Effects of finite β and plasma current on the reversed shear Alfvén eigenmode (RSAE)

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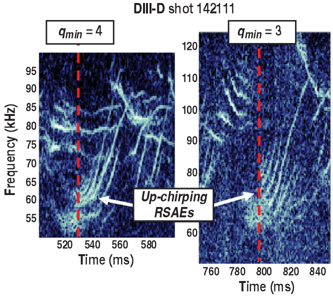
Introduction

- RSAE: shear Alfvén wave in the toroidal geometry, localized near $q_{\min}$, driven unstable by energetic particles (fast ions), frequency up-chirping
- RSAE in local linear ideal MHD limit in simple geometries is quite well-understood [Berk *et al.*, PRL 2001; Breizman *et al.*, PoP 2003, 2005; etc.]
- Global effects, kinetic effects, nonlinear effects, etc. are still worth studying
- RSAE simulations by global gyrokinetic toroidal code (GTC), with focus on finite β and plasma current effects

$$\omega_{\mathrm{RSAE}} \approx \frac{v_A}{R}\left|\frac{m}{q_{\min}} - n\right|$$



Experimental spectrogram showing frequency up-chirping of RSAEs driven by energetic particles [Tobias *et al.*, PRL 2011]

Outline

1. Gyrokinetic simulation model in GTC
2. Simulations in a simple geometry
   - Zero-β limit and benchmark with XHMGC
   - Ion finite-β and FLR effects
   - Equilibrium current effect
3. Simulations of DIII-D discharge #142111 at 750ms

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Gyrokinetic simulation model in GTC

- Thermal ions and fast ions are simulated using gyrokinetic PIC approach:

$$(\partial_t + \dot{\boldsymbol{X}} \cdot \nabla + \dot{v}_\parallel \partial_{v_\parallel})[f_0(\boldsymbol{X}, \mu, v_\parallel) + \delta f(\boldsymbol{X}, \mu, v_\parallel, t)] = 0$$

  [Brizard and Hahm, RMP 2007]

- Electrons are simulated using the electromagnetic fluid-kinetic hybrid model [Lin and Chen, PoP 2001; Holod *et al.*, PoP 2009]. In this work, only the adiabatic fluid part is used and they are described by the fluid equation:

$$\begin{aligned} 0 \quad = \quad & \partial_t \, \delta n_e - \delta \boldsymbol{B} \cdot \nabla \left(\frac{c}{4\pi e B_0} \boldsymbol{b}_0 \cdot \nabla \times \boldsymbol{B}_0 \right) + \boldsymbol{B}_0 \cdot \nabla \left(\frac{n_{0e} \, \delta u_{\parallel e}}{B_0} \right) \\ & + B_0 \boldsymbol{v}_E \cdot \nabla \left(\frac{n_{0e}}{B_0} \right) - n_{0e} (\delta \boldsymbol{v}_{*e} + \boldsymbol{v}_E) \cdot \frac{\nabla B_0}{B_0} \\ & + \frac{c \nabla \times \boldsymbol{B}_0}{B_0^2} \cdot \left[- \frac{\nabla \, \delta P_{\parallel e}}{e} + n_{0e} \nabla \, \delta\phi \right] \end{aligned}$$

- Gyrokinetic Poisson's equation [Lee, JCP 1987] & Ampère's law:

$$\begin{aligned} \frac{Z_f^2 n_f}{T_f}(\delta\phi - \delta\tilde{\phi}_f) + \frac{Z_i^2 n_i}{T_i}(\delta\phi - \delta\tilde{\phi}_i) \quad &= \quad \sum_{\alpha=e,i,f} Z_\alpha \, \delta n_\alpha \\ \frac{c}{4\pi}\{\nabla \times [\nabla \times (\delta A_\parallel \boldsymbol{b}_0)] \cdot \boldsymbol{b}_0\} \boldsymbol{b}_0 \quad &= \quad \sum_{\alpha=e,i,f} \delta \boldsymbol{J}_{\alpha\parallel} \end{aligned}$$

- Blue: finite-β effects. Red: plasma equilibrium current effects.

Reduction to ideal MHD, and RSAE equation

- With appropriate approximations made, the gyrokinetic model reduces to the ideal MHD equation:

$$\frac{\omega(\omega-\omega_{*P})}{v_A^2}\nabla_\perp^2\,\delta\phi - i\boldsymbol{B}_0\cdot\nabla\left\{\frac{\boldsymbol{b}_0\cdot\nabla\times[\nabla\times(k_\parallel\,\delta\phi\boldsymbol{b}_0)]}{B_0}\right\}$$
$$-\frac{i\omega}{c}\delta\boldsymbol{B}_\perp\cdot\nabla\left(\frac{\boldsymbol{b}_0\cdot\nabla\times\boldsymbol{B}_0}{B_0}\right)-i\omega\frac{4\pi}{c}\nabla\cdot\left(\frac{\boldsymbol{b}_0}{B_0}\times\nabla\cdot\delta\mathbb{P}\right)=0$$

- In a tokamak with concentric circular flux surfaces and in single n, m limit, the equation reduces to the RSAE equation:

$$\frac{1}{r}\frac{\mathrm{d}}{\mathrm{d}r}\left(r\Lambda\frac{\mathrm{d}}{\mathrm{d}r}\delta\hat{\phi}\right)-\frac{m^2}{r^2}\Lambda\,\delta\hat{\phi}-\frac{D}{r}\delta\hat{\phi}=0$$

- Λ reflects the Alfvén continuum [Zonca *et al.*, PPCF 1996]:

$$\Lambda=\frac{\omega^2}{v_A^2}-k_\parallel^2-\left(\frac{7}{4}+\frac{T_e}{T_i}\right)\frac{2v_i^2}{v_A^2R_0^2}$$

- D determines whether an eigenmode (RSAE) exists near the $q_{\min}$ continuum extremum [Breizman *et al.*, PoP 2005]:

$$D=k_\parallel\frac{\mathrm{d}k_\parallel}{\mathrm{d}r}+rk_\parallel\frac{\mathrm{d}^2k_\parallel}{\mathrm{d}r^2}-3k_\parallel\frac{\mathrm{d}k_\parallel}{\mathrm{d}r}-rk_\parallel\frac{\mathrm{d}^2k_\parallel}{\mathrm{d}r^2}+D_f+D_p+D_t$$

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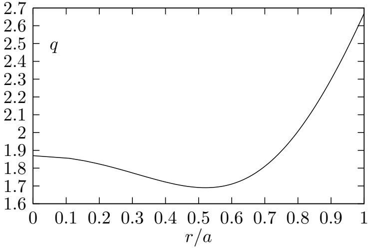
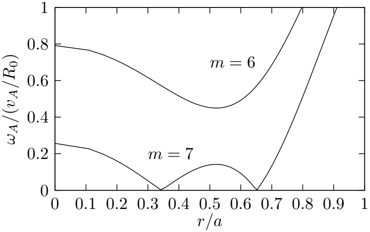
Simulation setup

- GTC: concentric circular flux surfaces
- XHMGC: shifted circular flux surfaces
- Uniform background plasma
- Equilibrium current artificially turned off



q-profile

$q_{\min} = 1.69$

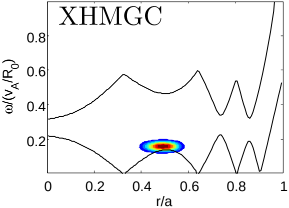


Alfvén continua estimation ($n = 4$)

$$\omega_A \approx \frac{v_A}{R_0}\left|n - \frac{m}{q}\right|$$

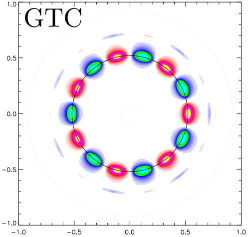
Antenna excitation of $(n, m, l) = (4, 7, 0)$ RSAE

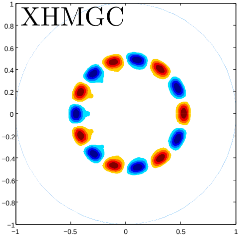
Alfvén continua and power spectrum:



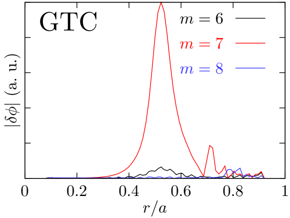
$$\omega_{\rm GTC} = 0.135 v_A/R_0$$
$$\omega_{\rm XHMGC} = 0.160 v_A/R_0$$

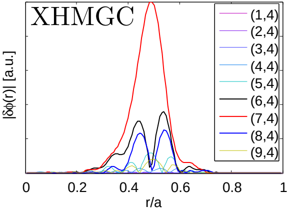
$\delta\phi$ poloidal contour:





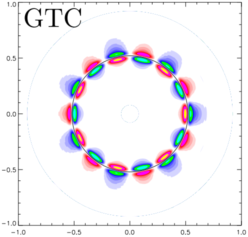
m-decomposed $\delta\phi$:



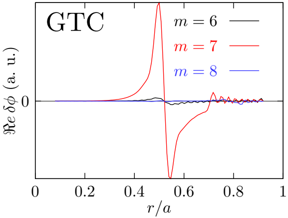


Antenna excitation of $(n, m, l) = (4, 7, 1)$ RSAE

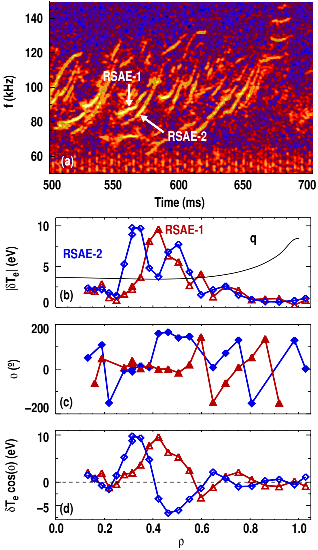
$\delta\phi$ poloidal contour:



m-decomposed $\delta\phi$:



$$\omega_{\mathrm{RSAE-2}} = 0.131 v_A/R_0 \qquad (l = 1)$$
$$\omega_{\mathrm{RSAE-1}} = 0.135 v_A/R_0 \qquad (l = 0)$$



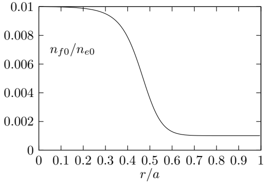
[Van Zeeland, NF 2009]

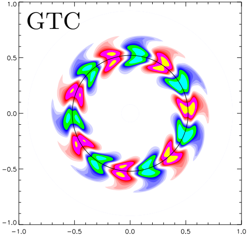
Drift-kinetic fast ion excitation of $n = 4$ RSAE

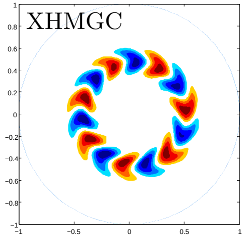
$$R_0/L_{n_{f0}} = 36.6,\ v_f/v_A = 0.3,\ \rho_f/a = 0.03,\ k_\perp \rho_f = 0.4,\ n_{f0}/n_{e0} = 0.01$$

$\delta\phi$ poloidal contour:

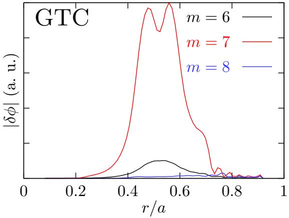
Fast ion density profile:

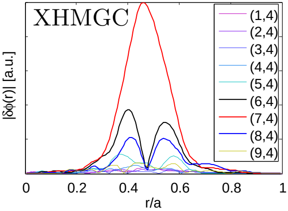






m-decomposed $\delta\phi$:



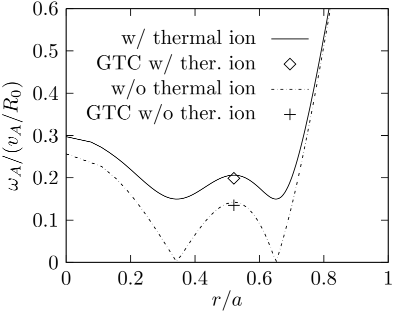


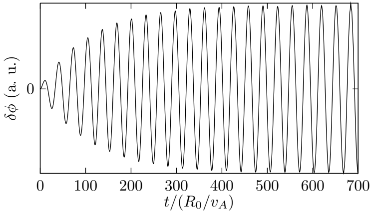
GTC: $(\omega_r, \gamma) = (0.107, 0.0159)\frac{v_A}{R_0}$
XHMGC: $(\omega_r, \gamma) = (0.145, 0.013)\frac{v_A}{R_0}$

Similar mode structure modification by fast ions is also seen in DIII-D experiment and TAEFL simulation [Tobias *et al.*, PRL 2011]

Antenna excitation of $(n, m, l) = (4, 7, 0)$ RSAE with drift-kinetic thermal ion, damping rate measurement

$v_i/v_A = 0.08$, $\rho_i/a = 0.008$, $k_\theta \rho_i = 0.1$





$$\delta\phi_{\rm sat} \propto \left[(\omega_0^2 - \omega_{\rm ant}^2)^2 + 4\gamma^2\omega_{\rm ant}^2\right]^{-1/2}$$

$$\frac{\omega_A^2}{v_A^2/R_0^2} \approx R_0^2 k_\parallel^2 + \left(\frac{7}{4} + \frac{T_e}{T_i}\right)\frac{2v_i^2}{v_A^2}$$

[Zonca *et al.*, PPCF 1996]

Antenna excitation of $(n, m, l) = (4, 7, 0)$ RSAE with drift-kinetic thermal ion, damping rate measurement

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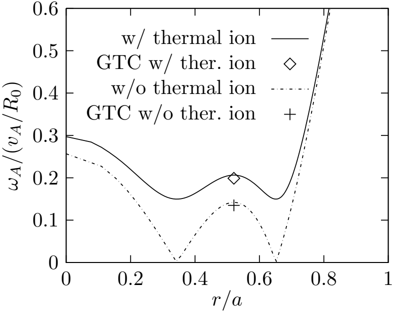
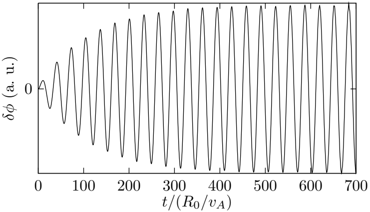
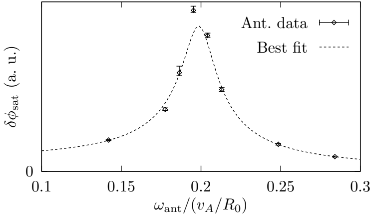


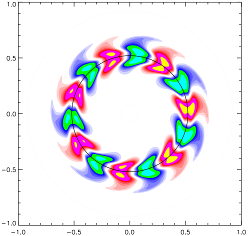
GTC antenna:
$(\omega_r, \gamma) = (0.199, 0.0106) v_A/R_0$
GTC initial perturbation:
$(\omega_r, \gamma) = (0.198, 0.011) v_A/R_0$
XHMGC initial perturbation:
$(\omega_r, \gamma) = (0.218, 0.017) v_A/R_0$



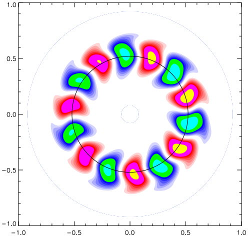
$$\delta\phi_{\mathrm{sat}} \propto [(\omega_0^2 - \omega_{\mathrm{ant}}^2)^2 + 4\gamma^2\omega_{\mathrm{ant}}^2]^{-1/2}$$



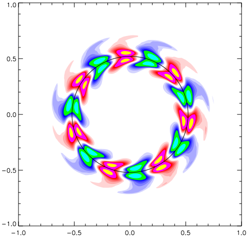
Kinetic thermal ion and FLR effects on the mode structure



MHD + drift-kinetic fast ions, no FLR, $\omega_r = 0.107 v_A/R_0$, $\gamma = 0.0159 v_A/R_0$



Drift-kinetic thermal & fast ions, no FLR, $\omega_r = 0.168 v_A/R_0$, $\gamma = 0.0174 v_A/R_0$

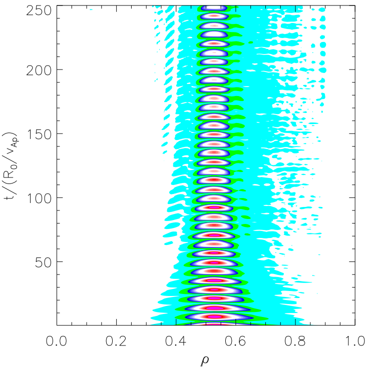


MHD + gyrokinetic fast ions, $\omega_r = 0.108 v_A/R_0$, $\gamma = 0.0090 v_A/R_0$

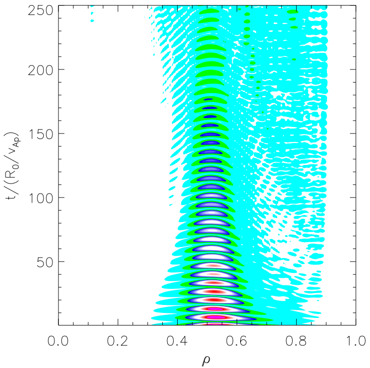
W. Deng *et al.*, Phys. Plasmas, **17**, 112504 (2010)

No RSAE with equilibrium current for this case

$\sqrt{\langle \delta\phi^2 \rangle_f}$ radial-time contour plots for $(n, m, l) = (4, 6, 0)$ RSAE:



Without equilibrium current
Eigenmode exists, oscillation at different location has same frequency.
Amplitude damps slowly.



With equilibrium current
Eigenmode doesn't exist, oscillation's frequency at different location is the local continuum frequency.
Amplitude damps quickly.

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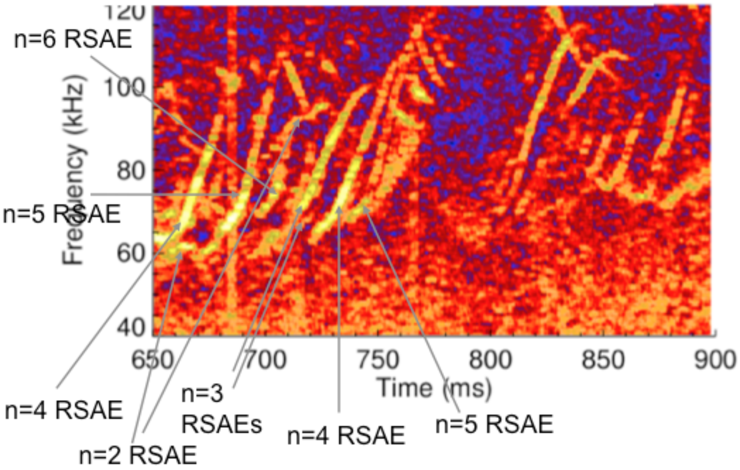
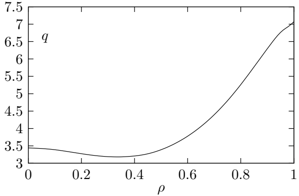
Experimental spectrum



[Van Zeeland *et al.*, PoP 2011 (in press); Tobias *et al.*, PRL 2011]

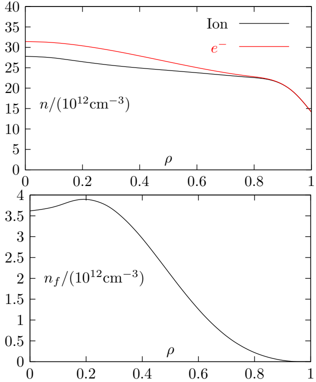
Equilibrium profiles

q-profile:

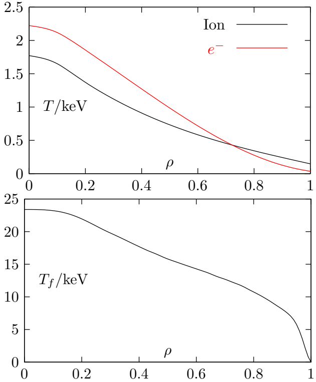


$q_{\text{min}} = 3.1828$

Density:



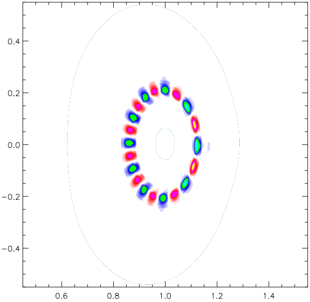
Temperature:



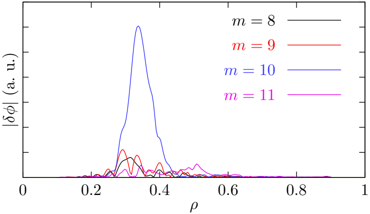
- $n = 3$ and $n = 4$ modes are being studied.
- Result comparisons with GYRO and TAEFL are in progress.
- Presented here are mostly GTC results. GYRO and TAEFL results are probably presented in E. Bass's and D. Spong's talks.

$n = 3$, zero-β ideal MHD, $m = 10$ initial perturbation

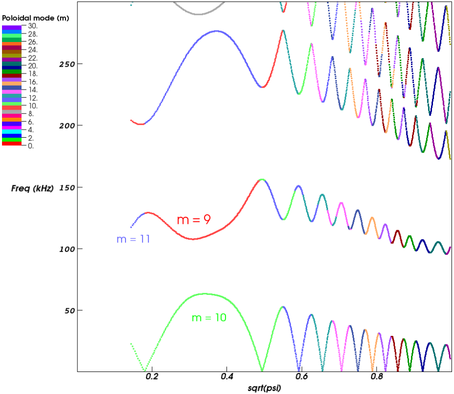
$\delta\phi$ poloidal contour plot:



m-harmonic decomposed $|\delta\phi|$:



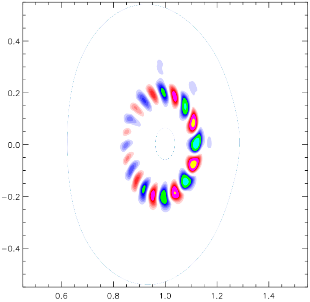
Alfvén continua w/o acoustic coupling:



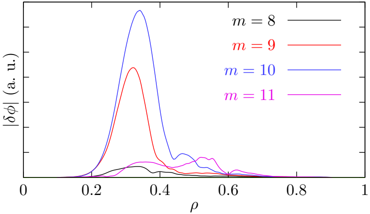
Mode frequency:
$\omega/(2\pi) = 73.8\text{kHz}$

$n = 3$, finite-β gyrokinetic plasma, fast ion excitation

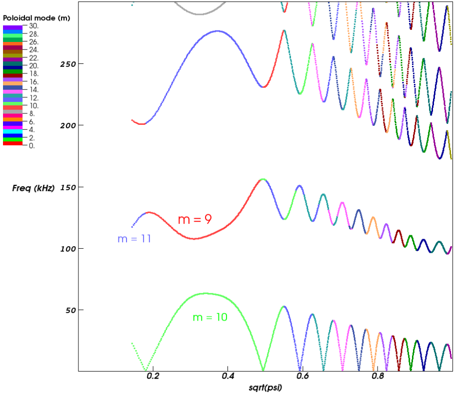
$\delta\phi$ poloidal contour plot:



m-harmonic decomposed $|\delta\phi|$:



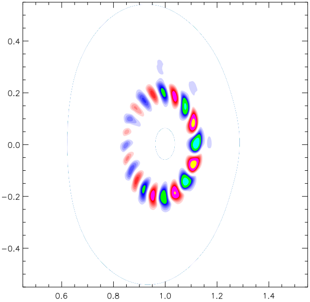
Alfvén continua w/o acoustic coupling:



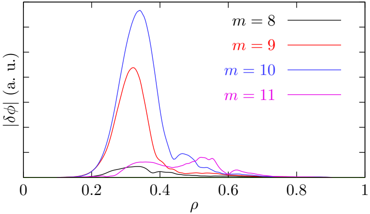
Mode frequency and growth rate:
$\omega_r/(2\pi) = 93.4\text{kHz}$, $\gamma/\omega_r = 0.067$

$n = 3$, finite-β gyrokinetic plasma, fast ion excitation

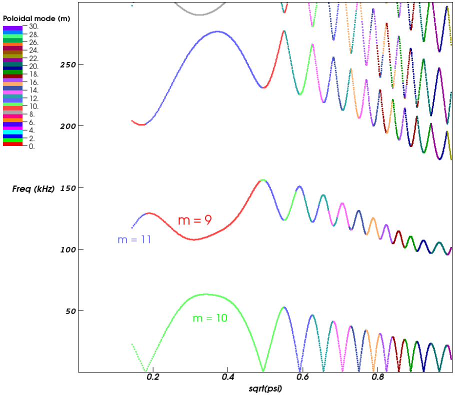
$\delta\phi$ poloidal contour plot:



m-harmonic decomposed $|\delta\phi|$:



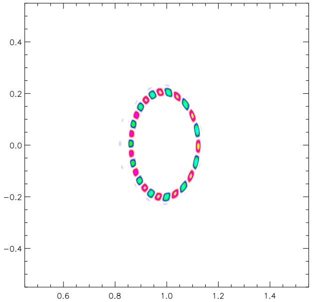
Alfvén continua w/o acoustic coupling:



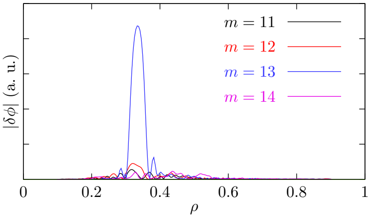
Mode frequency and growth rate:
$\omega_r/(2\pi) = 93.4$kHz, $\gamma/\omega_r = 0.067$
RSAE→TAE transition [Breizman *et al.*, PoP 2003]

$n = 4$, zero-β ideal MHD, $m = 13$ initial perturbation

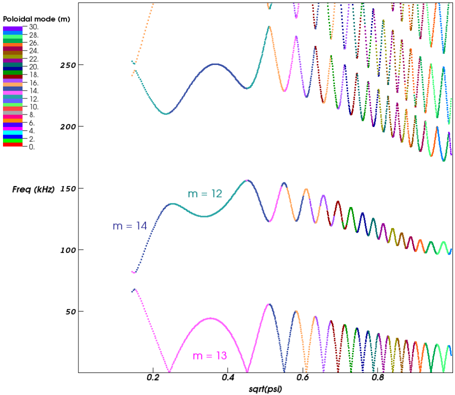
$\delta\phi$ poloidal contour plot:



m-harmonic decomposed $|\delta\phi|$:



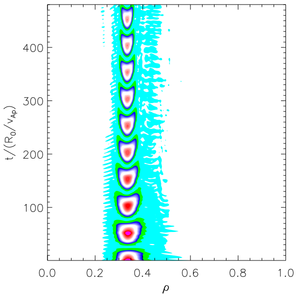
Alfvén continua w/o acoustic coupling:



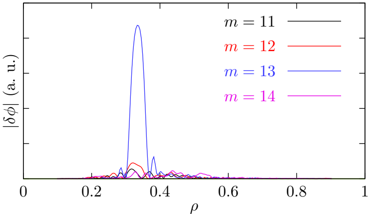
Frequency:
$\omega/(2\pi) = 45.1\text{kHz}$

$n = 4$, zero-β ideal MHD, $m = 13$ initial perturbation

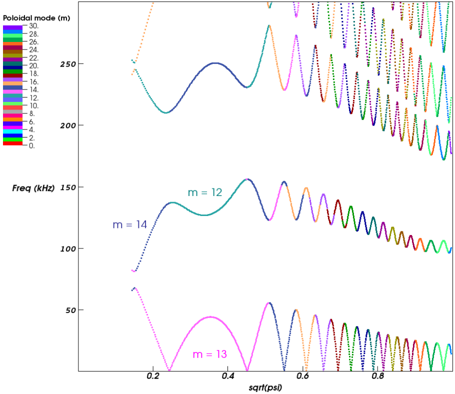
$\sqrt{\langle \delta\phi^2 \rangle_f}$ radial-time contour:



m-harmonic decomposed $|\delta\phi|$:



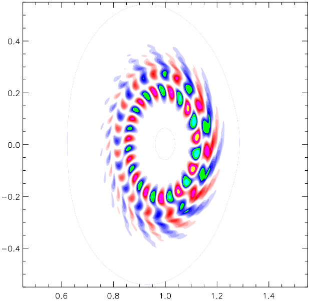
Alfvén continua w/o acoustic coupling:



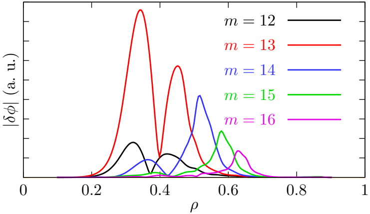
Frequency:
$\omega/(2\pi) = 45.1$kHz
Not an eigenmode, just continuum oscillation.

$n = 4$, finite-β gyrokinetic plasma, fast ion excitation

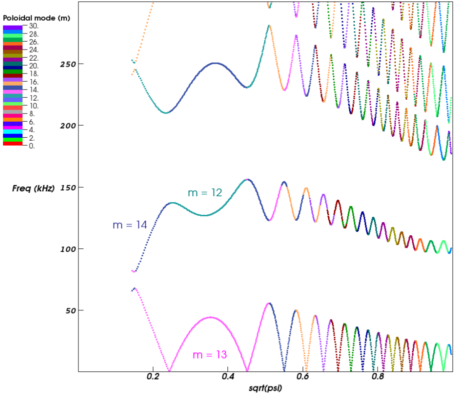
$\delta\phi$ poloidal contour plot:



m-harmonic decomposed $|\delta\phi|$:



Alfvén continua w/o acoustic coupling:



RSAE frequency and growth rate:
$\omega_r/(2\pi) = 79.2\text{kHz}$, $\gamma/\omega_r = 0.118$

Summary

- Electromagnetic gyrokinetic simulation model used in GTC is presented and can be shown to reduce to ideal MHD theory with appropriate approximations made.
- In a simple geometry
  - GTC simulation results are benchmarked with XHMGC and reasonable agreements are obtained. The discrepancy is probably due to the difference in geometry and fast ion model difference between the two codes.
  - Finite β raises the Alfvén continuum and thus raises the RSAE frequency.
  - Thermal ion kinetic effects introduce ion damping and modify the RSAE mode structure.
  - Fast ion FLR effect lowers the RSAE growth rate.
  - In the ideal MHD uniform background plasma limit without toroidal coupling, the RSAE doesn't exist with plasma current effect.
- Simulations of DIII-D discharge #142111 at 750ms successfully reveal some RSAEs and TAEs. Comparisons with GYRO and TAEFL results are in progress.