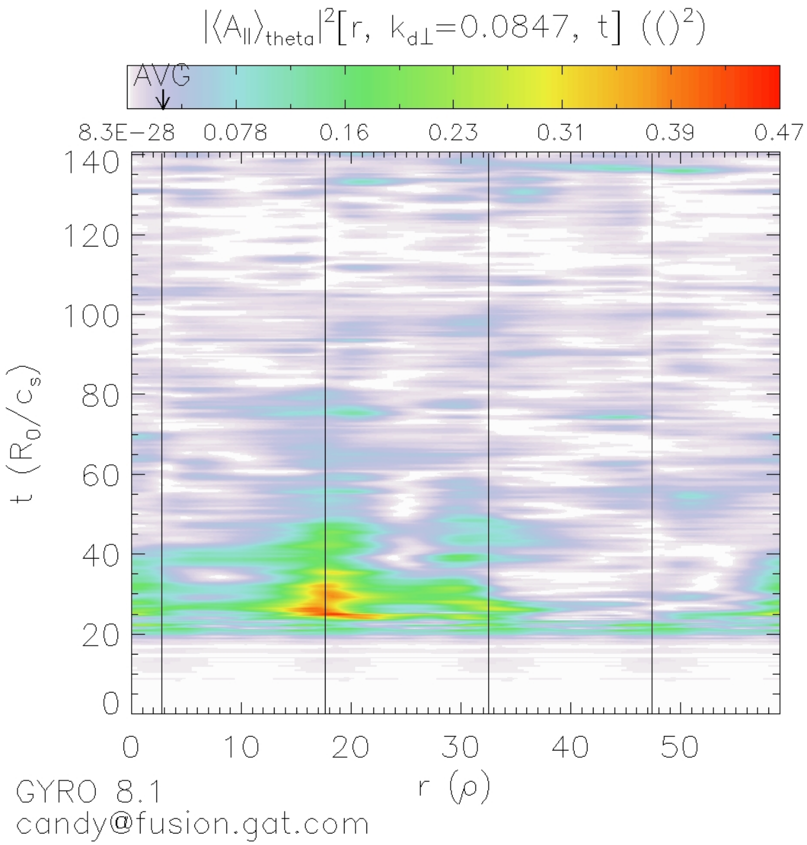
ITG turbulence drives magnetic reconnection

- GYRO uses flux coordinates:
  - (r,ζ) are field line labels
  - θ (poloidal angle) measures position along ***B***
- Project out resonant $\delta A_{||}$ component by taking 1-turn θ-average, $\langle \delta A_{||} \rangle_\theta$
- Magnetic reconnection occurs when resonant intensity,

$$\langle \delta A_{||} \rangle_\theta^2 (k_{\perp 0}, t)$$

is finite at rational surface

$\Rightarrow$ These simulations exhibit turbulence-driven reconnection



Data from GYRO simulation at β_e=0.1%

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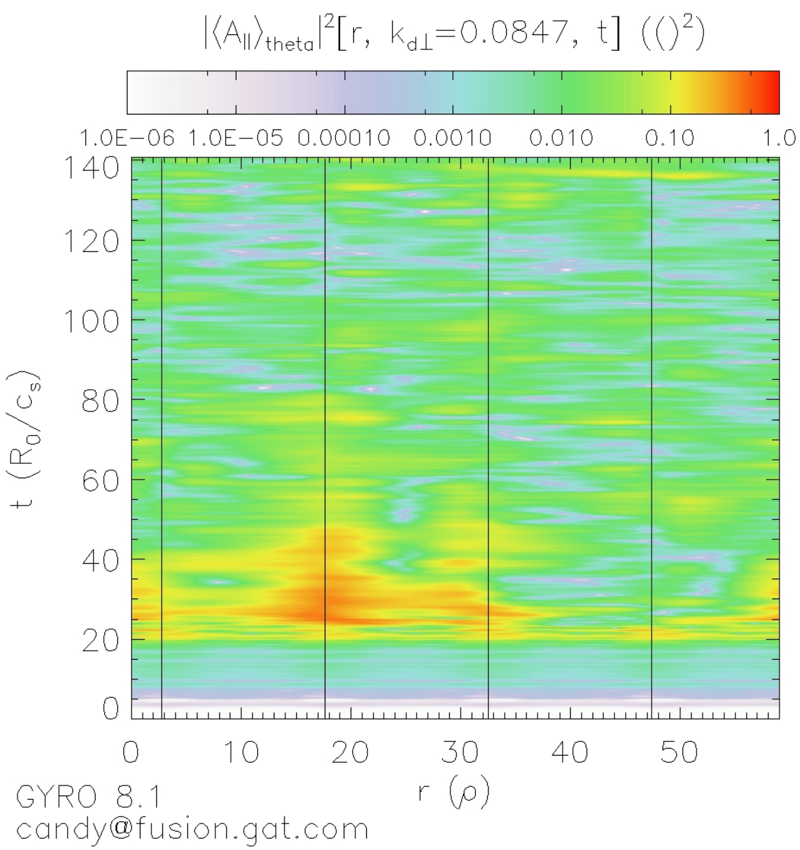
as before, but log-scale
(intensity does not vanish at late times)



Data from GYRO simulation at β_e=0.1%

Mode parity and magnetic surface integrity

- Ballooning modes (like ITG)
  - $\delta\phi$ is even in s
  - $\Rightarrow$ $\delta A_{||}$ is odd in s

$$\Rightarrow \int \frac{ds}{B} \delta A_{||} = 0$$

  - Ballooning modes don't cause magnetic reconnection
- Micro-tearing modes
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  - $\delta\phi$ is odd in s
  - Micro-tearing modes do cause magnetic reconnection
  - $\Rightarrow$ No reconnection if micro-tearing modes are stable???

$\Rightarrow$ Even electrostatic modes have implied magnetic parity!

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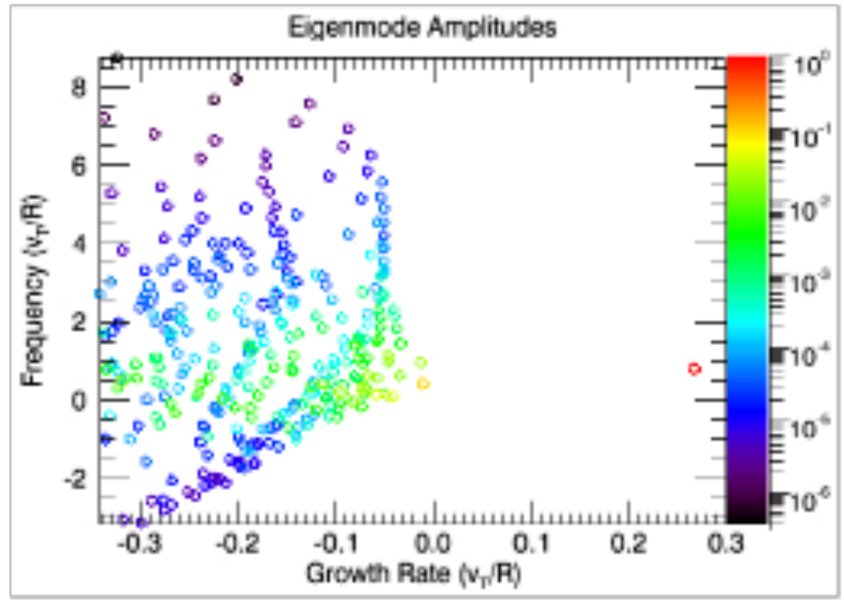

FIG. 4: A plot of the amplitudes, time averaged over the nonlinear state, of the 315 (of 8192) least damped eigenmodes (orthogonalized) of the linear gyrokinetic operator for Fourier mode $k_y = 0.3$, $k_x = 0.0$.

from D. Hatch et a, PRL **106**, 115003 (2011)

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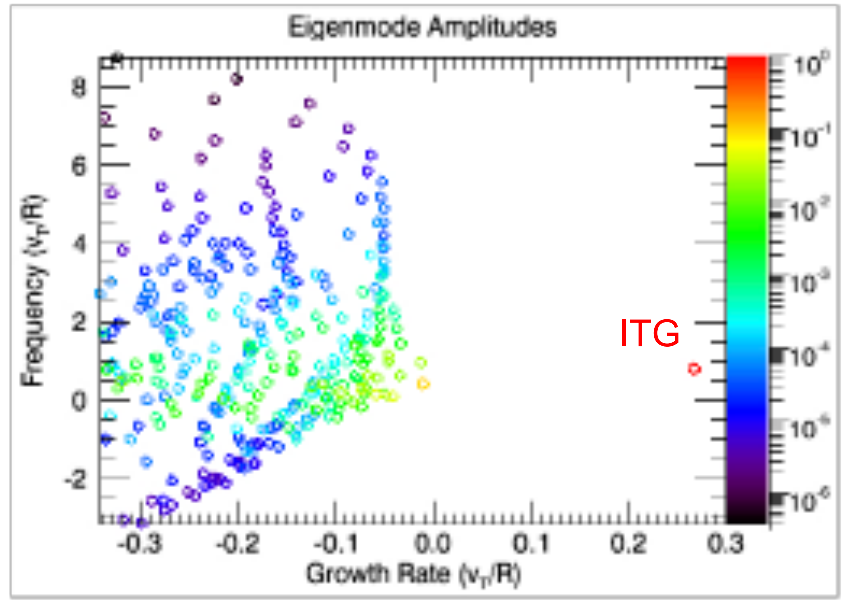

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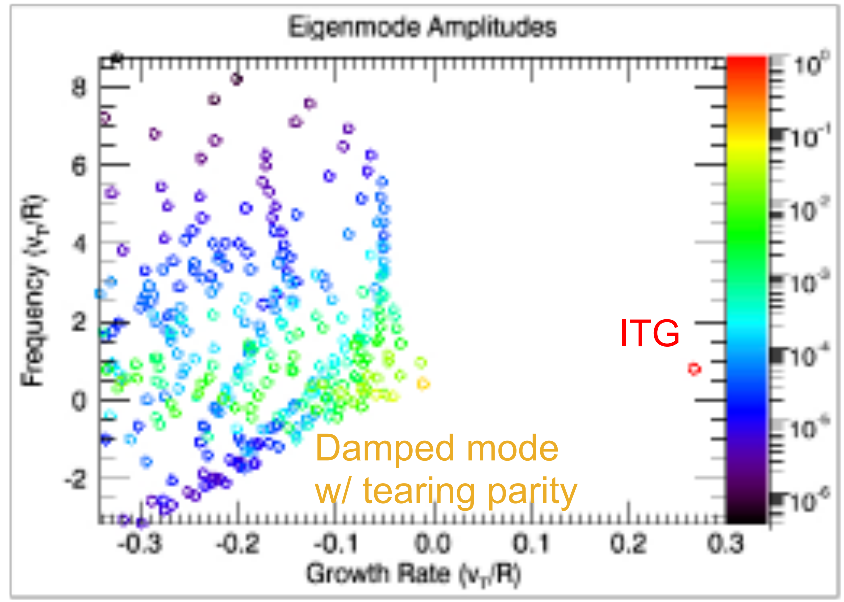

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How odd *φ(s)* generates even *A*$_{||}$*(s)*

- $j_{||}$ from non-adiabatic electron response

$$\delta f_e = \frac{e\varphi}{T_e} f_0 + h_e$$

$$\left(\frac{\partial}{\partial t} + v_{||}\frac{\partial}{\partial s}\right) h_e = -\frac{e}{T_e} f_0 \frac{\partial \varphi}{\partial t} + \frac{\nabla\varphi \times B}{B^2} \bullet \nabla f_0$$

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- And $A_{||}$ is even … but also small for $\beta_e << 1$

Poincaré surface-of-section and microstructure of *B*-field

- Integrate magnetic field-line trajectories
  - Many poloidal cycles (3000)
  - Many initial positions (100)
- Poincare surface-of-section formed by putting one • in (x,y)-plane each time a field line crosses the outboard mid-plane (3000 crossing/field line, so •'s merge into lines …).
- Possible outcomes:

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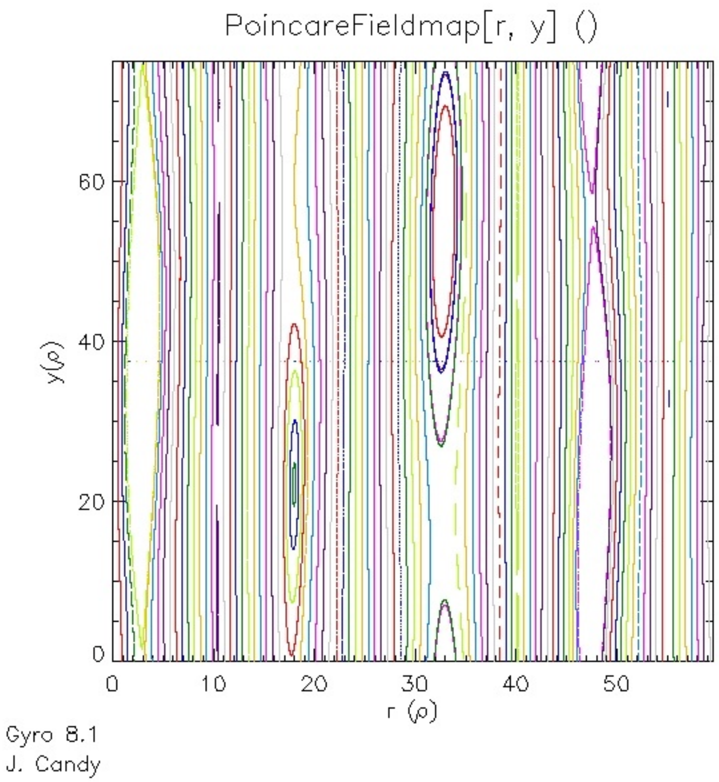
Surface-of-section plot showing regular ***B***-field



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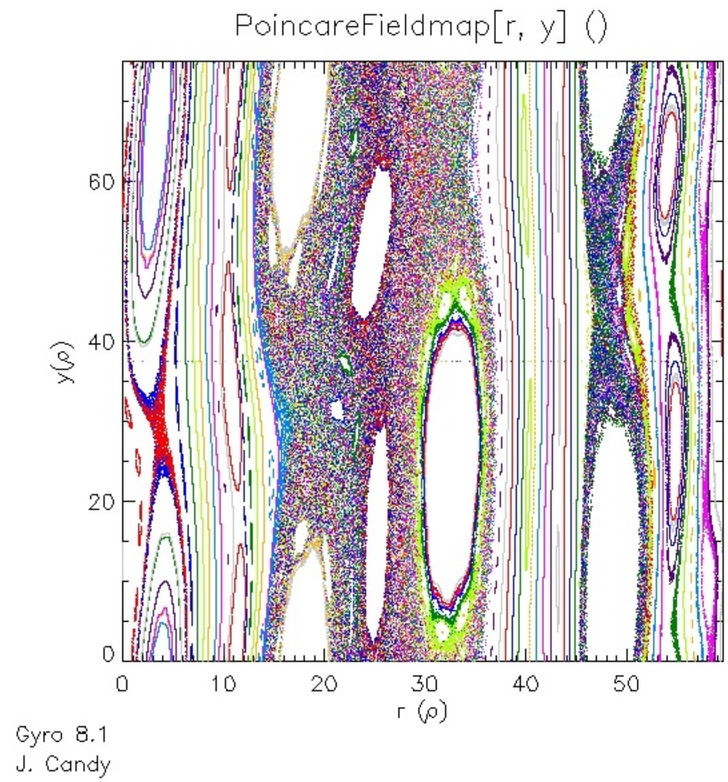
Surface-of-section plot

with isolating surfaces



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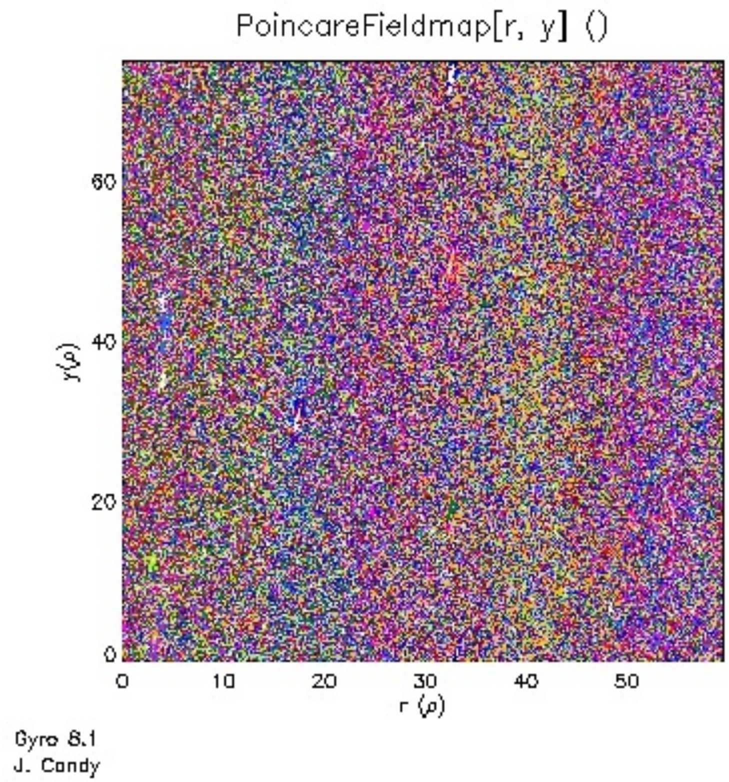
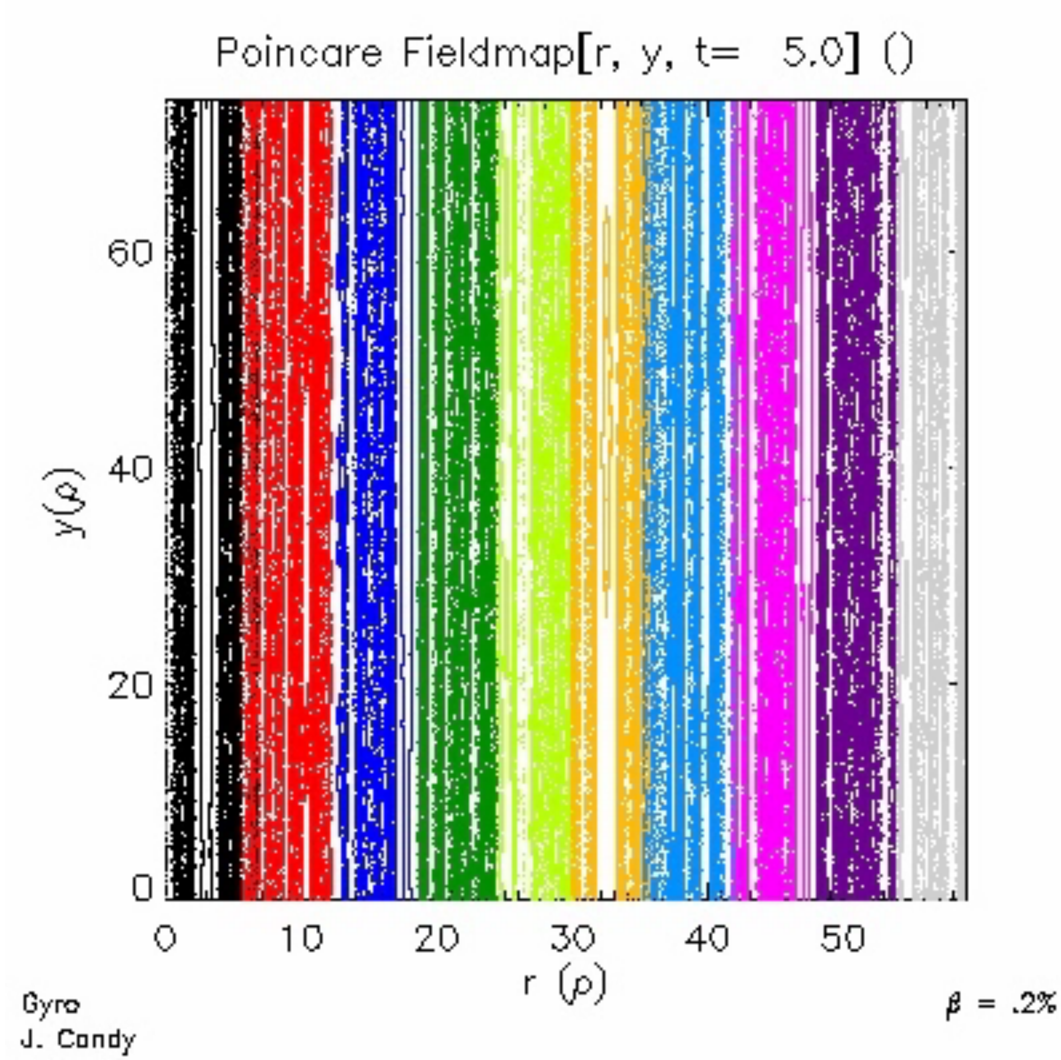


Cyclone base case, Beta=0.2%



Why is micro-field stochastic? Do islands of like order overlap?

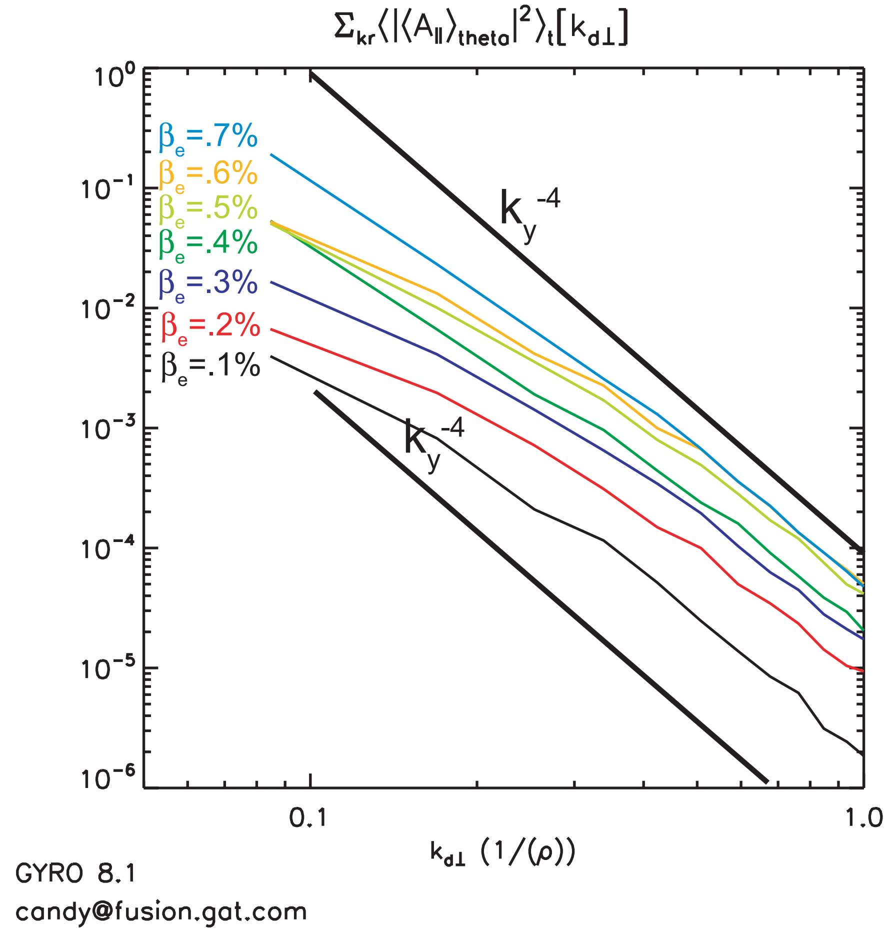
- Separation between rational surfaces of given order (k_y):

$$\Delta x(k_y) = \frac{1}{k_y \hat{s}}$$

- Island width is given by:

$$w(k_y) = 4\sqrt{\frac{qR}{\hat{s}B_T}}\left|\left\langle \delta A_{\|}(k_y)\right\rangle_\theta\right|^{1/2}$$

- High-order islands overlap if $S_{\delta A}(k_y) \approx |\delta A_{||}|^2$ falls off slower than k_y^{-4}
- $S_{\delta A}\left(k_y\right) \sim k_y^{-4}$ for all GYRO simulations in this β_e-scan



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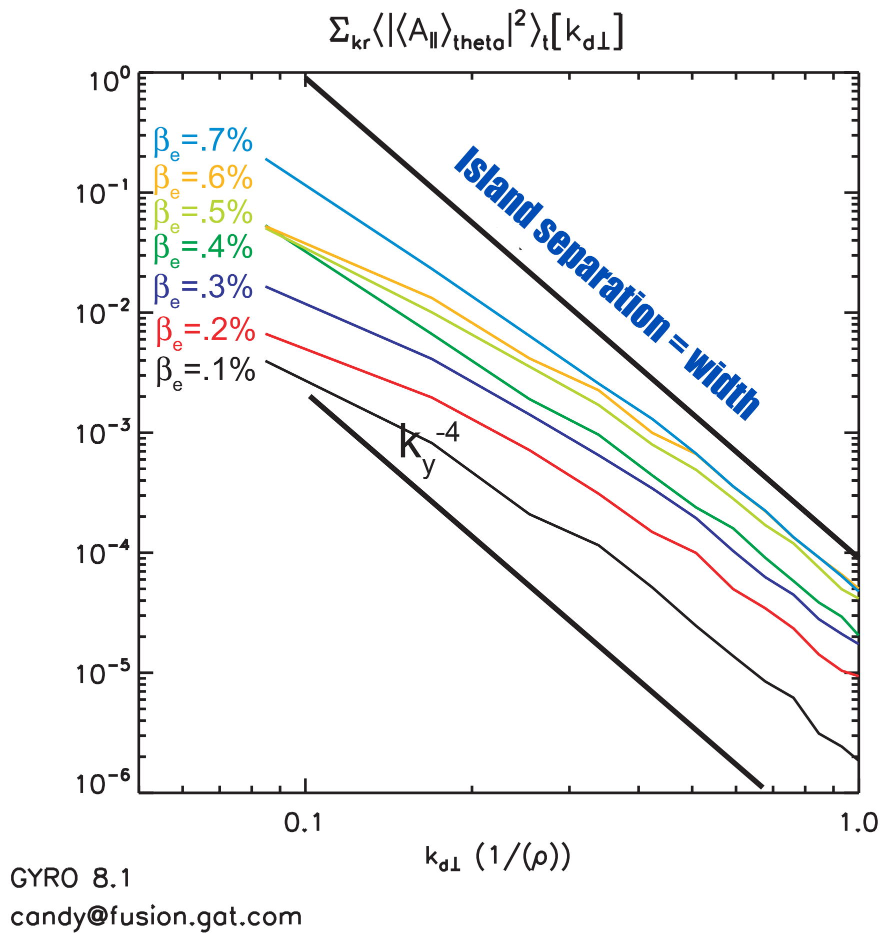


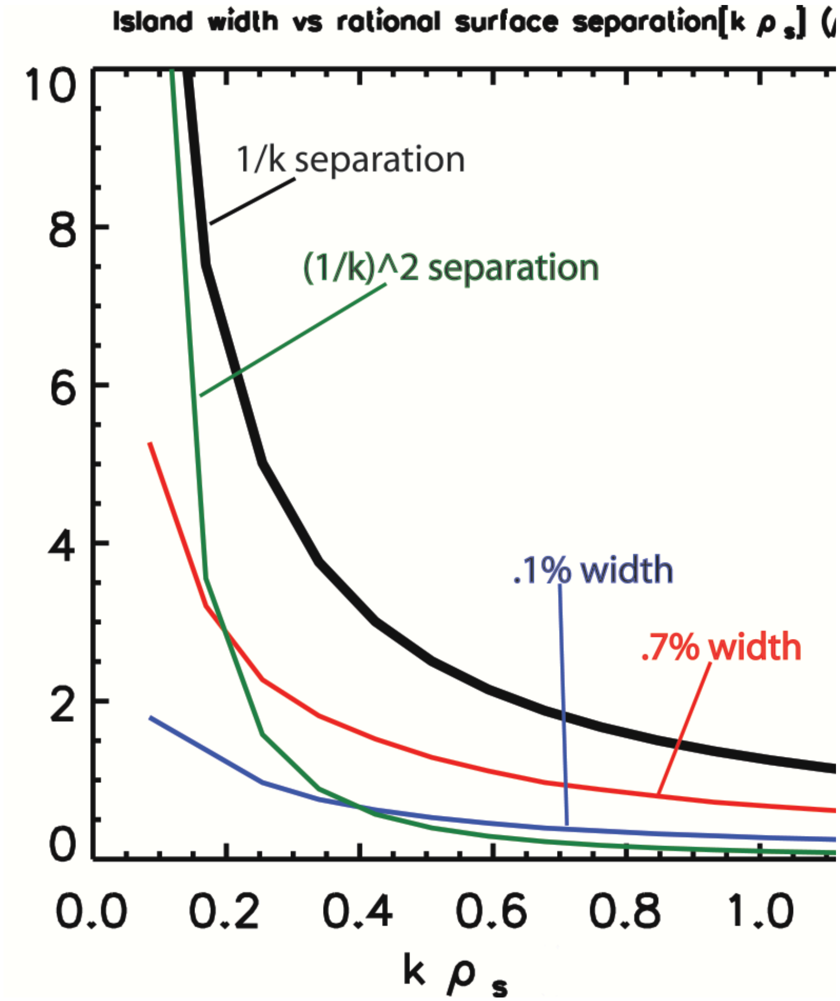
What about islands of neighboring orders?

- Minimum rational surface separation for neighboring k_y

$$\Delta x \approx \frac{\Delta k_y}{k_y^2 \hat{s}}$$

$\Rightarrow$As k_y increases, Island separation falls off faster than width

$\Rightarrow$Islands will always overlap at high k_y

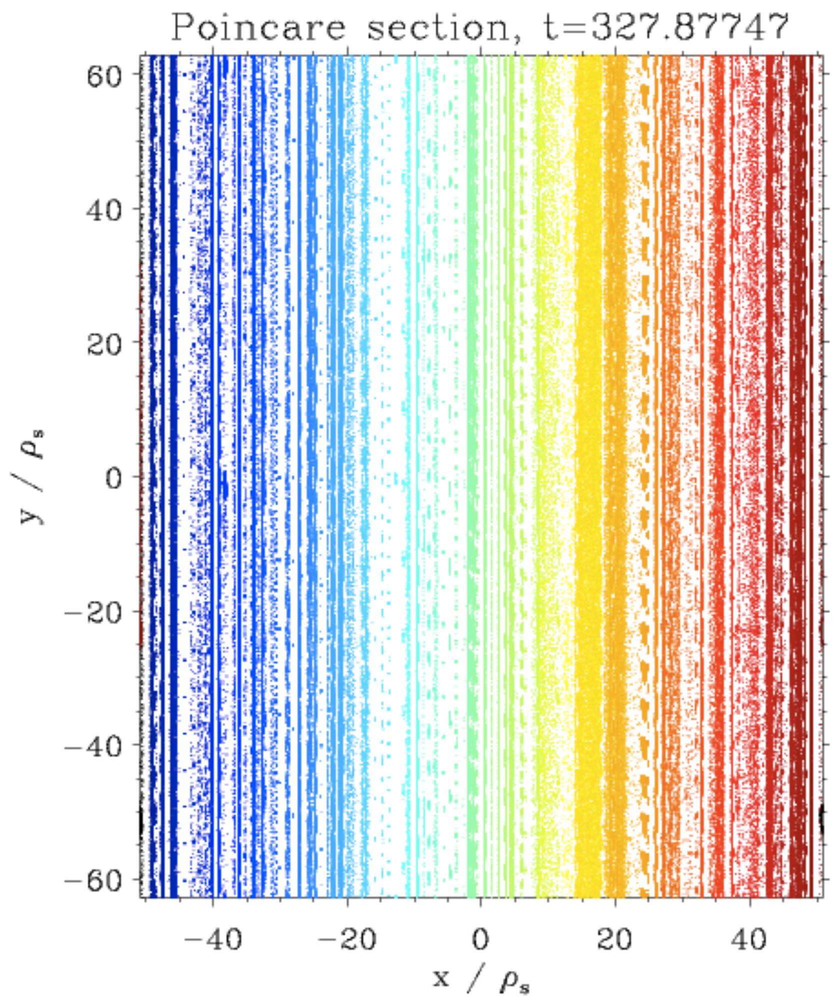


Can overlap of neighboring k_y island produce global magnetic stochasticity?

- A single pair of neighboring k_y-modes produces bands of stochasticity separated by isolating surfaces
- Many k_y-pairs can occur, producing stochastic bands at different radii
- Generally, these bands do not overlap (so stochastic almost everywhere)
- Except near low order surfaces

⇒ WE expect general stochasticity with islands of stability near O-points of low-order islands

GENE data retaining 2 highest *k_y-modes*
(with thanks to M.J. Püschel)



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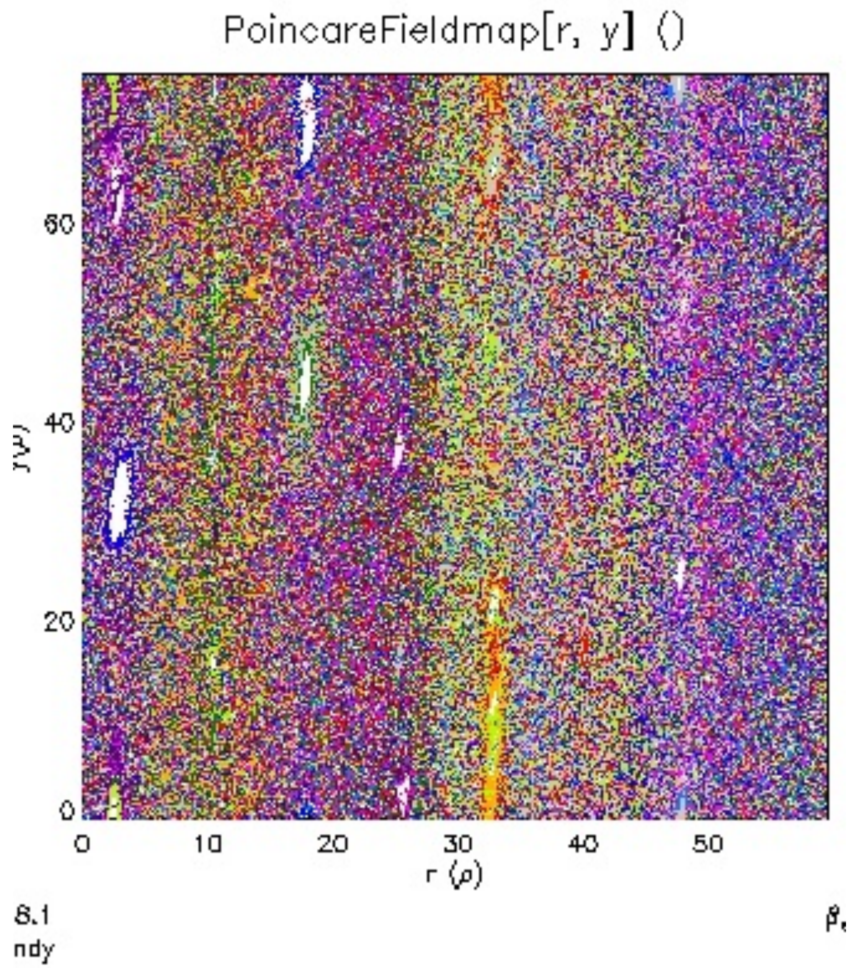
which is just what we see!



Is micro-scale magnetic stochasticity ubiquitous?

CON:

- Data shown is from CYCLONE base case
  - Heat transport larger than typical exp't by 50x

  $\Rightarrow |\varphi^2|, |A_{||}^2|$ also about 50x too large (!)

PRO:

- $\langle A_{||}^2(k_y)\rangle \sim k_y^{-4}$ is a consequence of non-linear physics, so this spectral index is (probably) a general result
- The fall-off probably extends out to $k_y\rho_e \sim 1$ (a factor of ~ 100 in k_y at $\beta_e = 10^{-3}$, or 10^8 in $\langle A_{||}^2\rangle(k_y)$)

$\Rightarrow$ More than compensates for 50x. With enough resolution, you will always be stochastic on micro-scale!