Inter-Species Energy Transfer and Turbulent Heating in Drift Wave Turbulence

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Outline

- *We reconsider the classic problems of calculating " net turbulent heating" and the inter-species transfer of energy in drift wave turbulence*

- Motivation: Transfer vs Transport → " Roles " in energy budget
  - Consider

  Net volumetric heating → Does turbulence heat a given volume of plasma?
  Physics of Electron → ion collisonless energy transfer channels

- Calculate and Estimate Energy Transfer Channels

  Electron cooling : quasilinear
  Ion heating : quasilinear, nonlinear, Ion Pol & Dia → Zonal flow

- Implication for ITER

  Turbulent vs collisional transfer
  Turbulent transport vs Turbulent transfer

- Results and Discussion

Motivation

- Transfer vs Transport

$$n\underbrace{\frac{\partial T_\alpha}{\partial t}+\nabla\cdot Q_\alpha}_{\text{Transport}}=\underbrace{\left\langle \tilde{E}\cdot\tilde{J}_\alpha\right\rangle}_{\text{collsionless transfer}}\underbrace{\mp n\nu\frac{m_e}{m_i}(T_e-T_i)}_{\text{Collisional transfer}}+\ldots$$ heat balance; $\alpha=e,i$

$\rightarrow$ Q heat flux, energy loss by turbulent transport

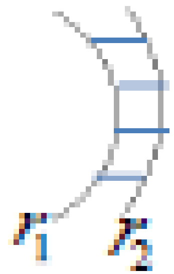
$\rightarrow\left\langle \tilde{E}\cdot\tilde{J}\right\rangle$ electron-ion collisionless energy transfer

$\rightarrow$ ITER: low collisionality, electron heated plasma

- Issues with $\left\langle \tilde{E}\cdot\tilde{J}\right\rangle=\sum_{\alpha=e,i}\left\langle \tilde{E}\cdot\tilde{J}_\alpha\right\rangle$
  - Is the net heating zero?(Manheimer 77)

- Periodical boundary condition ,no boundary term exist

$$\left\langle \tilde{E}\cdot\tilde{J}\right\rangle=0$$



But $\int dr\left\langle \tilde{E}\cdot\tilde{J}\right\rangle=\underbrace{-\tilde{\varphi}\tilde{J}_r\Big]_{r_1}^{r_2}}_{\text{Surface term survives!}}+\cancel{\int dr(\nabla\cdot\tilde{J}\tilde{\varphi})}\neq 0$ → Boundary effects in a finite annular region

Surface term survives! → Net heating

- *Another perspective*: Poynting theorm

$$\frac{\partial W}{\partial t}+\vec{\nabla}\cdot\vec{S}+\left\langle \vec{E}\cdot\vec{J}\right\rangle = 0$$

$W \equiv$ Wave energy density $S \equiv$ wave energy density flux

- At steady state

$$\int_{r_1}^{r_2} dr\left\langle \tilde{E}\cdot\tilde{J}\right\rangle = -S_r\Big]_{r_1}^{r_2}$$

$$S_r = V_{gr,r}\varepsilon_\omega = -2\frac{\rho_s^2 k_r k_\theta \varepsilon_\omega V_* \varepsilon_\omega}{\left(1+k_\perp^2\rho_s^2\right)^2}$$

➡ wave Energy flux differential ➡ *net heating*

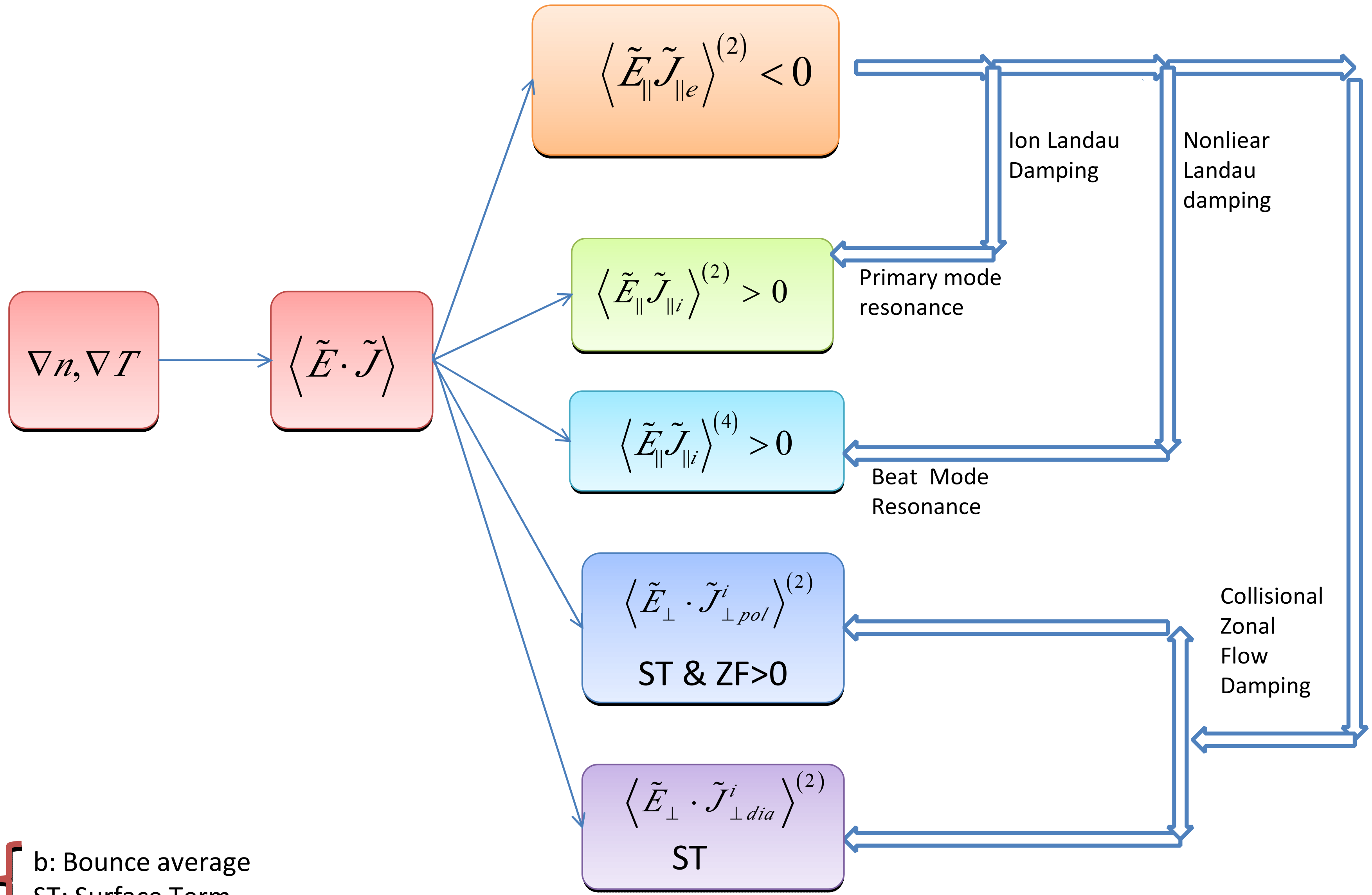
$$\left\langle \tilde{V}_r\tilde{V}_\theta\right\rangle = \sum -k_r k_\theta \left|\tilde{\varphi}\right|^2$$

★ *We need reconsider both the turbulent heating and energy transfer chanels in an annular region!*

- Collisionless, inter-species energy transfer
  - Where does the net energy transfer go?
  - How is energy transferred form electrons to ions (**turbulent transfer channels**)?
  - How reconcile with saturation mechanisms?
  - Role of ZF in heating?

  - **ZF is important to saturation, so must enter energy transfer as well!?**
    - **Zonal flow frictional damping is another energy trasfer channel**
    - **Nonlinear damping (considered in future) is another possibility**

Turbulent Energy flow Channels



b: Bounce average
ST: Surface Term

- Necessary Correspondence: Nonlinear Saturation and Energy Transfer
  - Nonlinear saturation in turbulent state implies energy transfer from source $(\nabla T_e, \nabla n)$ to sink
  - Schematically, saturation implies some balance condition must be satisfied

i.e. $$0 = \gamma = \gamma_{\substack{Linear \\ electron}} + \gamma_{\substack{Linear \\ ion}} + \gamma_{\substack{Zonal \\ Flow}} + \gamma_{NLLD} + \ldots$$

>0 <0 <0 <0

  - Channels for electron → ion energy transfer **must** be consistent with saturation balance

In particular:

- If zonal flows control saturation, they **must** contribute to energy transfer
- As zonal flows are nonlinearly generated (Reynolds stress), we should consider other nonlinear heating channels, as well,for completeness

Quasilinear Turbulent Heating in Drift Wave

- Calculate $\langle \tilde{E}_{\|} \tilde{J}_{\|e} \rangle^{(2)}$ in quasilinear theory
  - DKE for electron
  - Take non-adiabatic electron distribution function

$$\tilde{g}_k = \frac{(\omega_* - \omega)}{\omega - k_z v_z} \frac{e\tilde{\varphi}_k}{T_e} \langle f_e \rangle, \quad \omega_{*e} = \frac{k_y \rho_s c_s}{L_n}, \quad \langle f_e \rangle \text{ is Maxwellian}$$

- $$\langle \tilde{E}_{\|} \tilde{J}_{\|e} \rangle^{(2)} = e \int dv v_z \tilde{E}_z \tilde{g}_k = \sum_k \pi n T_e \left| \frac{e\tilde{\varphi}_k}{T_e} \right|^2 \frac{\omega}{|k_z| V_{the}} (\omega - \omega_{*e}) \langle f_e \rangle_{\frac{\omega}{k_z}}$$

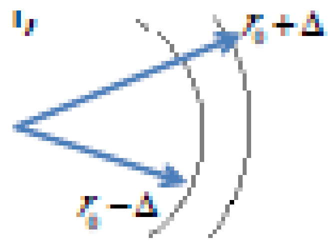
  - $\omega = \frac{\omega_{*e}}{1 + k_\perp^2 \rho_s^2}$, $\langle \tilde{E}_{\|} \tilde{J}_{\|e} \rangle^{(2)} < 0$ **the electrons cool via inverse electron Landau damping**

- Similarly, calculate $\langle \tilde{E}_{\|} \tilde{J}_{\|i} \rangle^{(2)}$ for ion
  - $$\langle \tilde{E}_{\|} \tilde{J}_{\|i} \rangle^{(2)} = \sum_k \pi n T_i \left| \frac{e\tilde{\varphi}_k}{T_i} \right|^2 \frac{\omega}{|k_z| V_{the}} \left(\omega + \frac{T_i}{T_e} \omega_{*e} \right) \langle f_i \rangle_{\frac{\omega}{k_z}}$$
  - $\langle \tilde{E}_{\|} \tilde{J}_{\|i} \rangle^{(2)} > 0$, **the ions gain energy via ion Landau damping**

Perpendicular Current Induced Turbulent Heating

- The turbulent heating induced by ion polarization current
  - $\left\langle \tilde{E}_\perp \cdot \tilde{J}^{pol}_{\perp i} \right\rangle = -\left\langle \vec{\nabla}_\perp \cdot \left(\tilde{\varphi} \tilde{J}^{pol}_{\perp i} \right) \right\rangle + \left\langle \tilde{\varphi} \vec{\nabla}_\perp \cdot \tilde{J}^{pol}_{\perp i} \right\rangle$
  - Defining a annular region

$$\langle \ldots \rangle = \int_0^{2\pi R} dz \int_0^{2\pi} r d\theta \int_{r_0-\Delta}^{r_0+\Delta} (\ldots) dr$$



- Net turbulent heating

$$\left\langle \tilde{E}_\perp \cdot \tilde{J}^{pol}_{\perp i} \right\rangle = n m_i A \left(\underbrace{\langle V_\theta \rangle \left\langle \tilde{V}_r \tilde{V}_\theta \right\rangle \Big]_{r-\Delta}^{r+\Delta}}_{\text{Surface term}} - \underbrace{\int_{r-\Delta}^{r+\Delta} dr \langle V_\theta \rangle \frac{\partial}{\partial r} \left\langle \tilde{V}_r \tilde{V}_\theta \right\rangle}_{} \right)$$

- Reynolds work on mean flow in annular
- Directly linked to zonal flow drive

- At steady state

$$\left\langle \tilde{E}_\perp \cdot \tilde{J}^{pol}_{\perp i} \right\rangle = \int_{r-\Delta}^{r+\Delta} dr \upsilon_{col} \langle V_\theta \rangle^2 > 0,$$

$\Rightarrow$ Zonal flow frictional damping is the fate of net electron-ion energy trasfer

- Diamagnetic current induced turbulent heating

$$\left\langle \tilde{E}_\perp \cdot \tilde{J}^{Dia}_{\perp i} \right\rangle = -nc \tilde{\varphi} \frac{B \times \nabla \tilde{p}}{B^2} \Big]_{r-\Delta}^{r+\Delta}$$

$\Rightarrow$ Heat flux differential $\Rightarrow$ Zonal flow

Nonlinear Turbulent Heating

- ZF coupling to $o\left(\left|\frac{e\tilde{\varphi}}{T}\right|^4\right)$, so need calculate parallel heating to $o\left(\left|\frac{e\tilde{\varphi}}{T}\right|^4\right)$
- Nonlinear turbulent heating $\longrightarrow$ perturbation theory (Dupree 68)

$$\left\langle \tilde{E}\cdot\tilde{J}\right\rangle^{(4)} = -\int dV m V_\parallel D_4 \frac{\partial}{\partial V_\parallel}\langle f\rangle$$

D_4 fourth order diffusion coefficient

- The nonlinear turbulent heating for ions

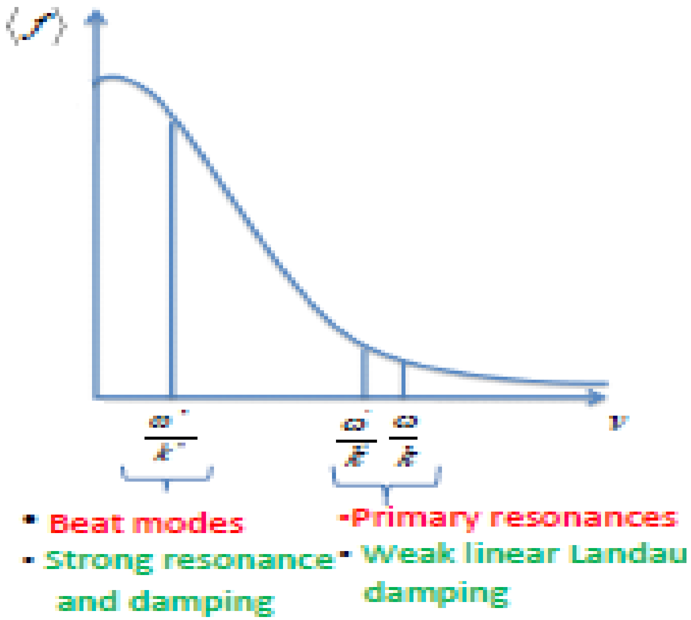
$$\left\langle \tilde{E}_\parallel\cdot\tilde{J}_{\parallel i}\right\rangle^{(4)} = \sum_{k,k'}\pi n T_i\left(\frac{e\tilde{\varphi}_k}{T_i}\right)^2\left(\frac{e\tilde{\varphi}_{k'}}{T_i}\right)^2\frac{k_z^{2}k_z'^{2}V_{thi}^3}{k_z''^{2}}\frac{\omega''^2}{\left|k_z''\right|}\left(\frac{k-k'}{(kv-\omega)(k'v-\omega')}\right)^2\langle f_i\rangle]]_{v=\frac{\omega''}{k''}} > 0$$

- The beat mode resonance

$$\omega'' = \omega\pm\omega',\ k'' = k\pm k'$$

- Nonlinear beat Landau resonance is a strong nonlinear effect !

$$\Rightarrow \left\langle \tilde{E}_\parallel\cdot\tilde{J}_{\parallel i}\right\rangle^{(4)} / \left\langle \tilde{E}_\parallel\cdot\tilde{J}_{\parallel i}\right\rangle^{(2)} \sim \left|\frac{e\tilde{\varphi}}{T}\right|^2 \exp\left(\frac{\omega^2}{k_\parallel^2 V_{thi}^2}\right)$$



Overview of Results

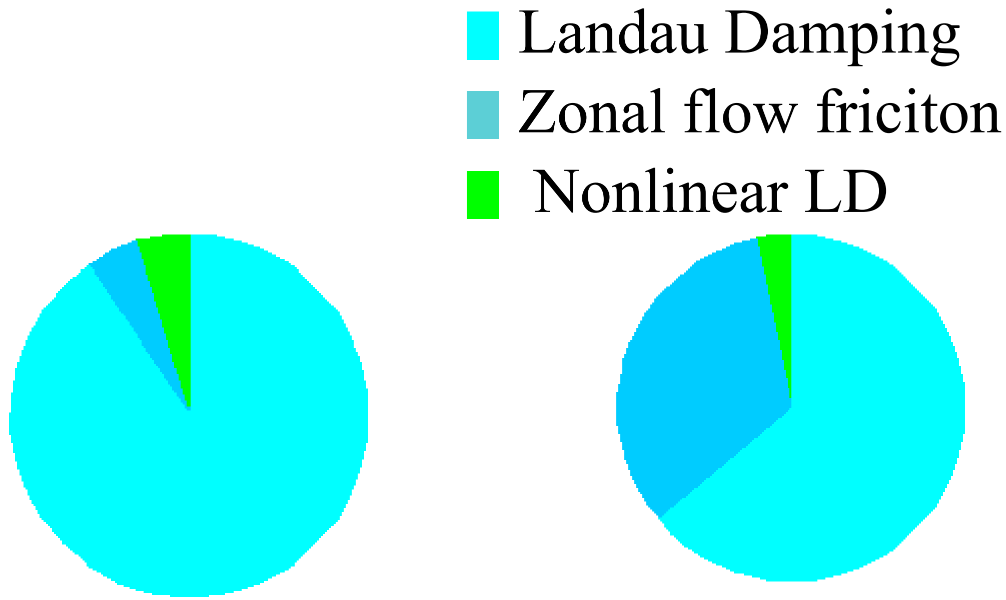
★ Estimation of the turbulent heating

Turbulent heating	analytical	Mixing length approximation for fluctuation levels $\frac{e\varphi}{T_e} \sim \rho_*$
$\left\lvert \left\langle \tilde{E} \cdot \tilde{J} \right\rangle_e^{(2)} \right\rvert$	$\left\lvert \frac{e\tilde{\varphi}}{T_e} \right\rvert^2 \frac{(\omega - \omega_{*e})\omega}{\lvert k_z \rvert V_{the}}$	$\rho_*^2 \frac{Rq\omega_*^2}{V_{the}} F_1(k_\perp \rho_s)$
$\left\langle \tilde{E} \cdot \tilde{J} \right\rangle_i^{(2)}$	$\left\lvert \frac{e\tilde{\varphi}}{T_i} \right\rvert^2 \frac{(\omega - \omega_{*i})\omega}{\lvert k_z \rvert V_{thi}} \exp - \left(\frac{\omega}{k_z V_{thi}} \right)^2$	$\rho_*^2 \frac{Rq\omega_*^2}{V_{thi}} F_2(k_\perp \rho_s)$
$\left\langle \tilde{E} \cdot \tilde{J} \right\rangle_i^{(4)}$	$\left\lvert \frac{e\tilde{\varphi}_k}{T_i} \right\rvert^2 \left\lvert \frac{e\tilde{\varphi}_{k'}}{T_i} \right\rvert^2 \frac{\omega''^2 k_z'^2 k_z^2 V_{thi}^3}{(k_z v - \omega)^2 (k_z' v - \omega')^2 \lvert k_z \rvert}$	$\rho_*^2 \rho_*'^2 \frac{V_{thi}^3}{L_n C_s^2} F_3(k_\perp \rho_s)$
$\left\langle \tilde{E} \cdot \tilde{J} \right\rangle_{pol}^{(2)}$	$m_i \nu_{col} \left\langle V_\theta \right\rangle^2$	$\rho_*^2 \nu_* \varepsilon^{3/2} m_i C_s^2 \frac{V_{thi}}{Rq}$

Basic comparision of channels

ITER Parameters R=6.2m, a=2m,q=2	
$Ratio = \frac{\langle \tilde{E} \cdot \tilde{J}_i \rangle}{\langle \tilde{E} \cdot \tilde{J}_e \rangle}$	Short wavelength
	$k_\perp \rho_s \sim 1$
$\langle \tilde{E} \cdot \tilde{J} \rangle_i^{(2)}$	1.56×10^{-2}
$\langle \tilde{E} \cdot \tilde{J} \rangle_{pol}^{(2)}$	$0.8 \nu_*$
$\langle \tilde{E} \cdot \tilde{J} \rangle_i^{(4)}$	$0.08 \times 10^4 \rho_*^2$

★ *Ratios of energy dissiaption channels at differernt collisionality*



$\nu_* = 10^{-3}$ $\rho_* = 10^{-3}$ $\nu_* = 10^{-1}$ $\rho_* = 10^{-3}$

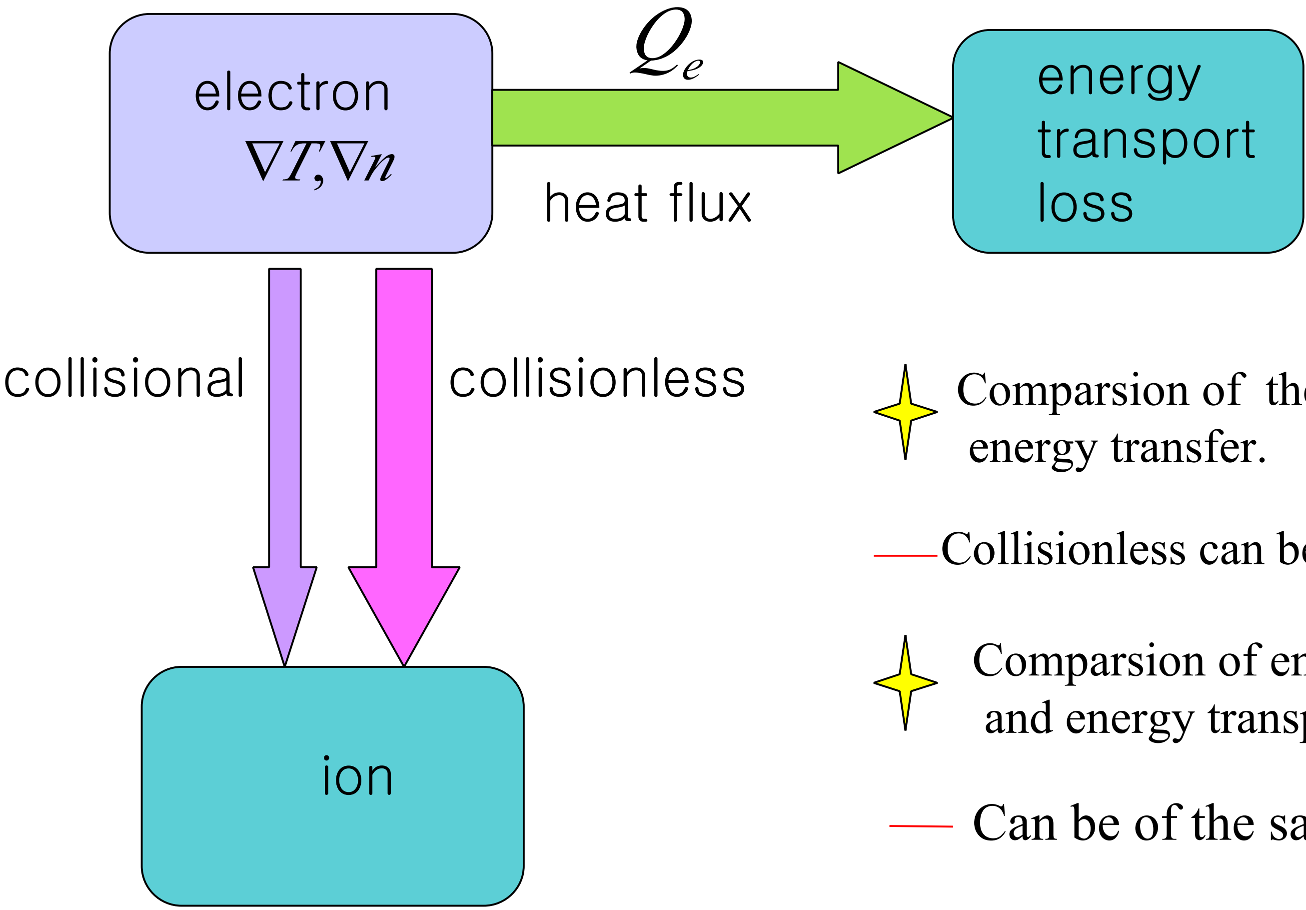
★ Zonal flow frictional damping can be a significant dissipation channel

★ "Collisionless drift wave" $\omega \gg \nu_* > 0$

Implication ➡ Bottom Line

- Electon turbulent energy transport

$$\frac{\partial T_e}{\partial t} + \nabla \cdot Q_e = \left\langle \tilde{E} \cdot \tilde{J}_e \right\rangle - n\nu \frac{m_e}{m_i}\left(T_e - T_i\right) \Rightarrow \text{Electron heat balance}$$



✦ Comparsion of the collisonal and collisionless energy transfer.

— Collisionless can be dominant at low collisionaltiy

✦ Comparsion of energy transfer in collisonless and energy transport by heat flux Q

— Can be of the same order

Collisionality

- Collisonality ν_* in ITER
  - dimensionless

$$\nu_* = \frac{\varepsilon^{-3/2} R q \nu_e}{V_{the}} \Rightarrow \nu_* \sim 10^{-3}$$

- Collisionality at crossover of collisional and collisionless coupling
  - Energy transfer in collision : $Q_i \approx \dfrac{n m_e \nu_e}{m_i} \tilde{T}_e$
  - Quasilinear trapped electron cooling in CTEM

$$\left\langle \bar{\tilde{E}} \cdot \bar{\tilde{J}}_e \right\rangle_b^{(2)} \simeq 4\pi^{1/2} \varepsilon^{1/2} n T_e \left(\frac{R}{2a} \right)^{3/2} \rho_*^2 \left(\omega - \omega_{*n} \right) \left\langle f_e \right\rangle \rrbracket_{E = \frac{\omega}{k_\theta} \frac{R T_e}{\rho_s C_s}}$$

  - **At crossover** : $Q_i \approx \left\langle \bar{\tilde{E}} \cdot \bar{\tilde{J}} \right\rangle_b^{(2)} \Rightarrow \nu_* \sim 10^{-2}$

★ The collisonless turbulent energy transfer then dominates inter-species coupling process

Transfer vs Transport

- The transfer and transport energy loss in CTEM
  - Compare the volume integral of the electron cooling to the surface integrated of the electron heat flux

$$A\tilde{Q}_e \rrbracket_{boundary} = \int d^3r \left\langle \tilde{E}\cdot\tilde{J} \right\rangle$$

  - The heat flux for electrons : $\tilde{Q}_e = \left\langle \tilde{v}_r \tilde{P}_e \right\rangle = -\frac{c}{B}\sum_k k_\theta \,\mathrm{Im}\, \tilde{P}_e^{(1)} \tilde{\varphi}$
  - The pressure fluctuation $\tilde{p}_e^{(1)} = \int d^3v \frac{1}{2} m v^2 \bar{\tilde{g}}_b$

$$\tilde{Q}_e = \sum 4\pi^{\frac{1}{2}} \varepsilon^{\frac{1}{2}} \left(\frac{R}{L_n}\right)^{\frac{5}{2}} \left|\frac{e\bar{\tilde{\varphi}}}{T_e}\right|^2 \frac{V_{the}^2 k_\theta n T_e}{\Omega_e} \left(\omega - \omega_{*n}\right) \left\langle f_e \right\rangle \rrbracket_{E=\frac{\bar{\omega}_{de} R T_e}{\omega_* L_n}}$$

The ratio

$$\frac{\Delta r \left\langle \tilde{E}\cdot\tilde{J} \right\rangle}{\tilde{Q}_e \rrbracket_{boundary}} \approx 2\frac{a}{R} \sim o(1)$$

The rate of energy lost by collisonless energy transfer is comparable as turbulent transport

Result and Discussion

- Net heating
  - Quasilinear turbulent energy transfer in drift wave
    - electron cooling and ion heating
  - Nonlinear ion heating by beat wave resonance
  - Energy flux differential gives rise to the net heaing ⇒ zonal flow
- Energy transfer channels
  - Identify a important energy transfer channels
  - Zonal flow frictional damping can be comparable to LD damping
- For low collisionity ITER plasma, collisionless enenrgy transfer can be a critical element of transport model analysis
  - Collisionless enenrgy transfer has same order as transport