

# Program for finding the upper bound on unstable Alfvén mode induced fusion alpha transport losses

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Global GYRO simulation of DIII-D shot 121717 with full physics: plasma shape, profile variation and ExB shear, collisions, and electromagnetic effects. Contours of electron density fluctuations. Shot has inverted q-profile for which  $q_{\min}$  is slightly less than 2.

TTF Meeting San Diego, CA

April 6-9, 2011

# ABSTRACT

Previous GYRO simulations have shown that reactor-scale fusion alpha transport from thermal plasma instabilities like ITG/TEM is likely to be insignificant [1]. Recent simulations of fixed gradient alpha transport induced by alpha driven local (very low-k but high-n) Alfvénic TAE/EPM turbulence embedded in very strong (moderate-k) ITG/TEM turbulence showed nonlinearly saturated states can exist at energetic particle (EP) pressures up to perhaps twice the TAE/EPM stability threshold with quasilinear (and likely intermittent) relaxation of the driving EP pressure gradient appearing at stronger EP drive[2]. However even the pre-relaxation level of EP transport is not significantly higher than the ITG/TEM induced level below the local linear TAE/EPM threshold EP pressure gradient. Since the global linear stability threshold will always exceed that for the local,  $-dP_{\alpha}^{loc-lin}/dr$  should provide an *upper bound on unstable Alfvén mode induced fusion alpha transport losses*: Given the MHD equilibrium and thermal plasma profiles, it is straightforward to calculate the local fusion energy deposition rate  $Q_{\alpha}(r)$  [MeV/sec/m<sup>3</sup>] from the classical slowing-down fusion alpha density profile  $n_{\alpha}^{class}(r)$  and effective alpha temperature profile  $T_{\alpha}^{class}(r)$  (which has a very weak gradient). Since  $-T_{\alpha}^{class} dn_{\alpha}^{class}/dr$  will be less than  $-dP_{\alpha}^{loc-lin}/dr$  beyond some outer radius,  $n_{\alpha}(r) = n_{\alpha}^{class}(r)$  for  $r > r_b$ . Integrating  $-dn_{\alpha}(r)/dr = [-dn_{\alpha}^{loc-lin}(r)/dr, -dn_{\alpha}^{class}(r)/dr]_{min}$  inward from  $r = r_b$ , the maximum  $n_{\alpha}(r)$  will be less than  $n_{\alpha}^{class}(r)$ . Since the effective alpha temperature should not deviate from  $T_{\alpha}^{class}(r)$ , the minimum fusion energy deposition rate to the thermal plasma is  $[n_{\alpha}(r)/n_{\alpha}^{class}(r)]Q_{\alpha}(r)$  from which an upper bound on alpha transport losses can be inferred. Physically accurate gyrokinetic profiles from TGLF projected ITER plasma profiles are easily obtained [3].

- [1] C. Estrada-Mila, J. Candy, and R.E. Waltz, "Turbulent Transport of Alpha Particles and Helium Ash in Reactor Plasmas," Phys. Plasmas **13**, 112303 (2006).  
[2] E. M. Bass and R.E. Waltz, "Gyrokinetic simulations of mesoscale energetic particle-driven Alfvénic turbulent transport embedded in microturbulence", Phys. Plasmas **17**, 112319 (2010)  
[3] J.E. Kinsey, G.M. Staebler, J. Candy, R.E. Waltz, and R.V. Budny, "ITER predictions using the GYRO verified and Experimentally verified TGLF Transport Model" submitted to Nuclear Fusion 2010.

\*Supported by the US Department of Energy under DE-FC02-04ER54698.

## Upper bound on fusion alpha confinement loss from Alfvén mode local linear threshold gradients: the upper bound may actually be small

- The “passive” transport or redistribution of the classical slowing down fusion alpha density from ITG/TEM turbulence is easily calculated directly from local gyrokinetic simulations (or a fitted TGLF model) and is not likely significant.
- However the transport from unstable (TAE/EPM) Alfvén modes will be so strong that the driving alpha pressure gradient can not significantly exceed the local linear threshold. The transport will be highly intermittent and difficult to directly simulate a time average:

• Better to look for an upper bound on the alpha energy transport loss or equivalently (and more importantly) a lower bound on the fusion energy deposition [MW/m<sup>3</sup>] to the thermal plasma:

$$Q^{heating}(r) = [n_{\alpha}(r)/n_{\alpha}^{class}(r)]Q^{fusion}(r) \quad Q^{fusion}(r) \sim E_{\alpha} C n_D(r) n_T(r) T_i^2$$

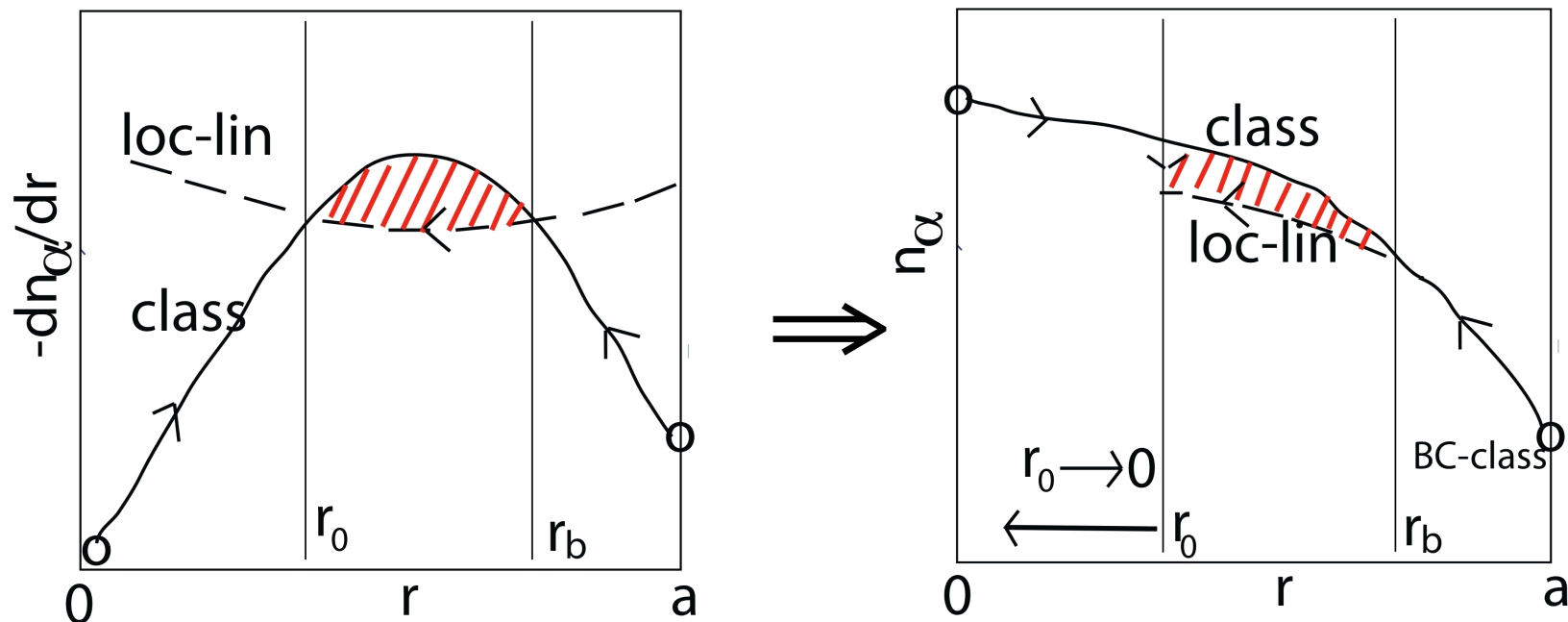
Key assumption: any turbulent processes don't distort the classical slowing down distribution

$$Power_{\alpha}^{tran}(r) = \int_0^r V'(r') dr' \{1 - [n_{\alpha}(r')/n_{\alpha}^{class}(r')]\} Q^{heating}(r')$$

$n_{\alpha}^{class}(r)$  is well known and we propose to find the minimum  $n_{\alpha}(r)$  from  $-P_{\alpha}^{local-lin}(r)$   
the local linear pressure gradient threshold

# Finding the minimum $n_\alpha(r)$ from the local linear threshold gradients

- The driving pressure gradient is mostly from the alpha density since the logarithmic temperature gradient is so weak:  $-dP_\alpha/dr = T_\alpha(-dn_\alpha/dr + n_\alpha 1/L_{T_\alpha}) \sim T_\alpha(-dn_\alpha/dr)$  (it is easy to correct for the neglected  $1/L_{T_\alpha} \ll 1/L_{n_\alpha}$ )
- If (as expected)  $-dP_\alpha/dr < -dP_\alpha^{loc-lin}/dr$   $r > r_b$  we can integrate the marginal profile inward from  $n_\alpha(r_b) = n_\alpha^{class}(r_b)$  with  $-dn_\alpha/dr = [-dn_\alpha^{loc-lin}/dr, -dn_\alpha^{class}/dr]_{min}$



- It's clear that  $r_0$  could extend to the origin, but the BC is on the  $n_\alpha^{class}$  profile.



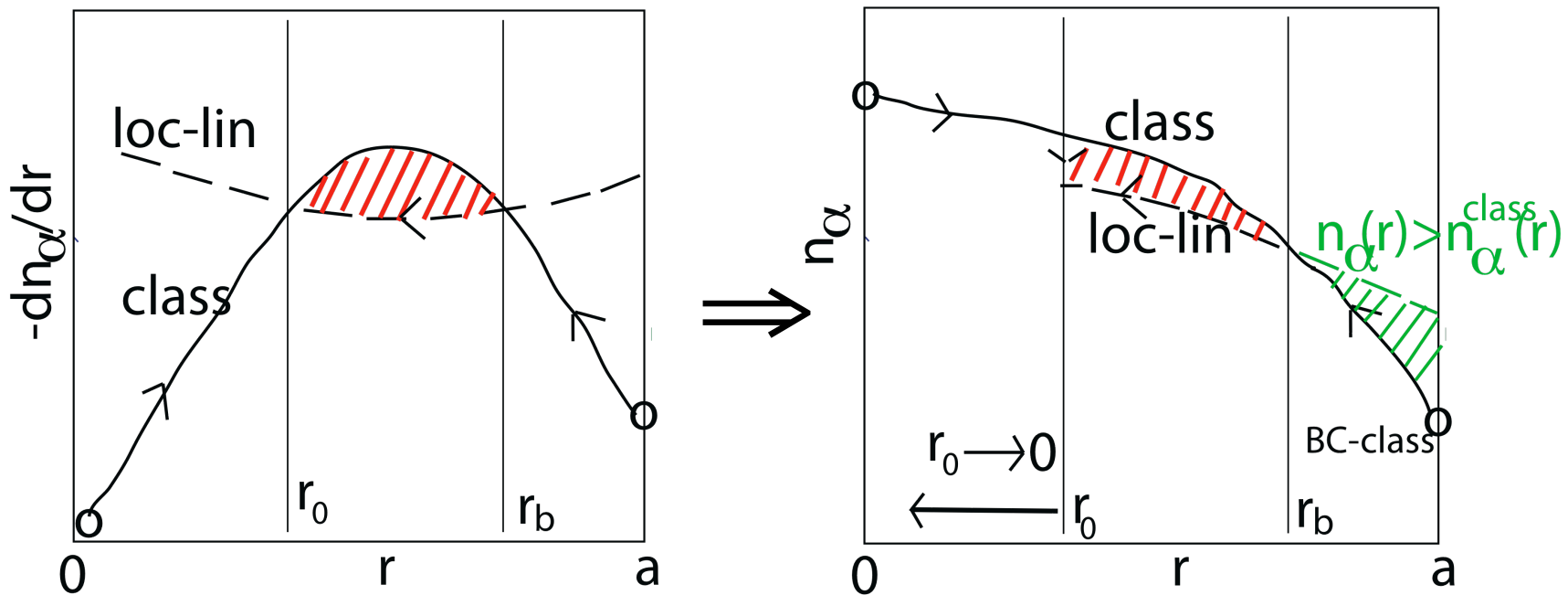
# Finding the minimum $n_\alpha(r)$ from the local linear threshold gradients

## Cont'd

- In the case  $-dP_\alpha/dr < -dP_\alpha^{loc-lin}/dr$   $r > r_b$ , we have ignored the likely possibility that some of the out going alphas will redeposit at least some of their energy outside  $r > r_b$  which means that the effective  $n_\alpha(r > r_b) > n_\alpha^{class}(r > r_b)$  so at the outer edge

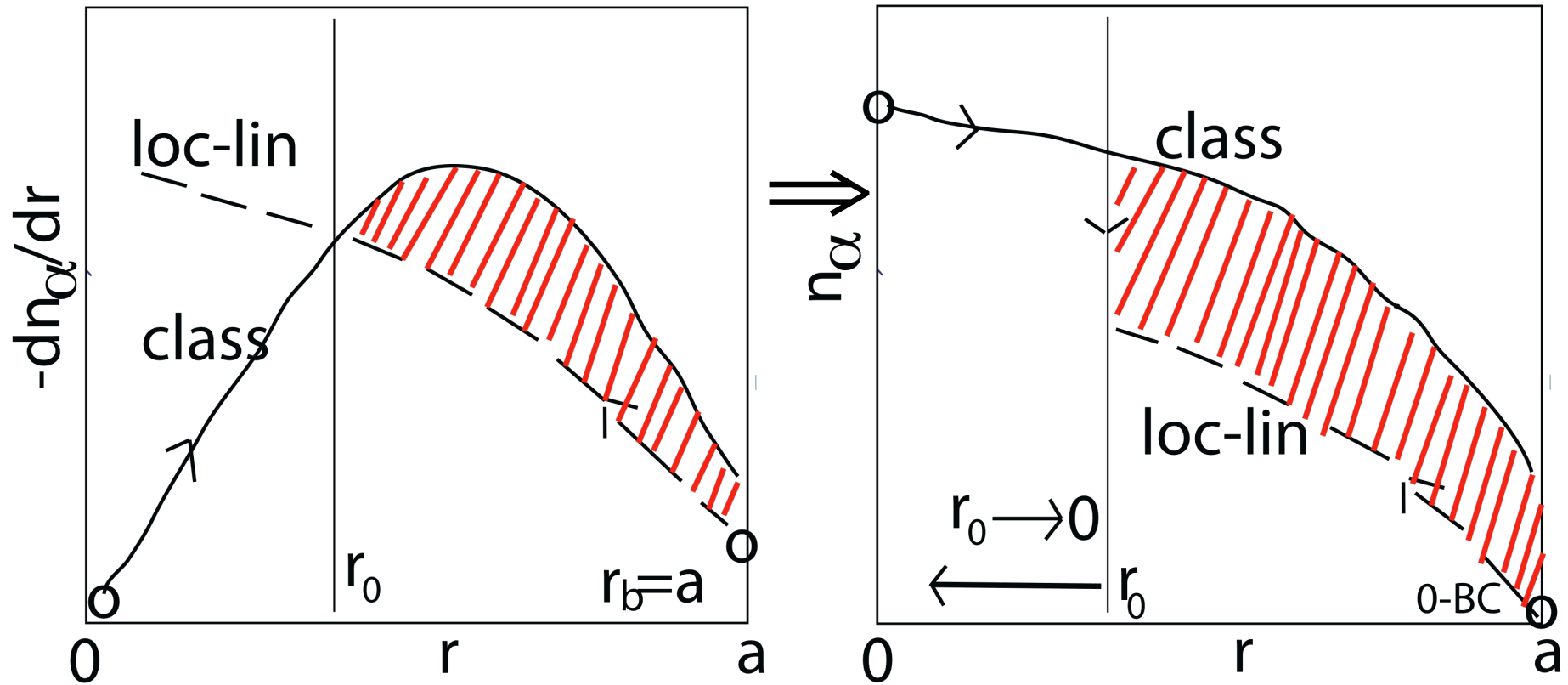
$$Q^{heating}_\alpha(r) = [n_\alpha(r)/n_\alpha^{class}(r)]Q^{fusion}_\alpha(r) > Q^{fusion}_\alpha(r)$$

..but this is not the worst case minimal plasma heating we seek



# Finding the minimum $n_\alpha(r)$ from the local linear threshold gradients Cont'd

- If  $-dP_\alpha^{loc-lin}/dr < -dP_\alpha^{class}/dr @r=a$  we must start the integration from 0-BC  $n_\alpha$  and the outer  $n_\alpha(r)$  will be much less than  $n_\alpha^{class}(r)$  and likely 0 in the worst case



- However since  $Q^{fusion}(r) \sim E_\alpha C n_D(r) n_T(r) T_i^2$  is highly peaked to central core, the increased loss of plasma heating  $Q^{heating}(r) = [n_\alpha(r)/n_\alpha^{class}(r)] Q^{fusion}(r)$  is not much bigger.

## Same minimum $n_\alpha(r)$ profiles should be found from actual solutions of “stiff (critical gradient) local radial diffusion” model

$$\frac{\partial f_\alpha(r, v)}{\partial t} = v_s v^{-3} \frac{\partial}{\partial v} [(v^3 + v_c^3) f_s] + S_0 / 4\pi v_c^3 \delta(v_c - v) + V^{-1}(r) \frac{\partial}{\partial r} [V(r) D(r) \frac{\partial f_\alpha}{\partial r}] + [?] v_d \frac{\partial}{\partial \lambda} (1 - \lambda^2) \frac{\partial f_\alpha}{\partial \lambda}$$

$$D(r) = D_{ITG-TEM} + D_{TAE-EPM}^{crit} (R/L_n^\alpha - R/L_n^{loc-lin})^{1/2} \quad D_{TAE-EPM}^{crit} \gg D_{ITG-TEM}$$

$$f_\alpha(r, v_{th}) = 0 \quad S_0(r) \sim C n_D(r) n_T(r) T_i^2(r)$$

- The small  $D_{ITG-TEM}$  just smoothes out the “sharp corners” on the linear threshold critical gradient profiles from  $D_{TAE-EPM}^{crit}$
- Note that  $D(r)$  is NOT a function of  $v$  (by assumption) so there should be no distortion of the slowing down distribution, i.e. the  $v$ - $r$  problem is separable  $f_\alpha(r, v) = n_\alpha(r) F_\alpha^{slow}$
- The TRANSP fast particle deposition code solves this kind of  $v$ - $r$  FP equation with  $D(r)$  as an empirically fit local- $r$  model to describe exp. TAE-EPM deprecated neutral beam fast particle density profiles.

## Why is this the “worst case” maximum loss & minimal plasma heating?

- The zonal flows from very strong ITG-TEM turbulence can nonlinearly saturate the low-level TAE/EPM alpha transport...but this only pushes the  $-dn_{\alpha}^{\text{crit}}/dr$  (quasilinear profile relaxation critical gradient or effective  $-dn_{\alpha}^{\text{loc-lin}}/dr$ ) higher. [Bass PoP2010]

- The local linear threshold  $-dn_{\alpha}^{\text{crit}}/dr$  from global modes is always higher than for local modes. Global linear modes are just “phased up” toroidally coupled local modes which have profile shear stabilization...and they take longer to form. Their growth rate is always lower than the maximum local mode rate. Their quasilinear “transport footprint” will not likely (?) extend much beyond that of the combined local modes. As rho-star in ITER gets smaller important n-mode numbers get 5x higher and “global” modes are more localized.

[How global modes are related to local modes being investigated...Bass TTF2011]

- When low-n (i.e. very low  $ky \cdot \rho_s$ ) local TAE-EPM modes get to  $-dn_{\alpha}^{\text{crit}}/dr$ , they act like n=0 zonal flows stabilizing the much higher-n ITG-TEM modes driving thermal plasma transport. Thus the thermal plasma energy confinement is slightly improved making the effective plasma heating larger than the minimum



## Why is this the “worst case” maximum loss & minimal plasma heating? Cont'd

- The TAE-EPM modes are velocity space resonance drive....maybe the quasilinear profile relaxation is not in r- space (i.e.  $-dn_{\alpha}^{\text{loc-lin}}/dr$ ) but at the local v-space resonance if collisions are not rapid enough to maintain the slowing-down v-distribution.... this only makes effective  $-dn_{\alpha}^{\text{loc-lin}}/dr$  higher (and local v-space distortion will not change heating rate much) [J. Lang, G.Y. Fu, Y.Chen PoP 2010]

- As noted earlier, we really should be considering the critical local linear pressure gradient  $\chi -dP_{\alpha}/dr = T_{\alpha}(-dn_{\alpha}/dr + n_{\alpha}1/L_T) \sim T_{\alpha}(-dn_{\alpha}/dr)$  not just the density gradient...however including the weak log temperature gradient is easily accounted for.

...similarly some recent algebraic analysis of the gyrokinetic dispersion relation seem to suggest that no matter how large  $-dP_{\alpha}/dr$  gets, there may be a minimal logarithmic pressure gradient  $-RdP_{\alpha}/dr/P_{\alpha}$  (hasn't been confirmed by GYRO linear analysis)....this again is an easily corrected (since we would use GYRO to determine  $-dn_{\alpha}^{\text{loc-lin}}/dr$  which is also dependent on the local  $n_{\alpha}(r)$  and  $T_{\alpha}(r)$  which doesn't change the principle but what is  $-dn_{\alpha}^{\text{loc-lin}}/dr$