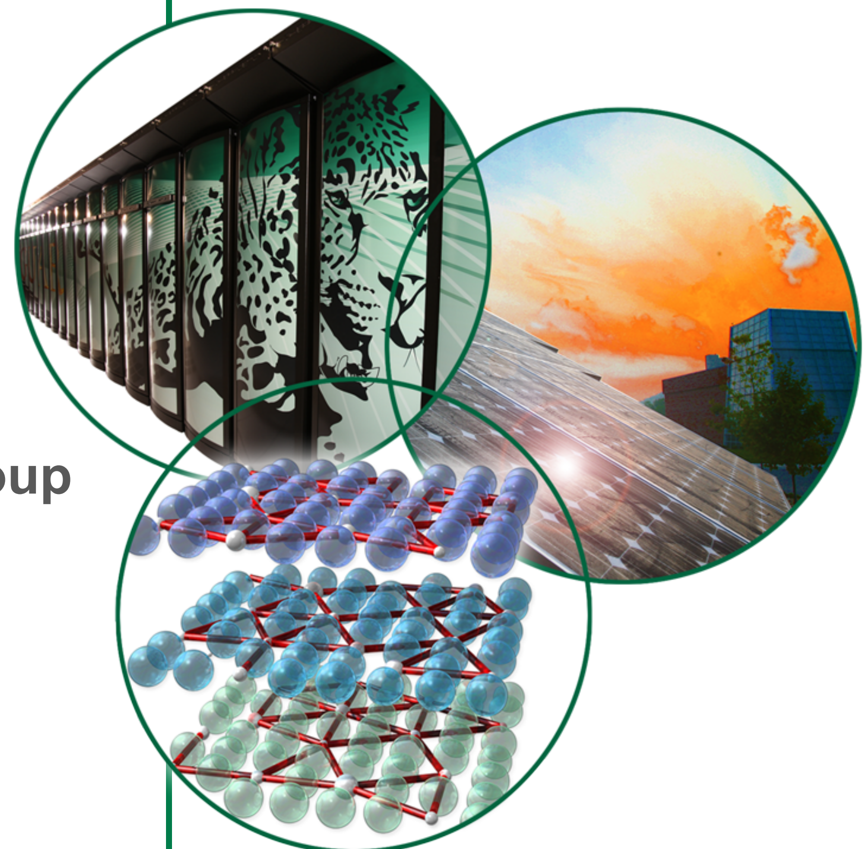
Recent development of energetic particle gyro-Landau fluid models



Don Spong – ORNL Fusion Theory Group

Transport Task Force Meeting

San Diego, California

April 6 – 8, 2011

U.S. DEPARTMENT OF ENERGY

OAK RIDGE NATIONAL LABORATORY

MANAGED BY UT-BATTELLE FOR THE DEPARTMENT OF ENERGY

Gyro-Landau fluid approach

- Hammett, Perkins (PRL, 1990)
- Landau damping in fluid model using improved equation of state
- dissipative term ~$|k_{||}|v_{th}$ introduces collisionless phase-mixing
- N-pole Padé approximation to Z-function
- $|k_{||}| \rightarrow |\nabla_{||}|$ non-local (global) operator
- Requires Fourier (spectral) representation
- Rapid/reduced dimensionality formulation for EP-driven Alfvénic modes (Spong, Carreras, Hedrick, POP 1992, 1994)
- Non-perturbative, actively benchmarking with other methods

Outline

- Basic TAEFL equations, solution method
- Recent applications
  - Validation with ECEI mode structure imaging from DIII-D
  - JET AE damping simulations
  - ITPA-EP group linear/nonlinear benchmarks
- Extensions of the basic model
  - Acoustic mode coupling
  - More general EP distributions
  - Initial value → eigenvalue methods
  - EP FLR effects

TAEFL is a hybrid MHD-energetic particle gyrofluid model based on the reduced MHD FAR code:

$$\frac{\partial \psi}{\partial t} = \nabla_{\parallel}\phi + \eta J_{\zeta} + \rho_i^2 \left(\frac{\pi}{2}\right)^{1/2} \left(\frac{v_A^2}{v_{Te}}\right) \left|\nabla_{\parallel}\right| \nabla_{\perp}^2 \psi$$

Ion FLR   e/i Landau damping terms

$$\frac{\partial U}{\partial t} = -\nabla_{\parallel}\left(\frac{J_{\zeta}}{B_{\zeta}}\right) + \sum_{thermal,fast} \hat{\zeta} \times \vec{\nabla}\left(\frac{R}{|B|}\right) \bullet \vec{\nabla} n_f T_{f0} + \omega_r \rho_i^2 \nabla_{\perp}^2 U - c_0 \frac{(\beta_e + \beta_i)}{\omega_r} \mathrm{Im}\left[X_e'' + X_i'' - \frac{(X_i' - X_e')^2}{2 + X_e + X_i}\right] \Omega_d^2(\phi)$$

Reduced MHD Ohm's law and vorticity equation

$$\frac{\partial n_f}{\partial t} = \frac{T_{f0}}{m_f \Omega_{cf}} \Omega_d(n_f) - n_{f0} \nabla_{\parallel} v_{\parallel,f} + \frac{q_f n_{f0}}{m_f \Omega_{cf}} \Omega_d(\phi) - \frac{q_f}{m_f \Omega_{cf} n_{f0}} \frac{dn_{f0}}{dr} \left(\frac{1}{r}\frac{\partial \phi}{\partial \theta} - \frac{\partial \psi_0 / \partial r}{R^2 B}\frac{\partial \phi}{\partial \zeta}\right)$$

EP Landau closure   EP ω* drive

$$\frac{\partial v_{\parallel,f}}{\partial t} = \frac{T_{f0}}{m_f \Omega_{cf}} \Omega_d(v_{\parallel,f}) - \left(\frac{\pi}{2}\right)^{1/2} v_{th,f} \left|\nabla_{\parallel}\right| v_{\parallel,f} - \frac{v_{th,f}^2}{n_{f0}} \nabla_{\parallel} n_f - \frac{q_f v_{th,f}^2}{m_f \Omega_{cf} R n_{f0}} \frac{dn_{f0}}{dr} \left(\frac{1}{r}\frac{\partial \psi}{\partial \theta} - \frac{\partial \psi_0 / \partial r}{R^2 B}\frac{\partial \psi}{\partial \zeta}\right)$$

Gyrofluid energetic particle closure moments

where $\Omega_d \propto \left(\hat{b} \times \vec{\nabla} B\right) \bullet \vec{\nabla}, \quad X_{e,i} = \xi_{e,i} Z(\xi_{e,i}), \quad \xi_{e,i} = \omega_r / \sqrt{2}\left|\nabla_{\parallel}\right| v_{th,e,i}$

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- Basic TAEFL equations, solution method
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  - **Validation with ECEI mode structure imaging from DIII-D**

MHD parity breaking – EP kinetic effects

- The standard MHD model has symmetry properties that allow use of reduced Fourier series:

$$\phi = \sum_{m,n} \phi_{mn,s}(\rho)\sin(m\theta + n\zeta); \qquad \psi = \sum_{m,n} \psi_{mn,c}(\rho)\cos(m\theta + n\zeta)$$

$$i.e., \text{ functions follow } g(\theta,\zeta) = \pm g(-\theta,-\zeta)$$

- However, a variety of effects can break this symmetry
  - Plasma flows, two-fluid effects, up-down equilibrium asymmetry (single-null divertor)
  - Kinetic effects, Landau resonance, drifts,…
  - Require full representations for all dynamical variables

$$Y = \sum_{m,n} \left[Y_{mn,c}(\rho)\cos(m\theta + n\zeta) + Y_{mn,s}(\rho)\sin(m\theta + n\zeta) \right]$$

TAEFL:hybrid MHD-EP gyrofluid model
without EP, 1st two equations maintain MHD symmetry

$$\frac{\partial \psi}{\partial t} = \nabla_{\parallel}\phi + \eta J_{\zeta}$$

$$\phi, U = \sum_{m,n} (\phi, U)_{mn,s} (\rho) \sin(m\theta + n\zeta); \qquad \psi, J_{\zeta} = \sum_{m,n} (\psi, J_{\zeta})_{mn,c} (\rho) \cos(m\theta + n\zeta)$$

$$\frac{\partial U}{\partial t} = -\nabla_{\parallel}\left(\frac{J_{\zeta}}{B_{\zeta}}\right)$$

TAEFL:hybrid MHD-EP gyrofluid model

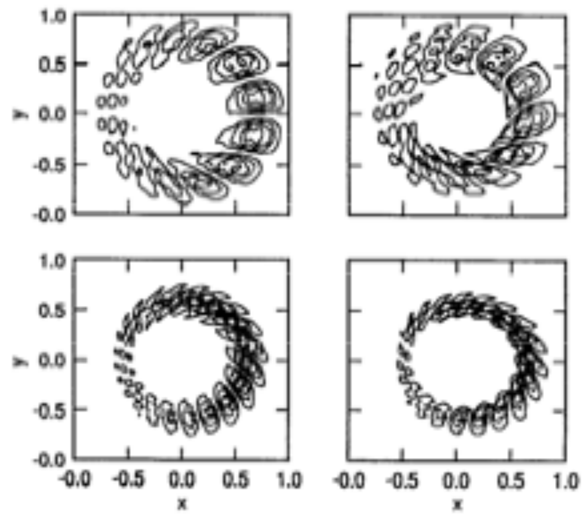
without EP, 1st two equations maintain MHD symmetry
with EP, symmetry is broken - non-perturbative effects in TAEFL

$$\frac{\partial\psi}{\partial t} = \nabla_{\parallel}\phi + \eta J_{\zeta}$$

$$\frac{\partial U}{\partial t} = -\nabla_{\parallel}\left(\frac{J_{\zeta}}{B_{\zeta}}\right) + \sum_{thermal,fast} \hat{\zeta}\times\vec{\nabla}\left(\frac{R}{|B|}\right)\bullet\vec{\nabla} n_f T_{f0}$$

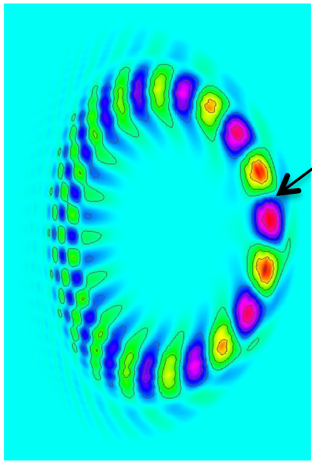
$$\frac{\partial n_f}{\partial t} = \frac{T_{f0}}{m_f\Omega_{cf}}\Omega_{df}\left(n_f\right) - n_{f0}\nabla_{\parallel}\mathrm{v}_{\parallel,f} + \frac{q_f n_{f0}}{m_f\Omega_{cf}}\Omega_{df}\left(\phi\right) + \frac{q_f}{m_f\Omega_{cf}}\Omega_{*f}\left(\phi\right)$$

$$\frac{\partial \mathrm{v}_{\parallel,f}}{\partial t} = \frac{T_{f0}}{m_f\Omega_{cf}}\Omega_{df}\left(\mathrm{v}_{\parallel,f}\right) - \left(\frac{\pi}{2}\right)^{1/2}\mathrm{v}_{\mathrm{th,f}}\left|\nabla_{\parallel}\right|\mathrm{v}_{\parallel,f} - \frac{\mathrm{v}_{\mathrm{th,f}}^2}{n_{f0}}\nabla_{\parallel}n_f - \frac{\mathrm{q_f v_{th,f}^2}}{m_f\Omega_{cf}R}\Omega_{*f}\left(\psi\right)$$



Linear TAEFL: D. Spong, et al., Phys. Fl. **B4** (1992) 3316.

$\nabla_{\parallel}$, Ω_{df}, Ω_{*f}, $\hat{\zeta}\times\vec{\nabla}\left(\frac{R}{|B|}\right)\bullet\vec{\nabla}$ operators change parities (i.e., sin $\rightarrow$ cos/cos $\rightarrow$ sin)

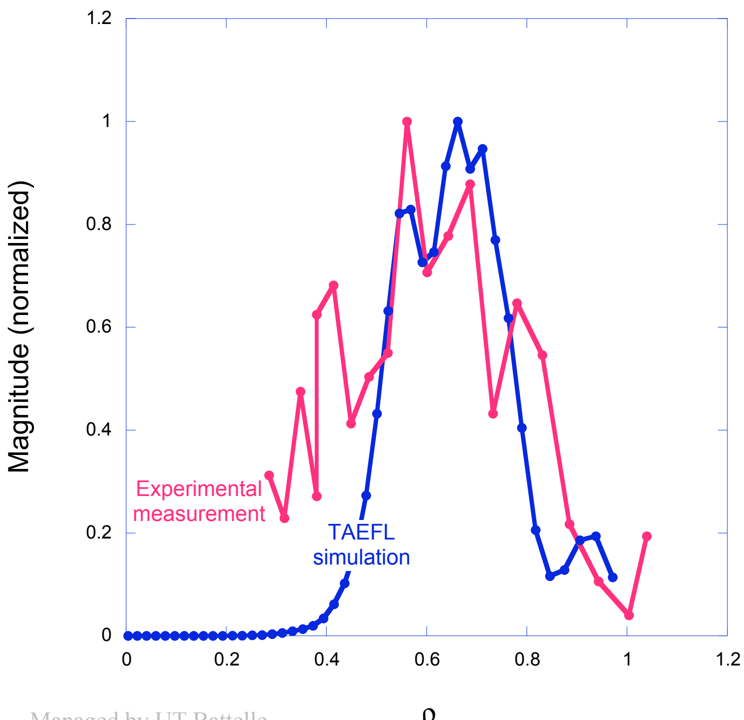


“twisted” form of mode structure (TAEFL perturbative effect) leads to observed phase variation

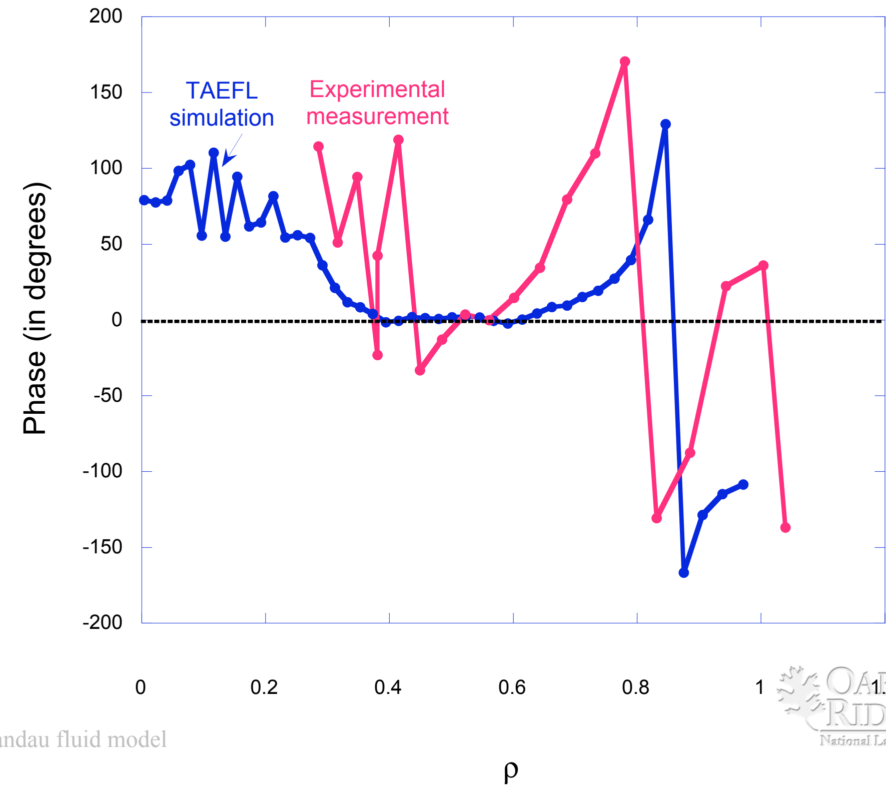
The consequence of MHD parity breaking is that mode structure at fixed toroidal plane will not be up-down symmetric

“Measurements, Modeling, and Electron Cyclotron Heating Modification of Alfv´en Eigenmode Activity in DIII-D,”
M A Van Zeeland, et al., submitted to Nuclear Fusion (2009)

magnitude comparison



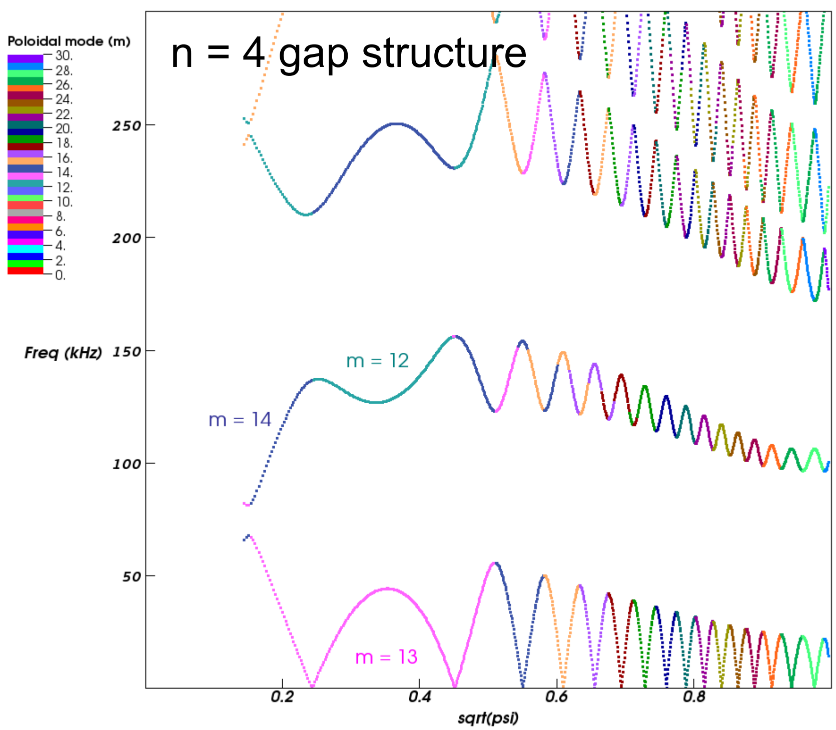
ECE phase comparison

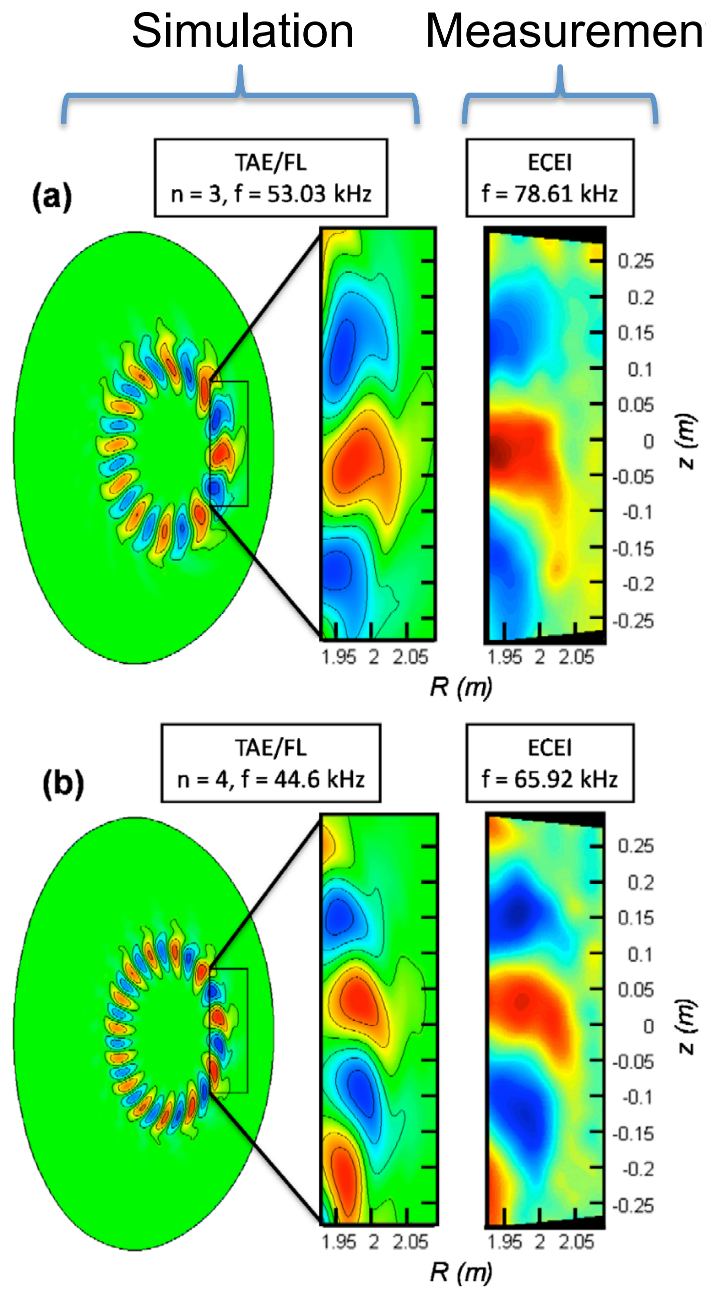


Recent ECEI measurements on DIII-D (B. Tobias, et al.) have provided direct imaging of TAE/RSAE mode structures

- these match well with eigenfunctions from the TAEFL gyrofluid model

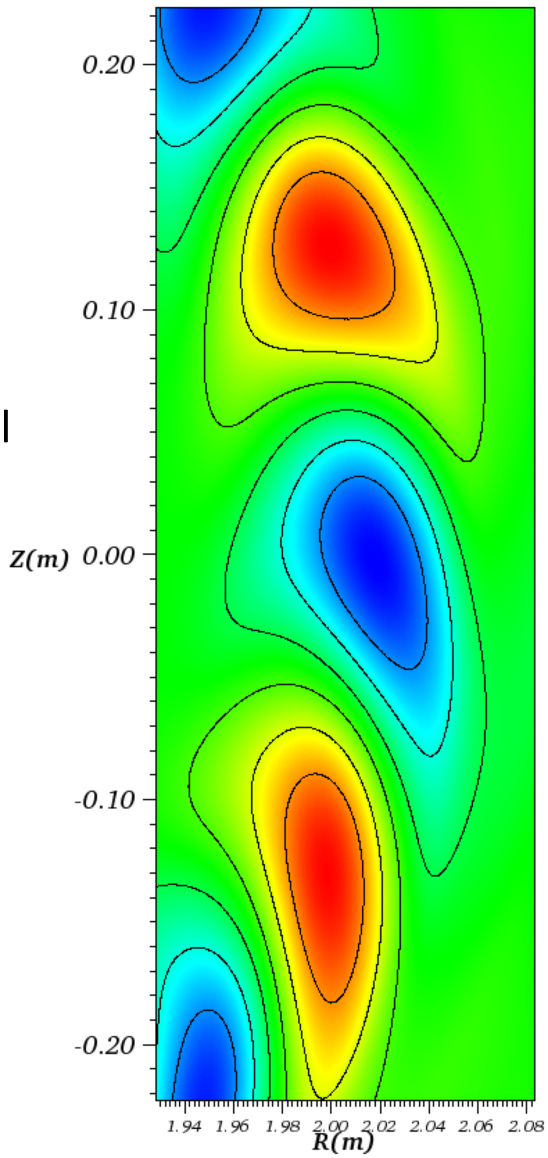
"Electron Cyclotron Emission Imaging of Fast Ion Diamagnetic Shearing in 2D Alfvén Eigenmode Structures," B. Tobias, et al. Phys. Rev. Lett. **106**, 075003 (2011).



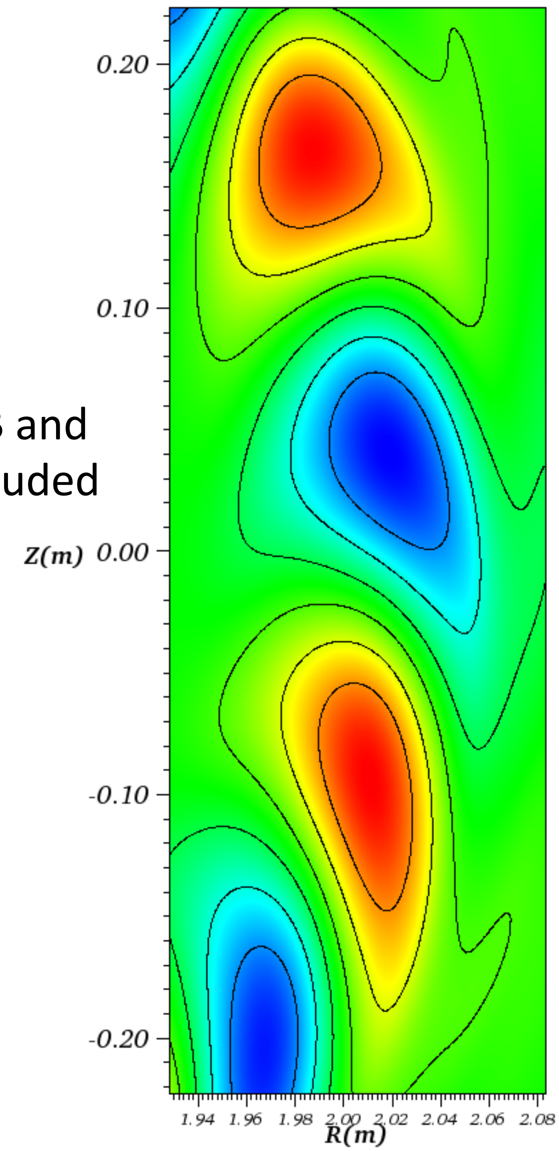


Inclusion of electric field shearing and rotation had minimal effect on mode structure

ExB and toroidal flow = 0

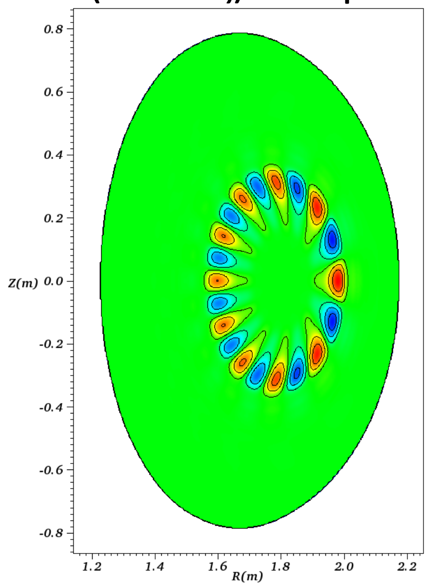


Experimental ExB and toroidal flows included

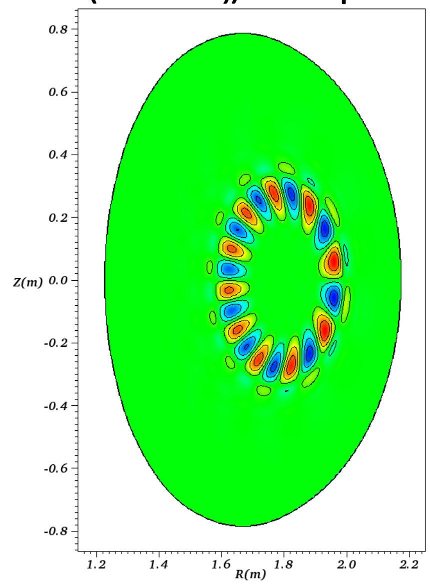


Magnitude/phasing between cos/sin components determines twist in eigenfunction. Driven by fast ion profiles/parameters

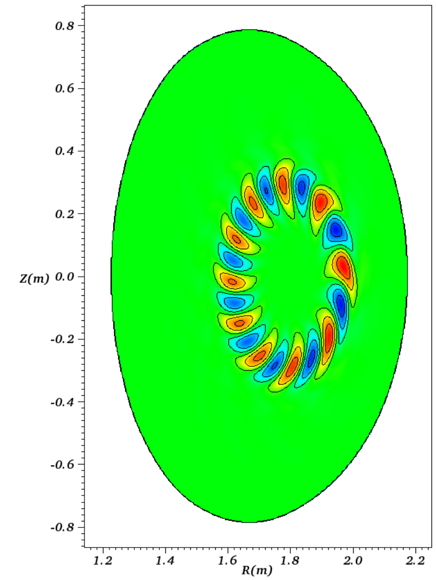
cos(mθ+nζ) component

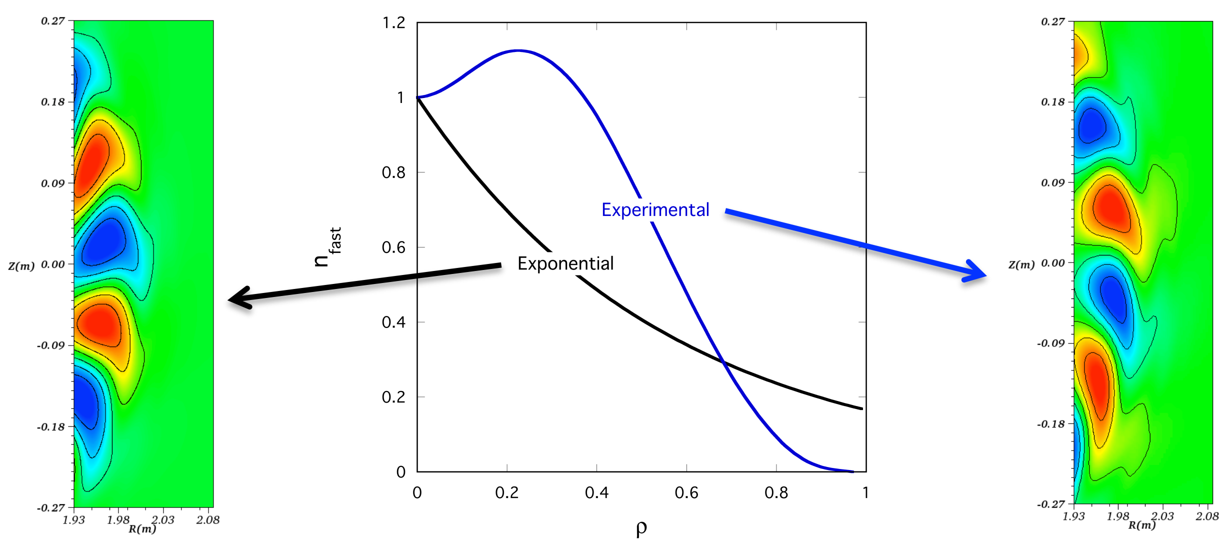


sin(mθ+nζ) component



TAEFL eigenfunction





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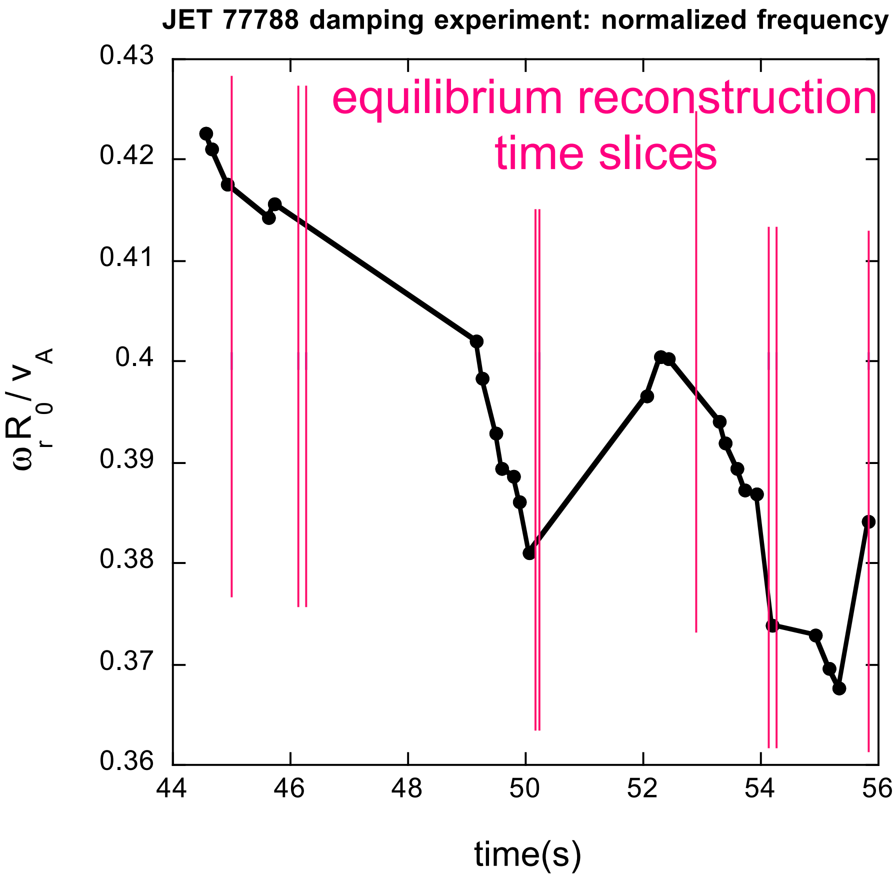
  - **JET AE damping simulations**

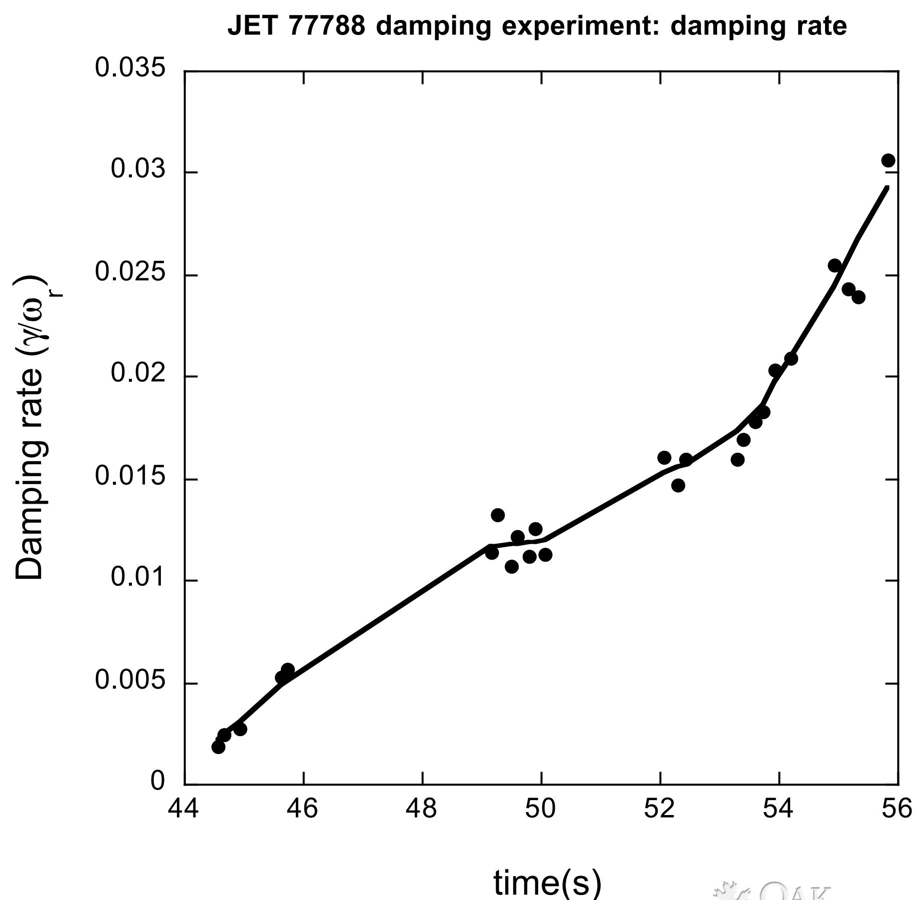
Damping effects included in TAEFL for modeling JET antenna Alfvén damping measurements:

- Continuum damping (contained in the Ohm's law and vorticity equation operators)
- Thermal ion Landau damping
- Thermal electron Landau damping
- Radiative damping (ion FLR terms)
- Resistivity

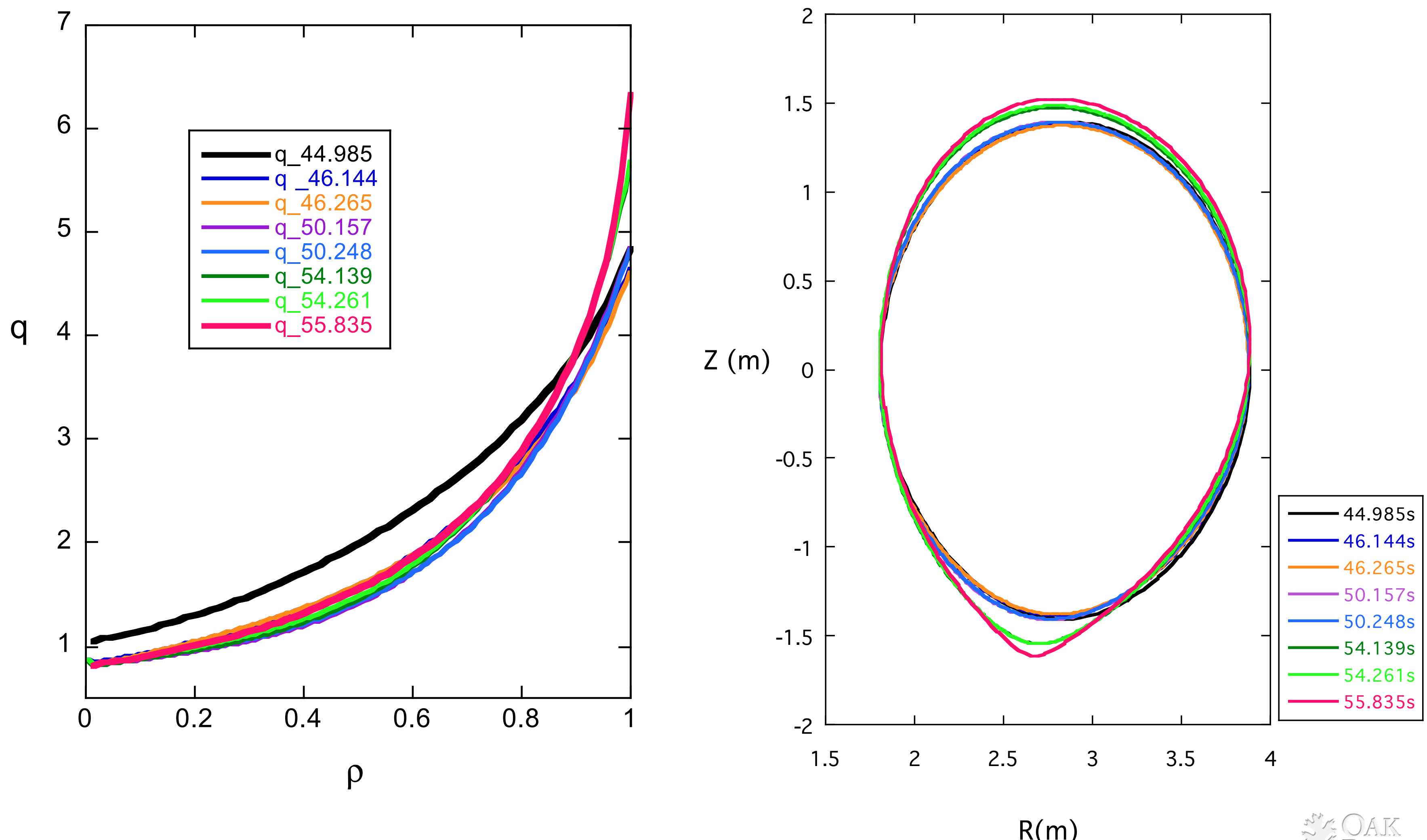
n = 3 Alfvén frequency antenna excitations in JET discharge #77788 are modeled

These JET data were provided by Theodoros Panis 9/25/2009

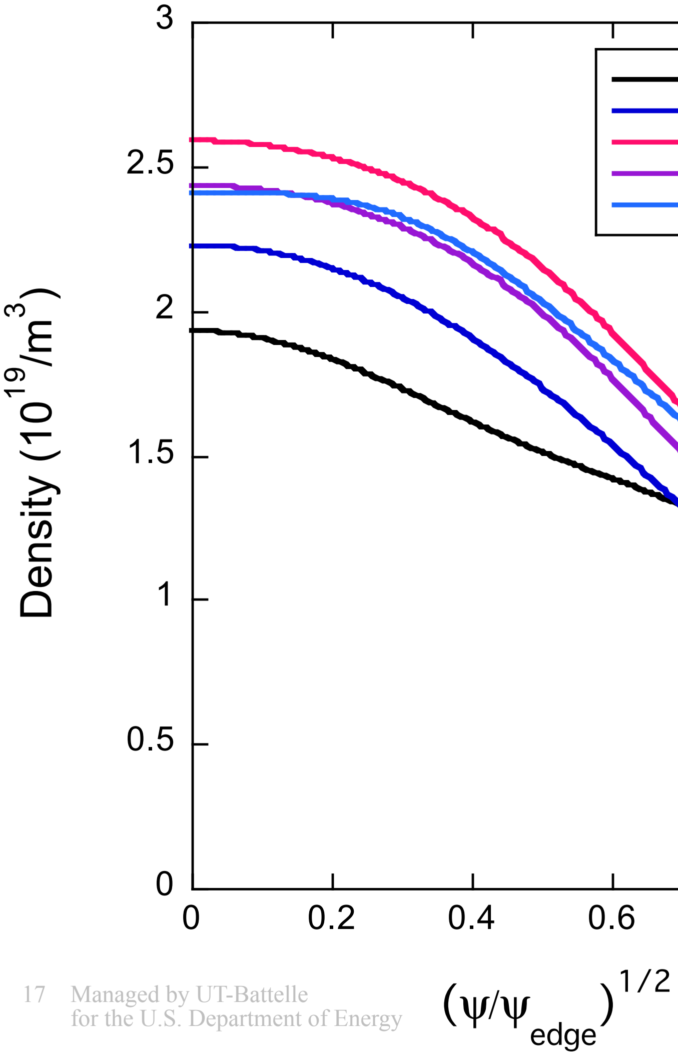


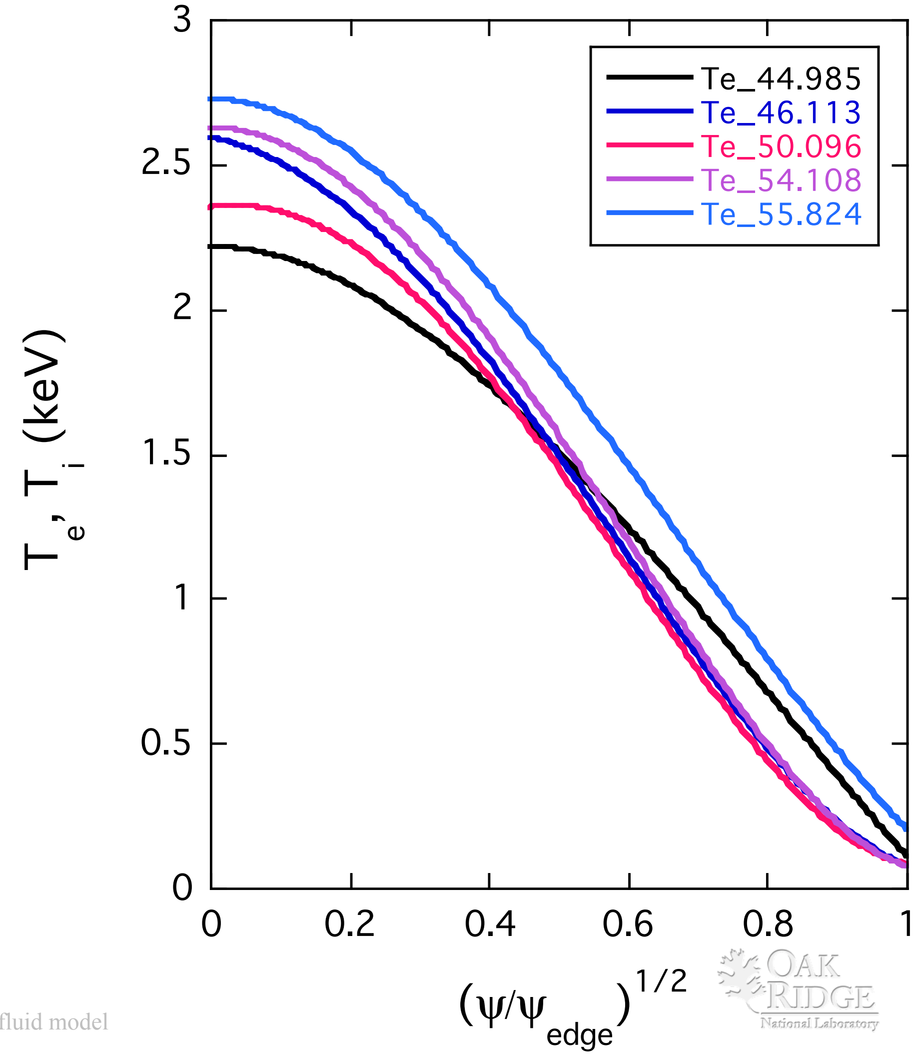


Evolution of q-profile and plasma shapes

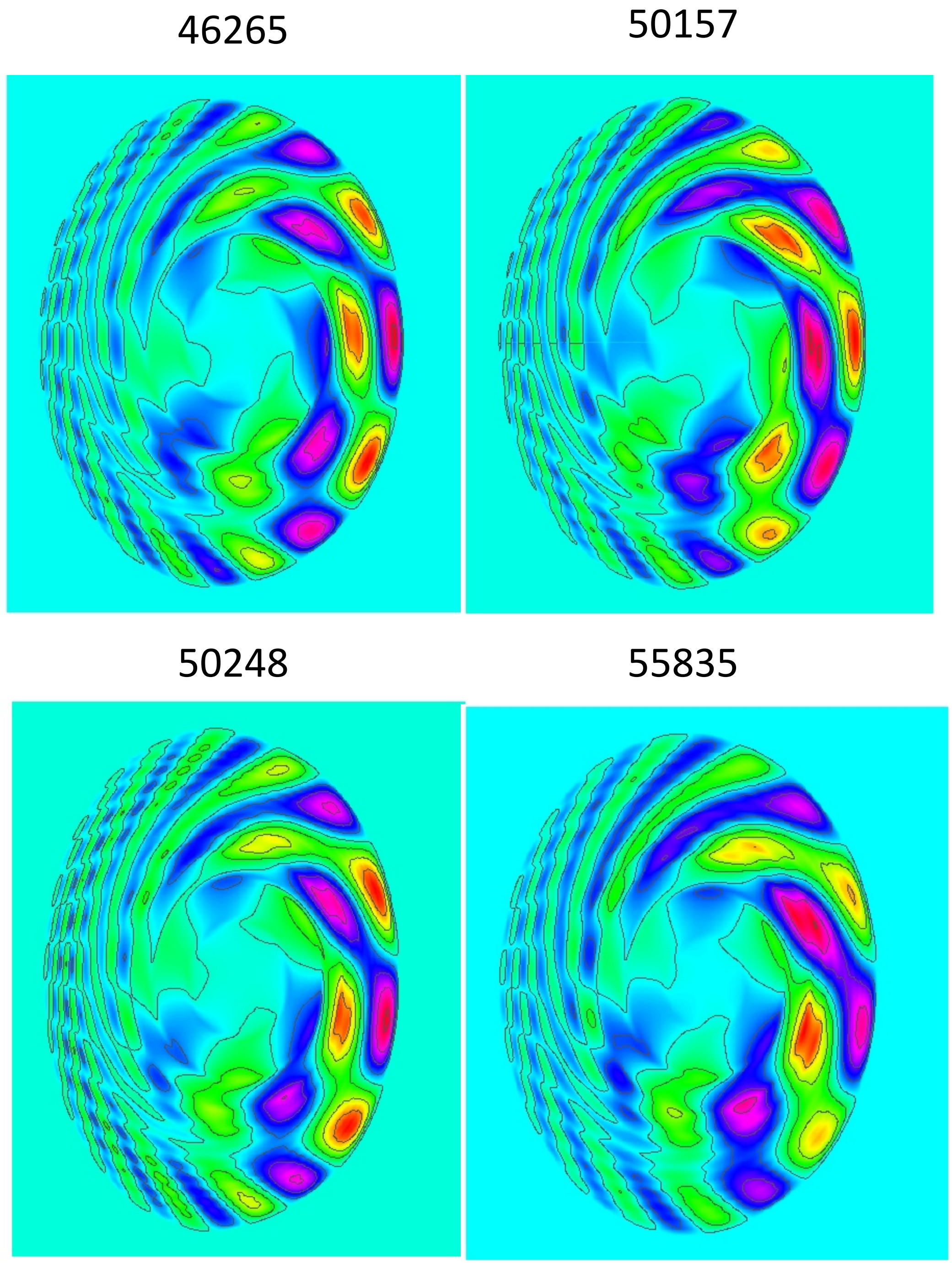


Evolution of plasma density and temperature profiles

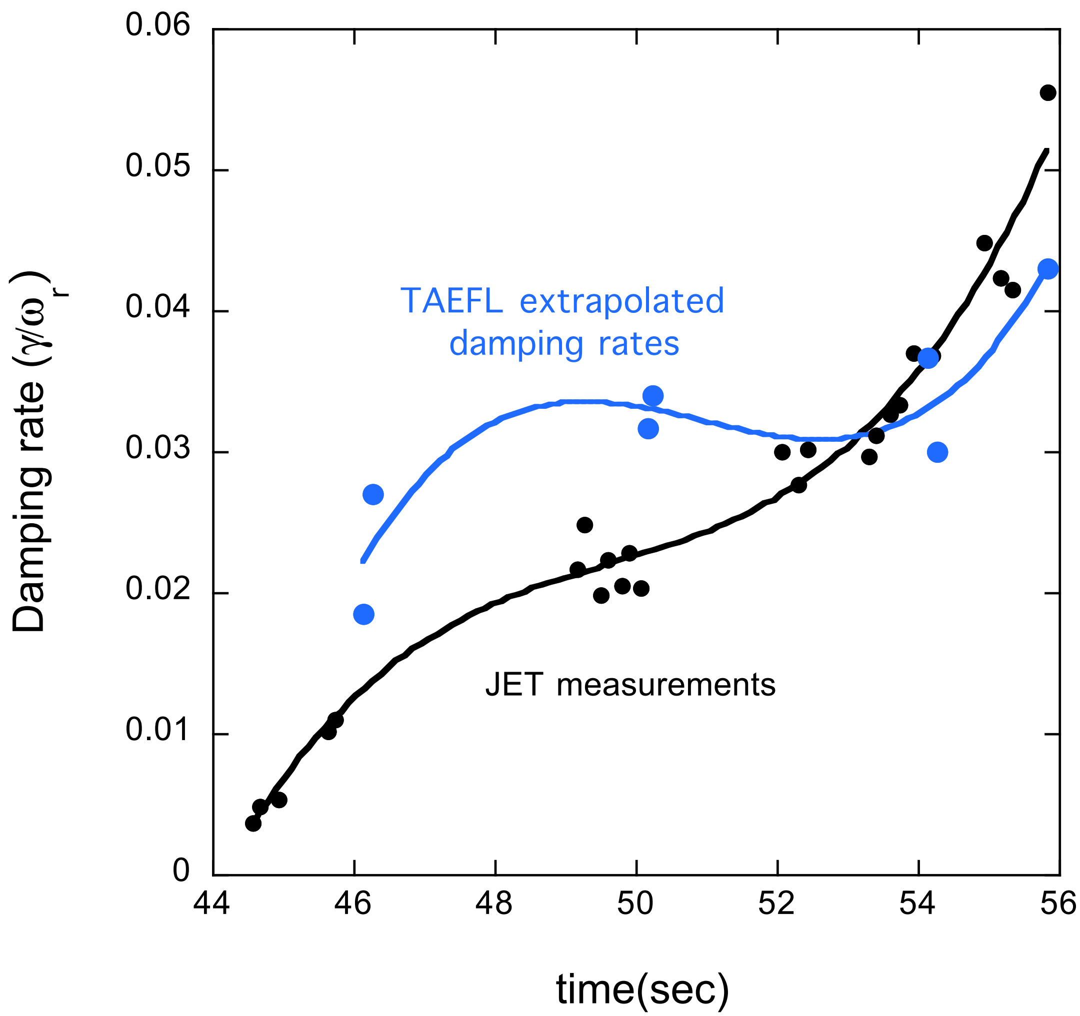




Eigenmode structures (shown at the $\zeta = 0$ plane)



β_{fast} threshold is determined from damping effects: damping inferred by extrapolation $\beta_{fast} = 0$



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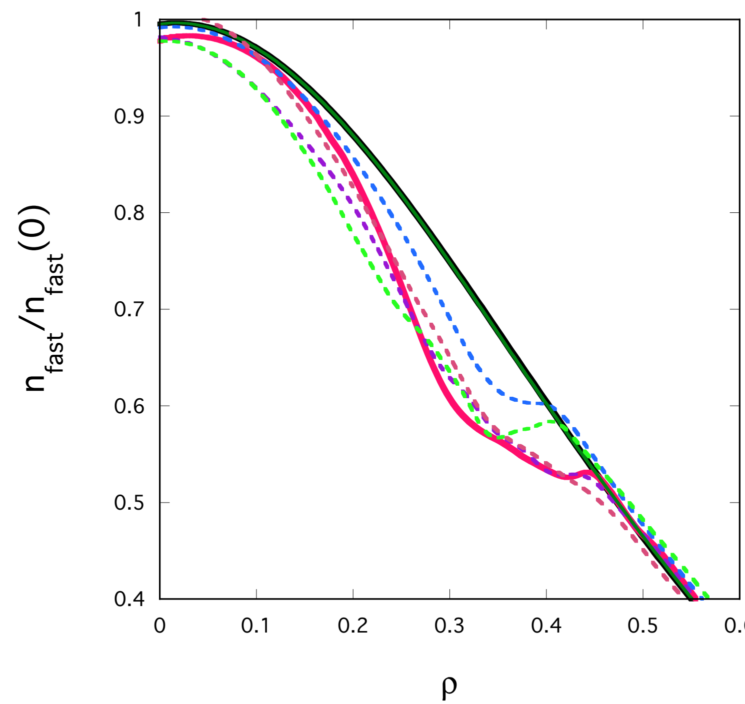
EPM nonlinear saturation study

- Benchmark test case chosen from previous case of S. Briguglio:
  - $\varepsilon = 1/10$, $q = 1.1 + 0.8116s^2$, $s = (1 - \psi/\psi_0)^{1/2}$, $m_H/m_i = 2$, $(T_H/m_H)^{1/2}/\Omega_h a = 0.01$, $(T_H/m_H)^{1/2}/v_A = 1$, $n_H(s) = n_{H0}\exp(-ks^2)$
  - In particular, we chose to study the EPM mode found at $k = 2.5$, $n_H(0)/n_{ion}(0) = 0.0025$ to 0.003 since several codes agreed on this
- Goal: study dependence of growth rates and nonlinear saturation levels on drive strength

TAEFL single and multi-toroidal mode runs

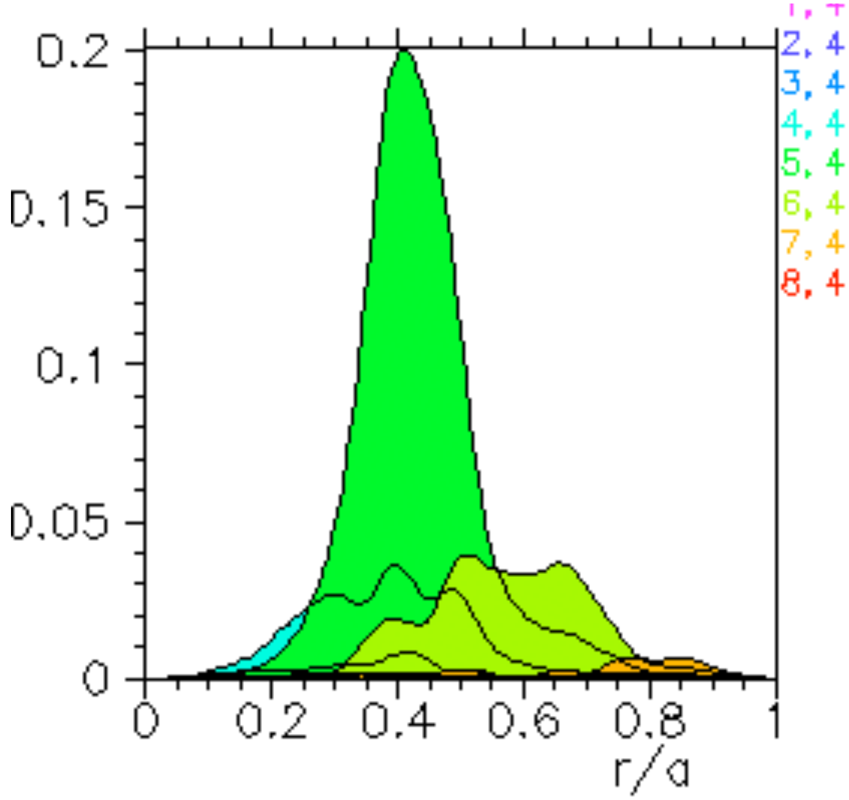
- TAEFL: reduced MHD model with gyro-Landau closure
  - Single-n nonlinear run
    - n = 4, m = 0 to 20; n = 0, m = 0 to 9
  - Multi – n nonlinear run
    - n = 0; m = 0 to 9   n = 2; m = 0 to 10
    - n = 3; m = 0 to 10   n = 4; m = 0 to 12
    - n = 5; m = 1 to 15   n = 6; m = 2 to 16



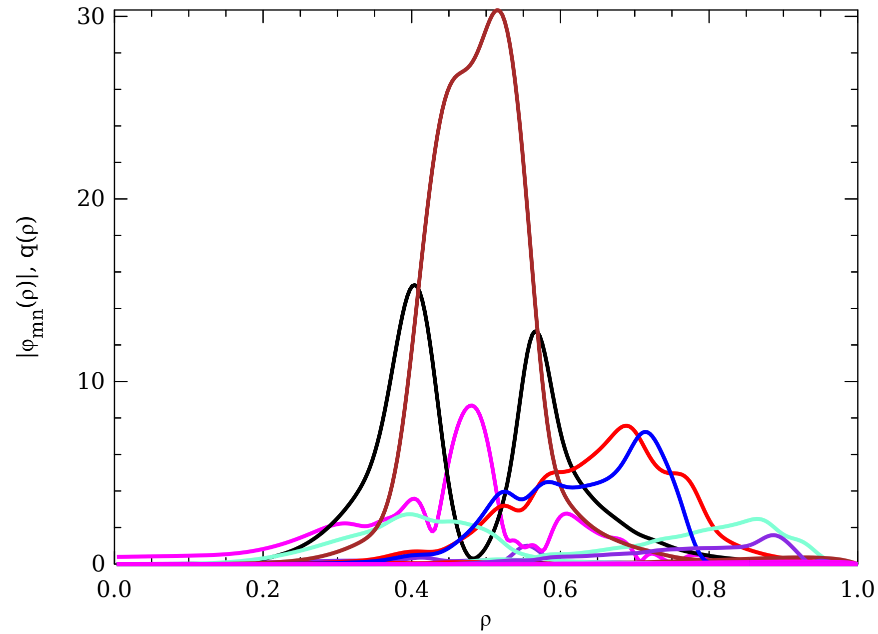
- Nonlinear saturation mechanisms
  - Nonlinear beating of n > 0 into n = 0
    - Flattening in n_{fast} profile, zonal flows, perturbed currents
  - Confinement of n_{fast} by convective eddies
    - Fluid version of particle-wave trapping
  - Energy cascades to smaller scales

Eigenfunction comparison

HMGC

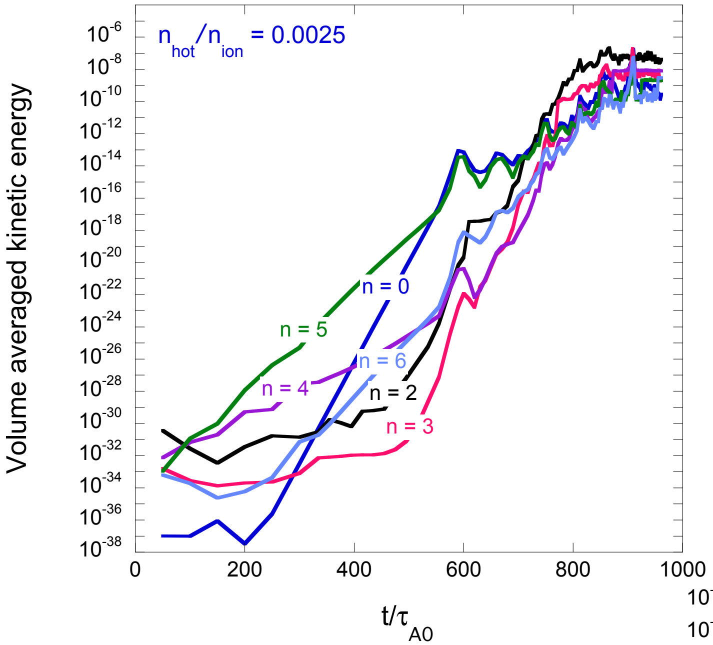


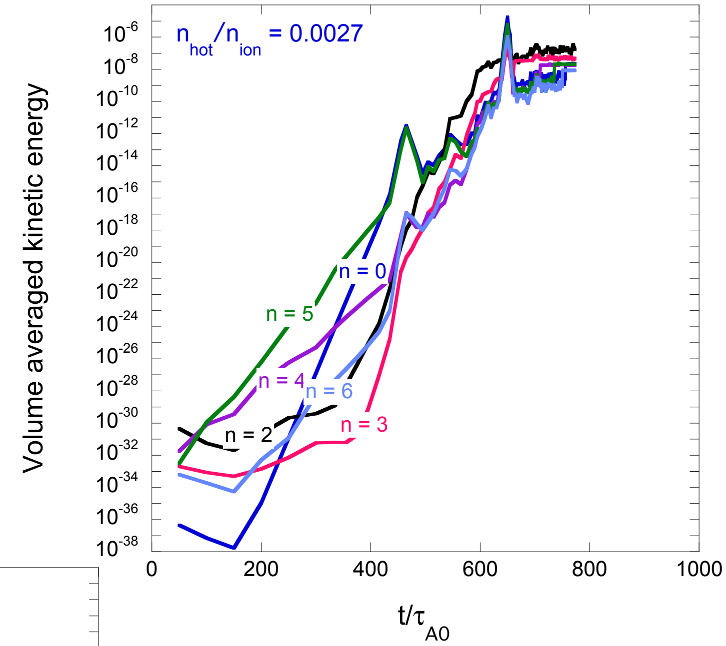
TAEFL

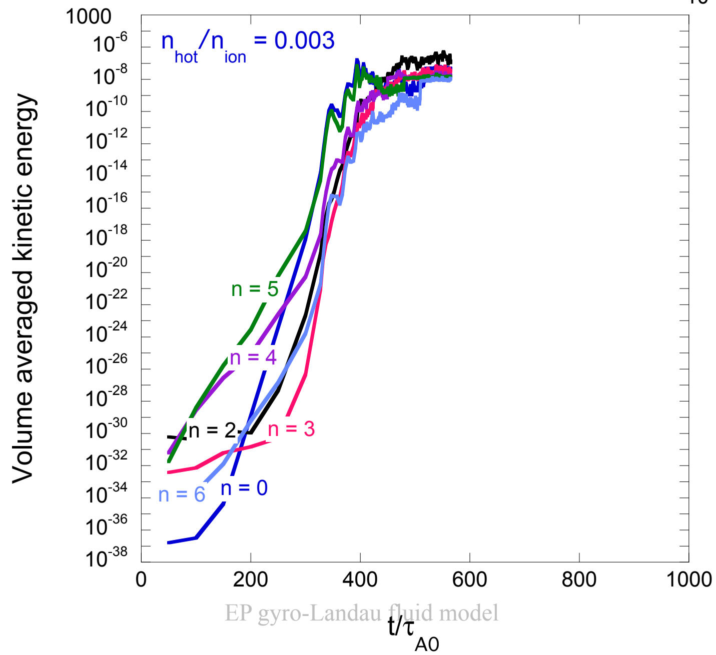


TAEFL multiple toroidal mode energy evolution

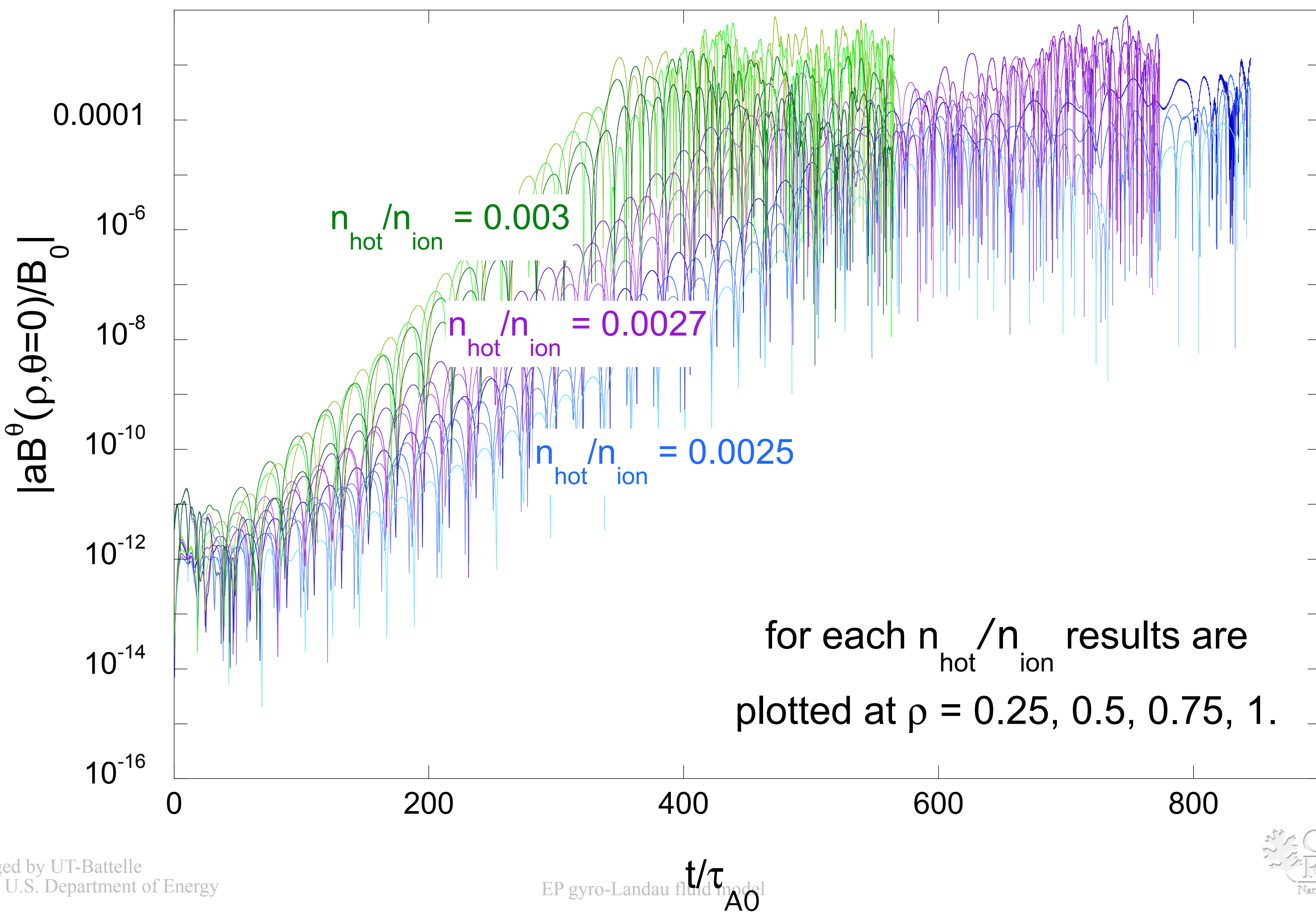
Kinetic energy evolution for multiple n simulations (n = 0, 2, 3, 4, 5, 6 included)



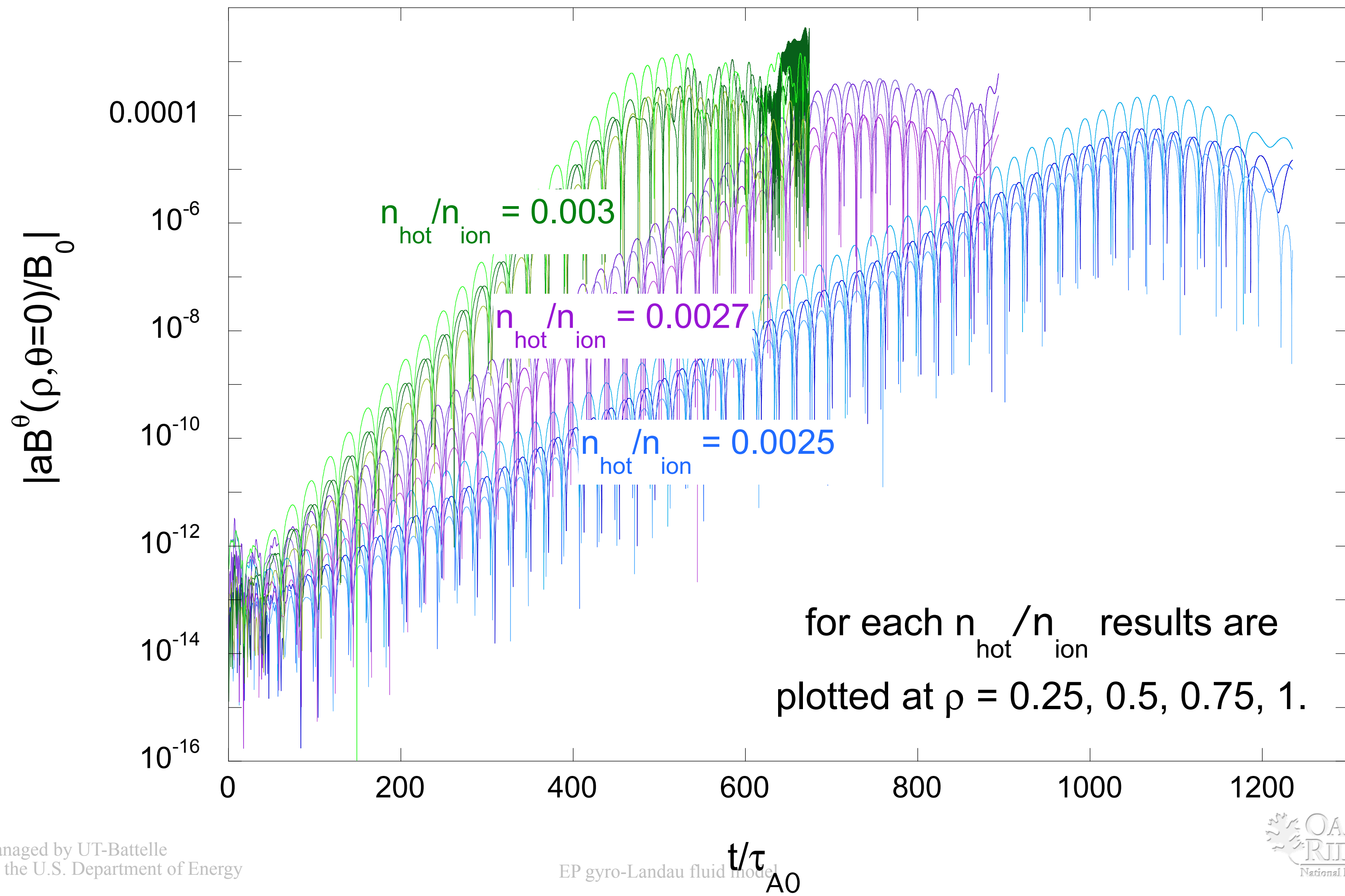




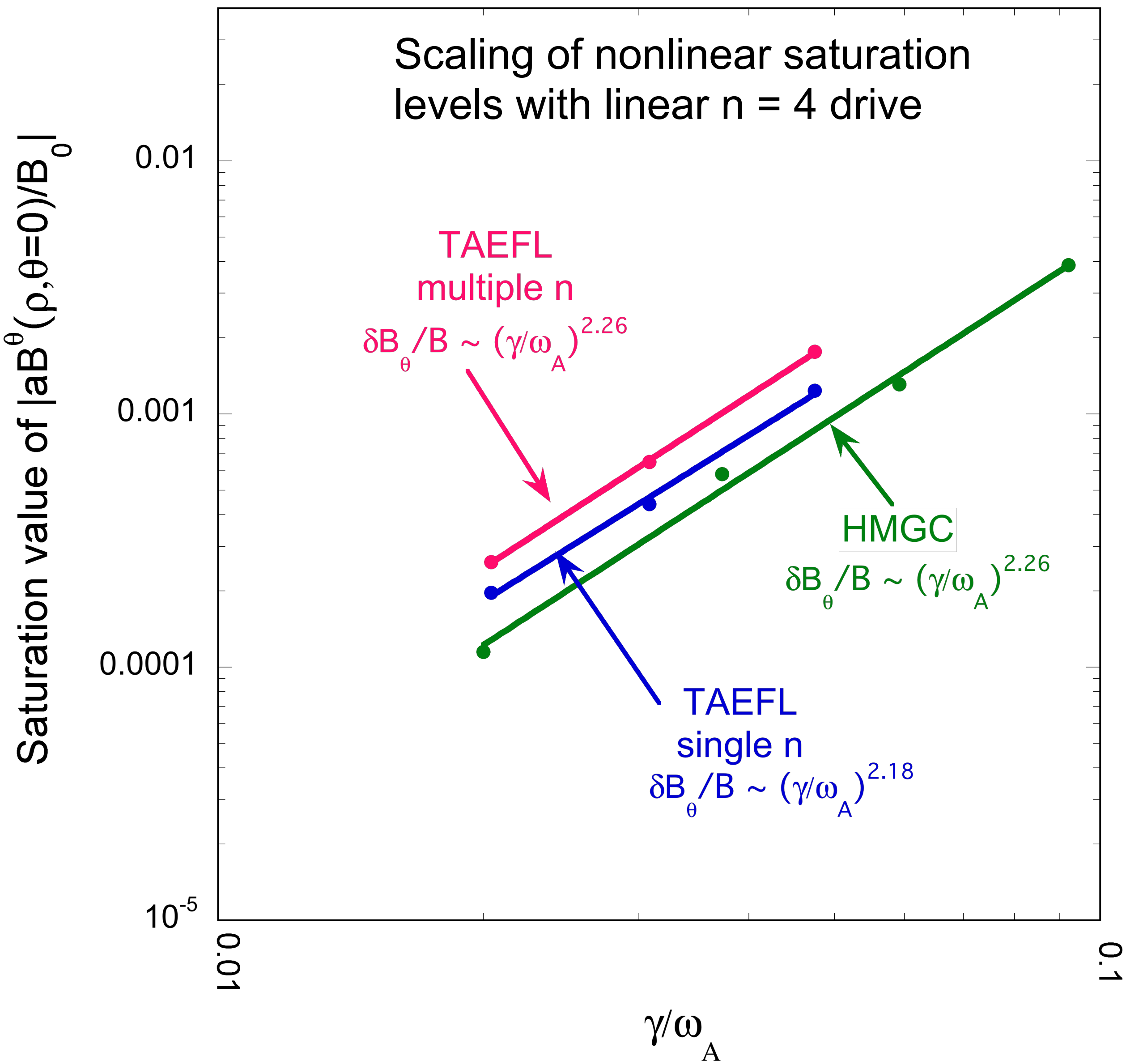
Multi-toroidal mode $\delta B_\vartheta/B_0$ evolution



Single toroidal mode $\delta B_\vartheta/B_0$ evolution



Comparison of nonlinear saturation levels



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  - **Acoustic mode coupling**
  - More general EP distributions
  - Initial value → eigenvalue methods
  - Extension to 3D configurations
  - EP FLR effects

TAEFL Equations with damping and acoustic terms (acoustic coupling in green):

$$\frac{\partial \psi}{\partial t} = \nabla_\parallel \phi + \eta J_\zeta + \rho_i^2 \left(\frac{\pi}{2}\right)^{1/2} \left(\frac{v_A^2}{v_{Te}}\right) \left|\nabla_\parallel\right| \nabla_\perp^2 \psi - \frac{1}{n_{th} e} \nabla_\parallel \left(n_{th} k T_e\right)$$

Ion FLR

e/i Landau damping terms

$$\frac{\partial U}{\partial t} = -\nabla_\parallel \left(\frac{J_\zeta}{B_\zeta}\right) + \hat{\zeta} \times \vec{\nabla}\left(\frac{R}{|B|}\right) \bullet \vec{\nabla} n_f T_{f0} + \hat{\zeta} \times \vec{\nabla}\left(\frac{R}{|B|}\right) \bullet \vec{\nabla} n_{th} T_{th} + \omega_r \rho_i^2 \nabla_\perp^2 U - c_0 \frac{\left(\beta_e + \beta_i\right)}{\omega_r} \mathrm{Im}\left[X_e'' + X_i'' - \frac{\left(X_i' - X_e'\right)^2}{2 + X_e + X_i}\right] \Omega_d^2(\phi)$$

Ohm's Law and vorticity equation

$$\frac{\partial n_f}{\partial t} = \frac{T_{f0}}{m_f \Omega_{cf}} \Omega_d\left(n_f\right) - n_{f0} \nabla_\parallel v_{\parallel,f} + \frac{q_f n_{f0}}{m_f \Omega_{cf}} \Omega_d(\phi) - \frac{q_f}{m_f \Omega_{cf} n_{f0}} \frac{d n_{f0}}{dr} \left(\frac{1}{r}\frac{\partial \phi}{\partial \theta} - \frac{\partial \psi_0 / \partial r}{R^2 B} \frac{\partial \phi}{\partial \zeta}\right)$$

EP Landau closure

EP ω* drive

$$\frac{\partial v_{\parallel,f}}{\partial t} = \frac{T_{f0}}{m_f \Omega_{cf}} \Omega_d\left(v_{\parallel,f}\right) - \left(\frac{\pi}{2}\right)^{1/2} v_{th,f} \left|\nabla_\parallel\right| v_{\parallel,f} - \frac{v_{th,f}^2}{n_{0f}} \nabla_\parallel n_f - \frac{q_f v_{th,f}^2}{m_f \Omega_{cf} R n_{f0}} \frac{d n_{f0}}{dr} \left(\frac{1}{r}\frac{\partial \psi}{\partial \theta} - \frac{\partial \psi_0 / \partial r}{R^2 B} \frac{\partial \psi}{\partial \zeta}\right)$$

Gyrofluid energetic particle closure moments

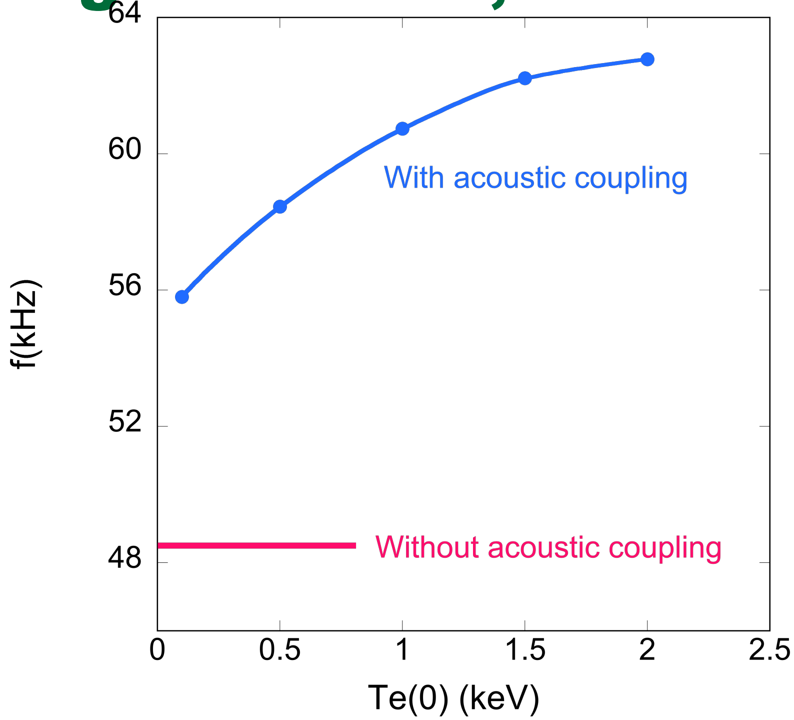
$$\frac{\partial n_{th}}{\partial t} = \frac{\vec{B} \times \vec{\nabla}\phi}{B^2} \bullet \vec{\nabla} n_{0,th} + \Gamma n_{0,th} \vec{\nabla} \bullet \left[\frac{\vec{B} \times \vec{\nabla}\phi}{B^2} + v_{\parallel,th} \hat{b}\right]$$

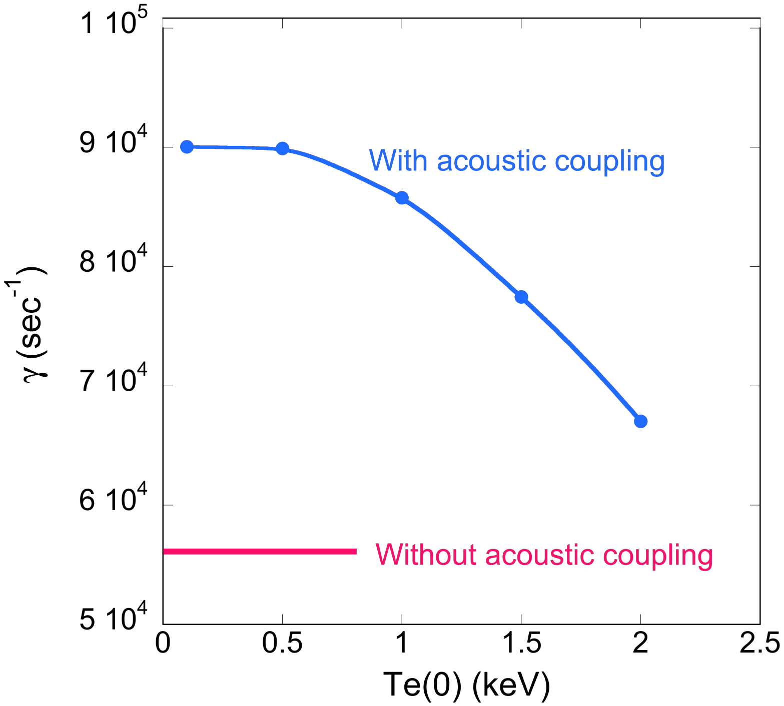
$$\frac{\Gamma}{c_s^2} \frac{\partial v_{\parallel,th}}{\partial t} = -\frac{1}{n_{0,th}} \left[\hat{b}_0 \bullet \vec{\nabla} n_{th} + \hat{b}_1 \bullet \vec{\nabla} n_{0,th}\right]$$

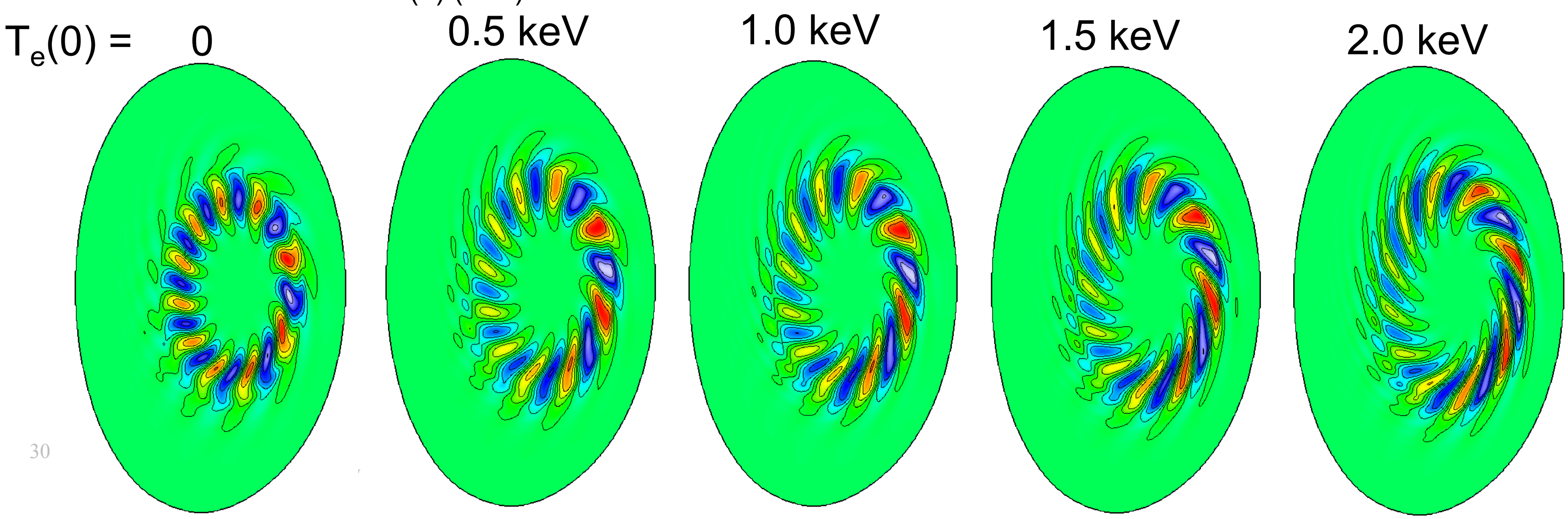
Acoustic-wave coupling

where $\Omega_d \propto \left(\hat{b} \times \vec{\nabla} B\right) \bullet \vec{\nabla}$, $X_{e,i} = \xi_{e,i} Z\left(\xi_{e,i}\right)$, $\xi_{e,i} = \omega_r / \sqrt{2} \left|\nabla_\parallel\right| v_{th,e,i}$

Acoustic coupling increases the frequency, lowers the growth rate, modifies mode structure:







More general EP distribution functions

- Structure of the highest order (closure) equation reflects the form of the fast ion distribution function
- Two-pole fit can be adapted to Maxwellian or slowing-down distribution function.

$$\frac{\partial n_f}{\partial t} = \frac{T_{f0}}{m_f\Omega_{cf}}\Omega_d\left(n_f\right) - n_{f0}\nabla_\parallel \mathrm{v}_{\parallel,\mathrm{f}} + \frac{q_f n_{f0}}{m_f\Omega_{cf}}\Omega_d\left(\phi\right) - \frac{q_f}{m_f\Omega_{cf}n_{f0}}\frac{dn_{f0}}{dr}\left(\frac{1}{r}\frac{\partial\phi}{\partial\theta} - \frac{\partial\psi_0/\partial r}{R^2B}\frac{\partial\phi}{\partial\zeta}\right)$$

$$\frac{\partial \mathrm{v}_{\parallel,f}}{\partial t} = \frac{T_{f0}}{m_f\Omega_{cf}}\Omega_d\left(\mathrm{v}_{\parallel,f}\right) - \sqrt{2}\mathbf{A}_1\mathrm{v}_{\mathrm{th,f}}\left|\nabla_\parallel\right|\mathrm{v}_{\parallel,f} - 2\mathbf{A}_0\frac{\mathrm{v}_{\mathrm{th,f}}^2}{n_{0f}}\nabla_\parallel n_f - \frac{2\mathbf{A}_0\mathrm{q_f}\mathrm{v}_{\mathrm{th,f}}^2}{m_f\Omega_{cf}Rn_{f0}}\frac{dn_{f0}}{dr}\left(\frac{1}{r}\frac{\partial\psi}{\partial\theta} - \frac{\partial\psi_0/\partial r}{R^2B}\frac{\partial\psi}{\partial\zeta}\right)$$

For $\mathrm{Z}(\xi) = Z^{(2)} = \dfrac{\xi - \mathbf{A}_1}{\mathbf{A}_0 + \mathbf{A}_1\xi - \xi^2}$ = two pole approximation to Z function, $\mathbf{A}_0 = \dfrac{1}{2}$, $\mathbf{A}_1 = \dfrac{-i\sqrt{\pi}}{2}$

For Maxwellian, $\mathrm{Z}_{Max.}(\xi) = \dfrac{1}{\sqrt{\pi}}\displaystyle\int du\frac{e^{-u^2}}{u-\xi}$, For a slowing-down distribution, $\mathrm{Z}_{SD}(\xi) = \dfrac{1}{\sqrt{\pi}}\displaystyle\int du\frac{1}{(u-\xi)(u^3+u_c^3)}$

SUMMARY: the TAEFL gyrofluid model of EP stability has recently been successfully applied to:

- Comparisons with DIII-D ECEI imaging of RSAE/TAE modes
- JET antenna AE damping measurements
- ITPA linear/nonlinear benchmark cases

Further improvements are underway in the areas of:

- Acoustic mode coupling
- More general EP distribution functions
- Initial value solution -> eigenvalue solution (allows study of sub-dominant modes)
- EP FLR effects