

# Finite Pressure Effects on Momentum Transport in a Toroidal Plasma

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MST

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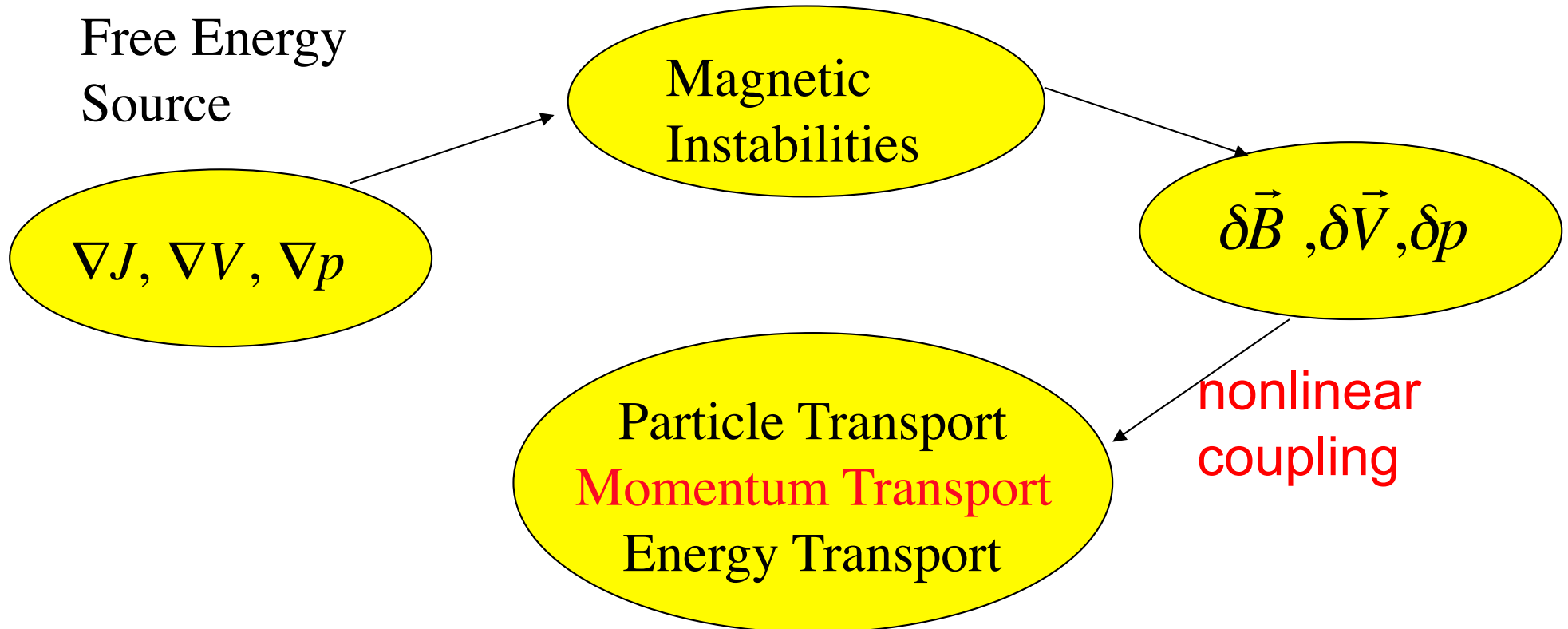
<sup>4</sup> *Laboratory for Laser Energetic*



(TTF 2011, San Diego, CA. April, 2011)

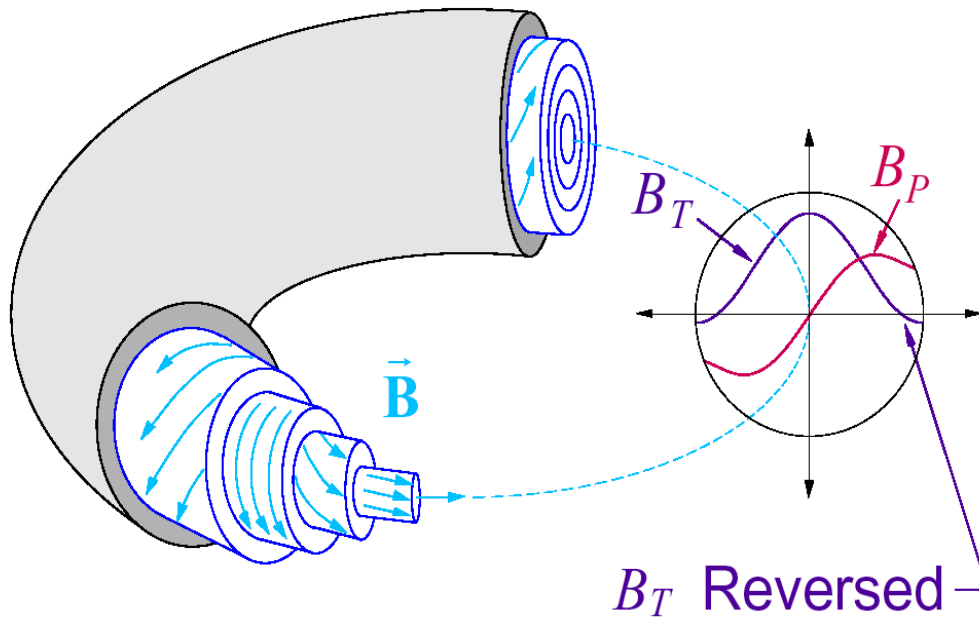


# Magnetic Fluctuations Play an Important Role in Momentum Transport

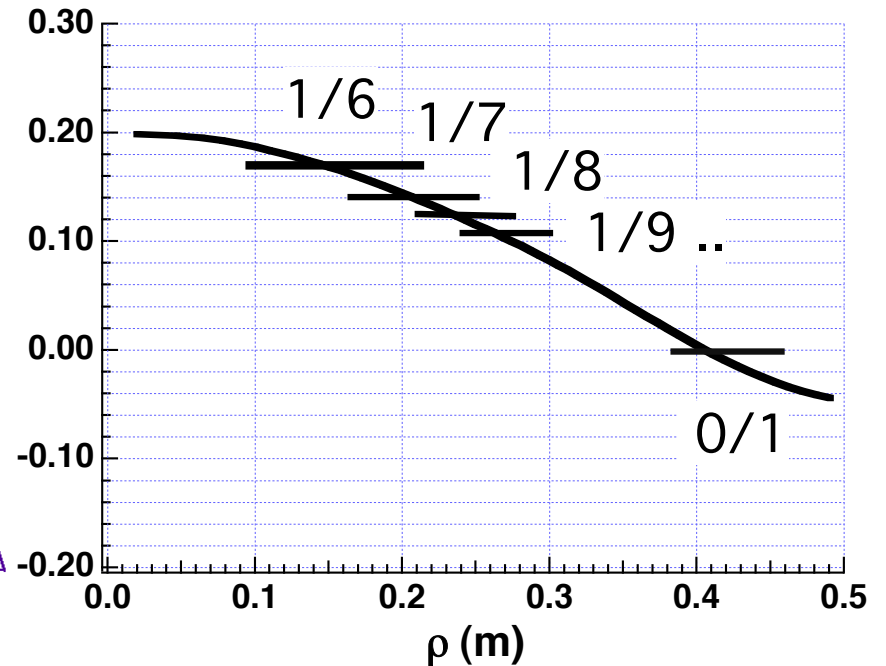


*In this talk, we focus on the interaction between tearing instability driven magnetic and pressure fluctuations, resulting in the momentum transport in MST.*

# Madison Symmetric Torus -Reversed Field Pinch



$$q(r) = \frac{r B_T}{R B_p}$$



$\beta \sim 6 - 7\%$

*Multiple Overlapping Tearing Modes* →

*Stochastic Magnetic Field on MST*

# Outline

Measurements of

Fluctuation-Induced Momentum Flux:

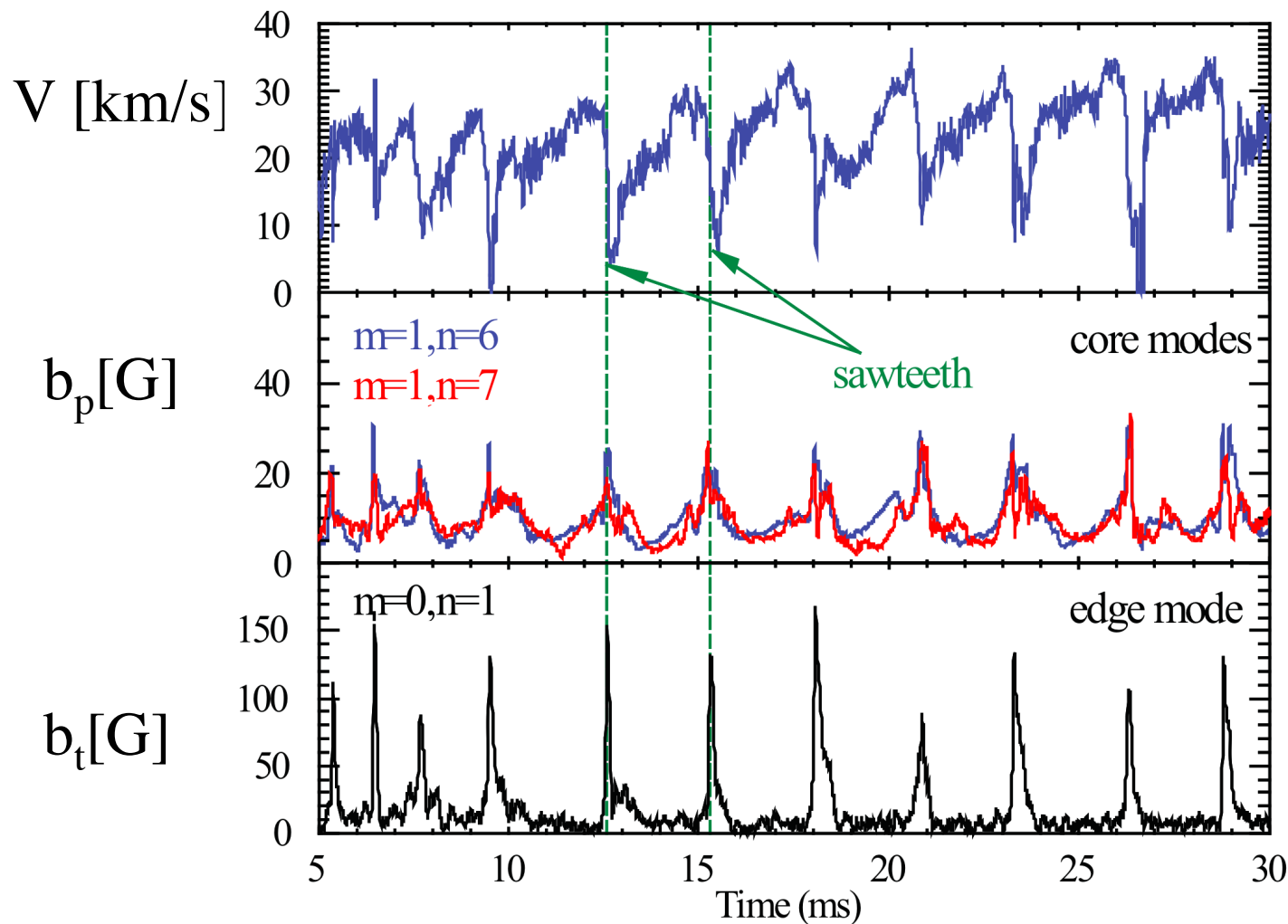
## (1) MHD effects on Momentum Transport

- Maxwell stress and Reynolds stress

## (2) Kinetic (pressure) effects on Momentum Transport

- Density fluctuations in a stochastic magnetic field

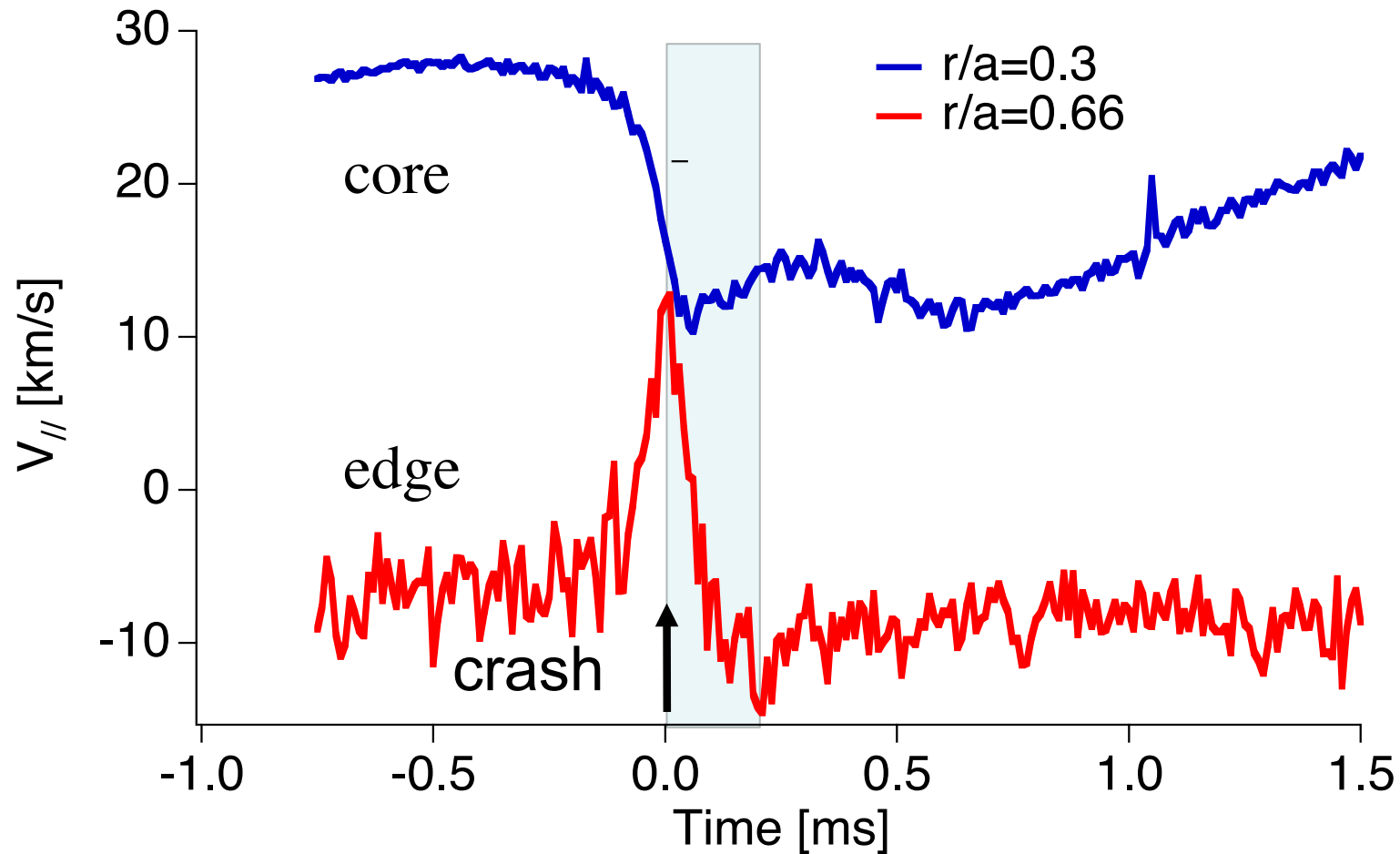
# Momentum Transport and Fluctuations in the MST



Toroidal flow in  
the core

*Momentum transport is strongly associated with magnetic fluctuation*

# Momentum Relaxation during a Sawtooth Event



*There is a coupling between the edge and the core flow*

*Parallel momentum is redistributed within the time scale ( $\sim 100 \mu\text{s}$ )  
much faster than classical ion-ion viscous transport time*

# Momentum transport by Fluctuations in a Torus

$$\rho \left( \frac{\partial}{\partial t} \vec{V} + \vec{V} \cdot \nabla \vec{V} \right) = \vec{J} \times \vec{B} - \nabla \cdot \vec{P}$$

Parallel momentum components

$$\rho \frac{\partial}{\partial t} \langle V_{\parallel} \rangle = \langle \delta \vec{J} \times \delta \vec{B} \rangle_{\parallel} - \rho \langle \delta \vec{V} \cdot \nabla \delta \vec{V} \rangle_{\parallel} - \langle \nabla \cdot \vec{P} \rangle_{\parallel}$$

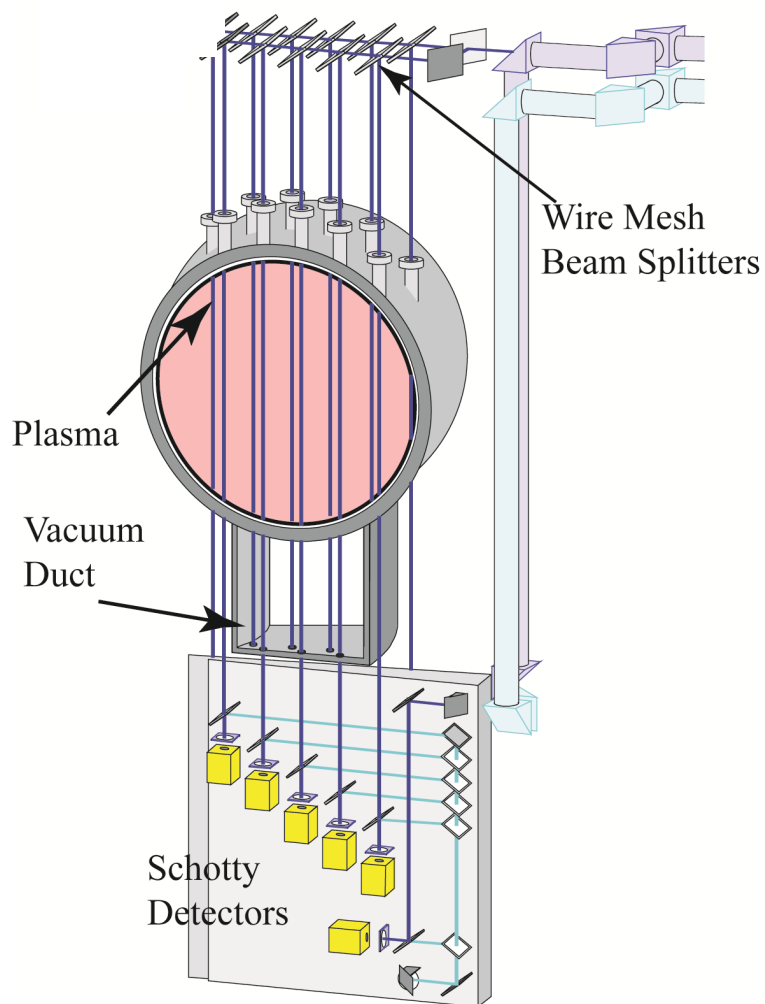
Flow  
Change

Maxwell  
Stress

Reynolds  
stress

Finite  
pressure  
effect

# Laser-based FIR system is used to measure density and magnetic fluctuations



## Interferometer:

$$\phi_{\text{int}}(x) \sim \int n_e dz \quad \Rightarrow n_e, \tilde{n}_e$$

## Differential interferometer ( $\Delta x$ 1mm):

$$\partial \phi_{\text{int}} / \partial x \quad \Rightarrow \nabla n_e, \nabla \tilde{n}_e$$

## Polarimeter:

$$\psi_{\text{pol}}(x) \sim \int n_e B_z dz$$

$$\delta \vec{B}, \rightarrow \delta \vec{J} \times \delta \vec{B}$$

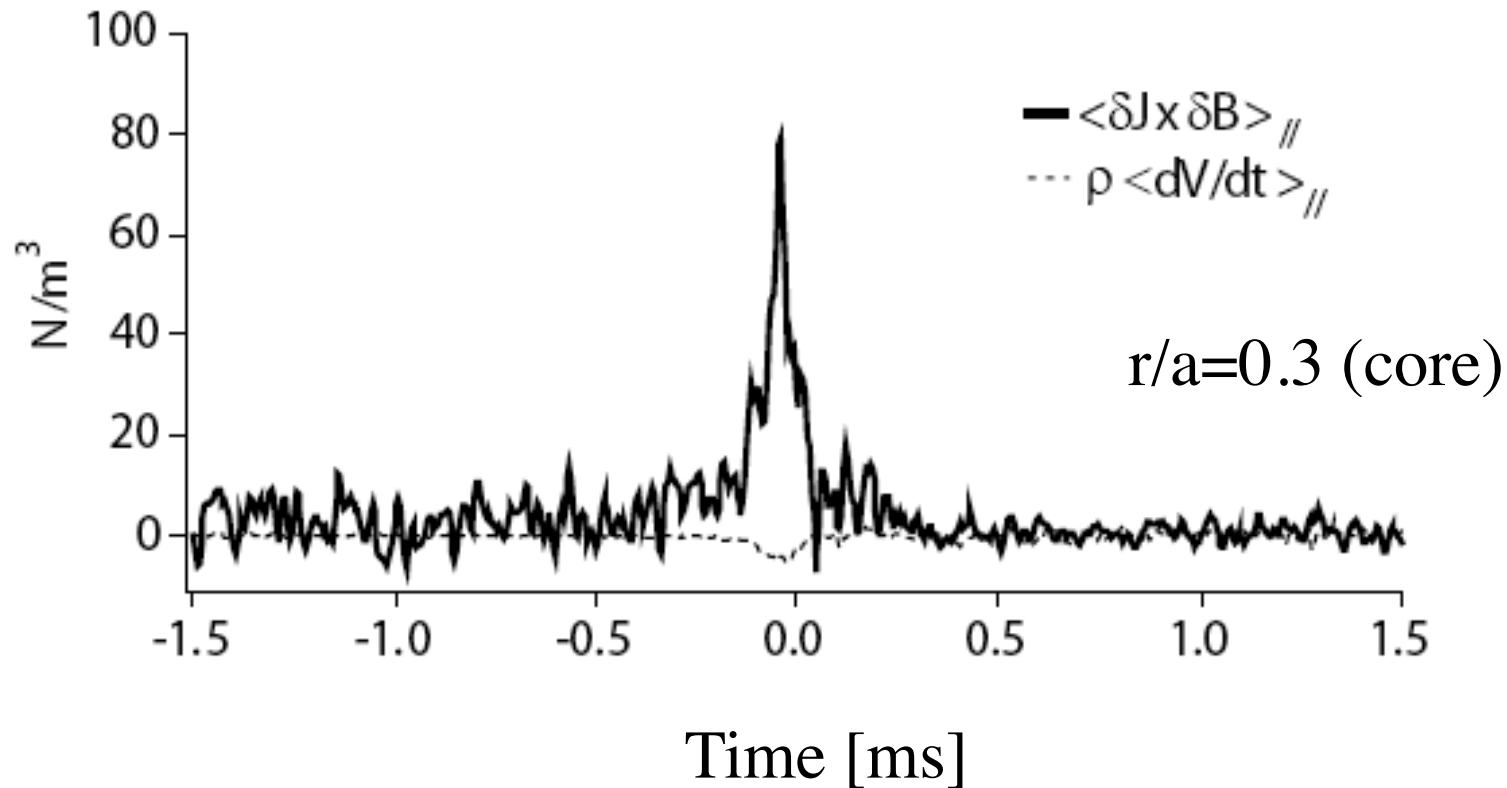
$$\rightarrow (\nabla \times \delta \vec{B}) \times \delta \vec{B}$$

## 32 magnetic coils: $\Rightarrow (m, n)$

**11 chords,  $\Delta x \sim 8$  cm, phase  $\sim 0.05^\circ$ ,  
time response  $\sim 1\mu\text{s}$**



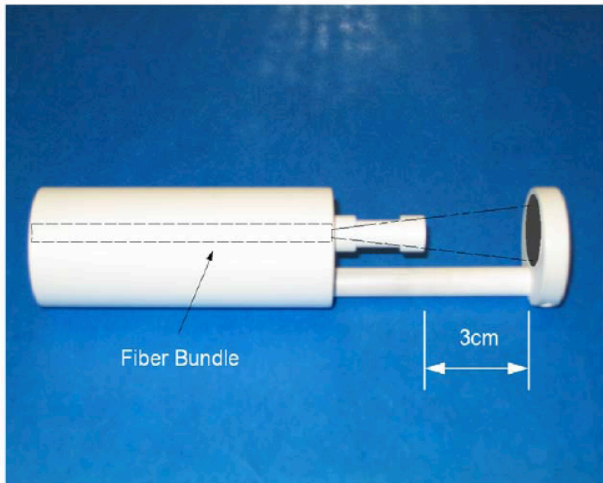
# In plasma core, measured Maxwell Stress is much larger than momentum change



$$\underbrace{\rho \frac{\partial}{\partial t} \langle V_{\parallel} \rangle}_{\text{---}} = \underbrace{\langle \delta \vec{J} \times \delta \vec{B} \rangle_{\parallel}}_{\text{---}} - \rho \langle \delta \vec{V} \cdot \nabla \delta \vec{V} \rangle_{\parallel} - \langle \nabla \cdot \vec{P} \rangle_{\parallel}$$

*Reynolds stress is expected to offset huge Maxwell Stress*

# Various Probes are used to measure Maxwell and Reynolds Stress



*Mach Probe +IDS (optical probe)  
measure velocity fluctuations*

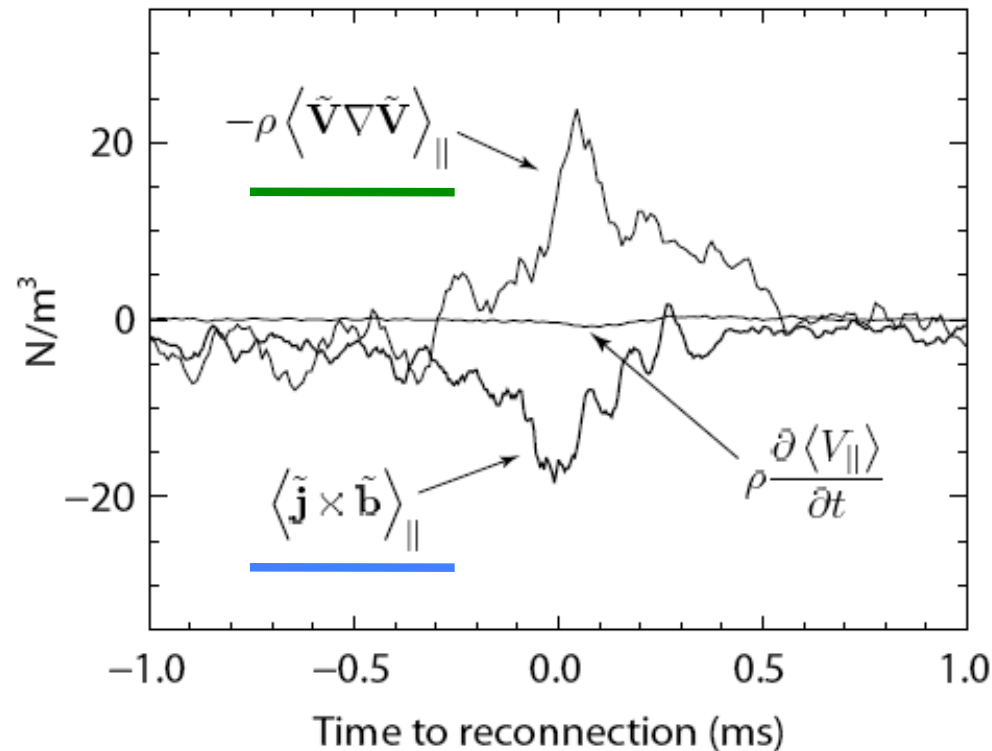
*(Reynolds Stress)*



*Torque probe measures all  
3 comp. of  $j$  and  $B$*

*(Maxwell Stress)*

# In plasma edge, both Maxwell and Reynolds stress are measured



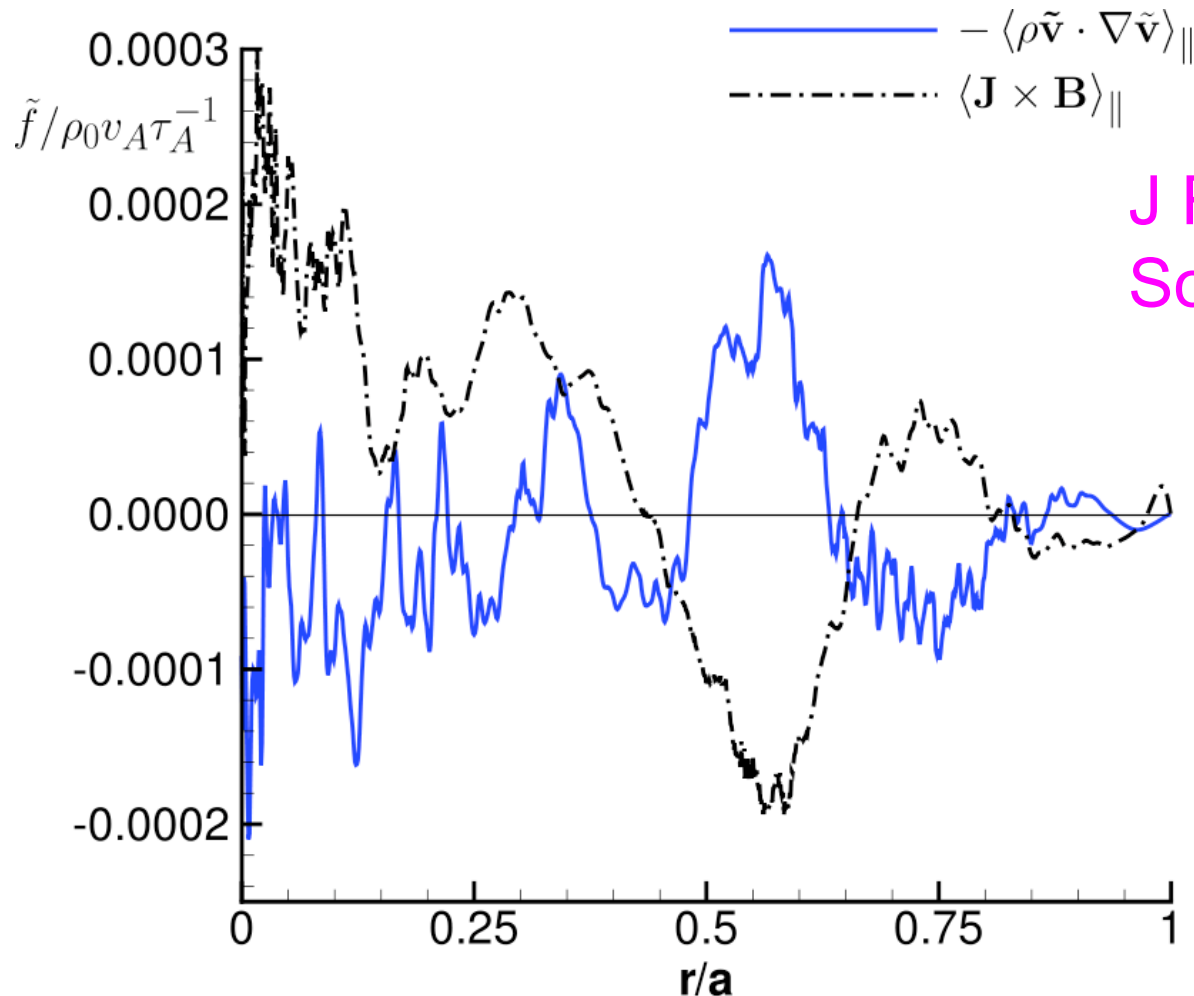
$r/a=0.85$  (edge)

*Kuritsyn, et al PoP 2009*

$$\rho \frac{\partial}{\partial t} \langle V_{||} \rangle = \underbrace{\langle \delta \vec{J} \times \delta \vec{B} \rangle_{||}}_{\text{Maxwell}} - \underbrace{\rho \langle \delta \vec{V} \cdot \nabla \delta \vec{V} \rangle_{||}}_{\text{Reynolds}} - \langle \nabla \cdot \vec{P} \rangle_{||}$$

*Maxwell and Reynolds stresses are in opposite directions and largely offset*

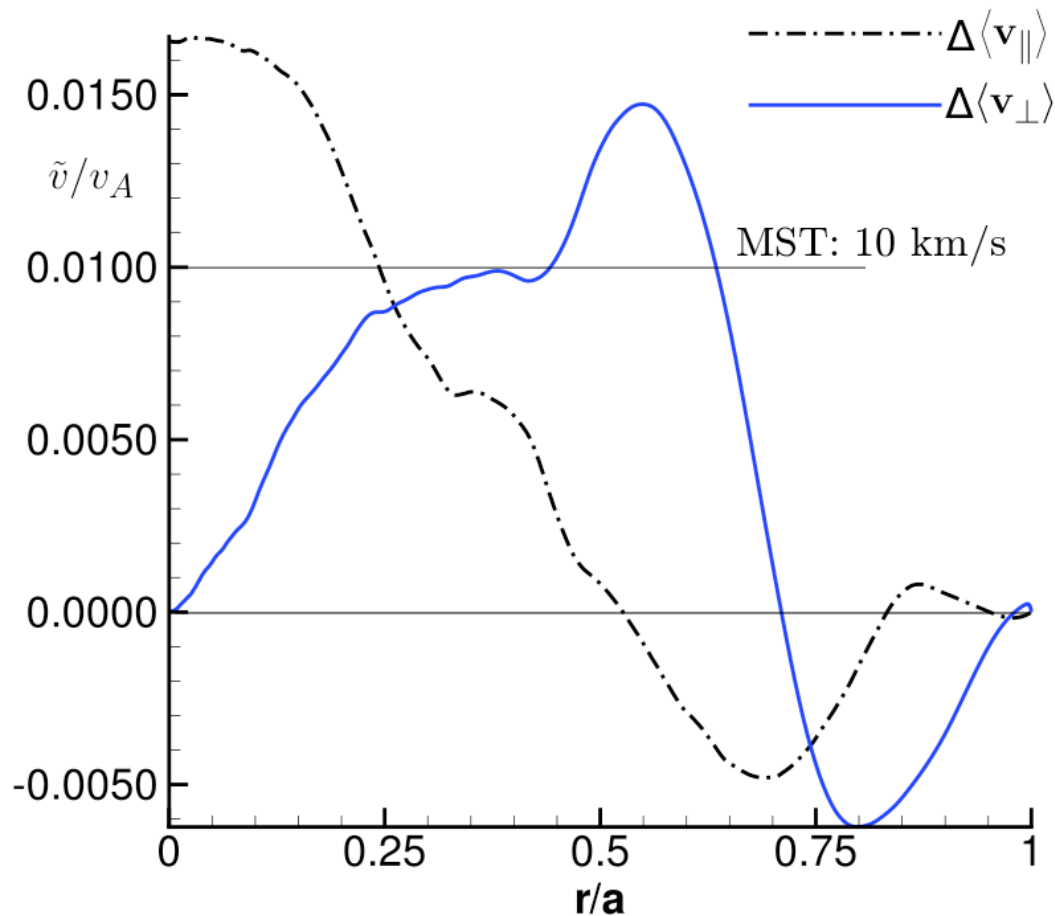
# Two-fluid NIMROD simulation for RFP has been carried out



J R King, C R  
Sovinec, V V Mirnov

*Reynolds and Maxwell stresses from fluctuations are large and tend to balance each other, similar to observations in MST*

# Parallel and Perpendicular Flows Appear as a Natural Consequence of Two-fluid Relaxation



**Two-fluid  
NIMROD  
simulation**

*Simulations produce a flow modification quantitatively similar to measurements on MST*

# Kinetic effects on Momentum beyond MHD

$$\rho \frac{\partial}{\partial t} \langle V_{\parallel} \rangle = \langle \delta \vec{J} \times \delta \vec{B} \rangle_{\parallel} - \rho \langle \delta \vec{V} \cdot \nabla \delta \vec{V} \rangle_{\parallel} - \langle \nabla \cdot \vec{P} \rangle_{\parallel}$$

$$\langle \nabla \cdot \vec{P} \rangle_{\parallel} = \nabla \cdot \langle p_{\parallel} \vec{b} \rangle - \langle p_{\perp} \nabla \cdot \vec{b} \rangle$$

$$\vec{P} = p_{\perp} \vec{I} + (p_{\parallel} - p_{\perp}) \vec{b} \vec{b}$$

$$\vec{b} = \frac{\vec{B}}{B}$$

$$Flux = \langle p_{\parallel} \vec{b} \rangle \cdot \vec{e}_r = \frac{\langle \delta p_{\parallel} \delta b_r \rangle}{B}$$

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$$\rho \frac{\partial}{\partial t} \langle V_{\parallel} \rangle = \langle \delta \vec{J} \times \delta \vec{B} \rangle_{\parallel} - \rho \langle \delta \vec{V} \cdot \nabla \delta \vec{V} \rangle_{\parallel} - \nabla \cdot \frac{\langle \delta p_{\parallel} \delta b_r \rangle}{B} \vec{e}_r + \dots$$

*Kinetic Stress*

# Parallel Pressure Fluctuation-Induced Momentum Transport

$$\rho \frac{\partial}{\partial t} \langle V_{\parallel} \rangle = \langle \delta \vec{J} \times \delta \vec{B} \rangle_{\parallel} - \rho \langle \delta \vec{V} \cdot \nabla \delta \vec{V} \rangle_{\parallel} - \nabla \cdot \frac{\langle \delta p_{\parallel} \delta b_r \rangle}{B} \vec{e}_r + \dots$$

$$Flux = \frac{\langle \delta p_{\parallel} \delta b_r \rangle}{B} = T_{\parallel} \frac{\langle \delta n \delta b_r \rangle}{B} + n \frac{\langle \delta T_{\parallel} \delta b_r \rangle}{B}$$



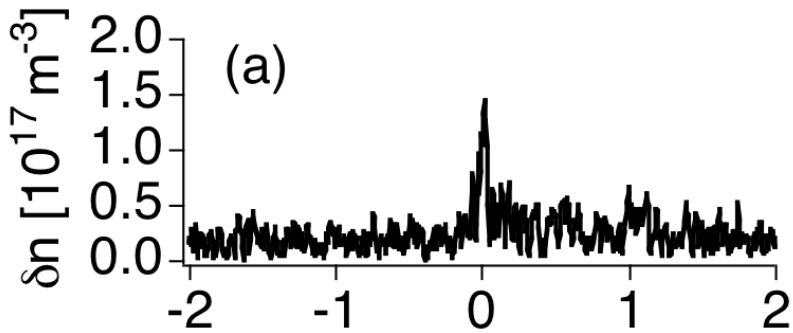
Density correlated with magnetic fluctuation

Temperature correlated with magnetic fluctuation

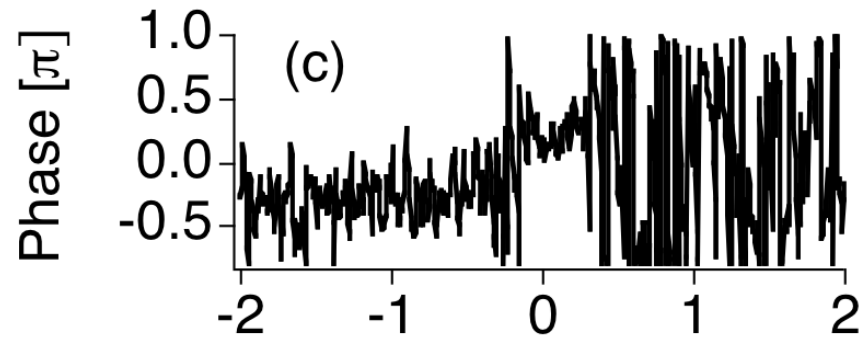
*Here, we focus on the role of density fluctuation on momentum transport*

# Measurement of Density Fluctuation-induced Momentum Flux

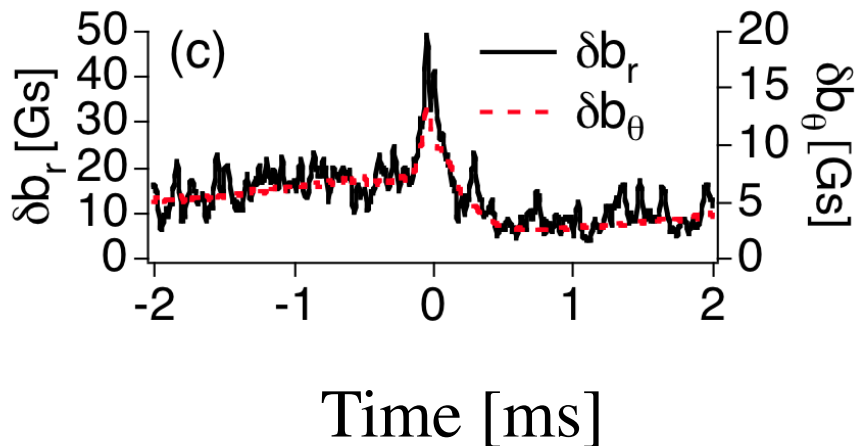
## Density Fluctuation



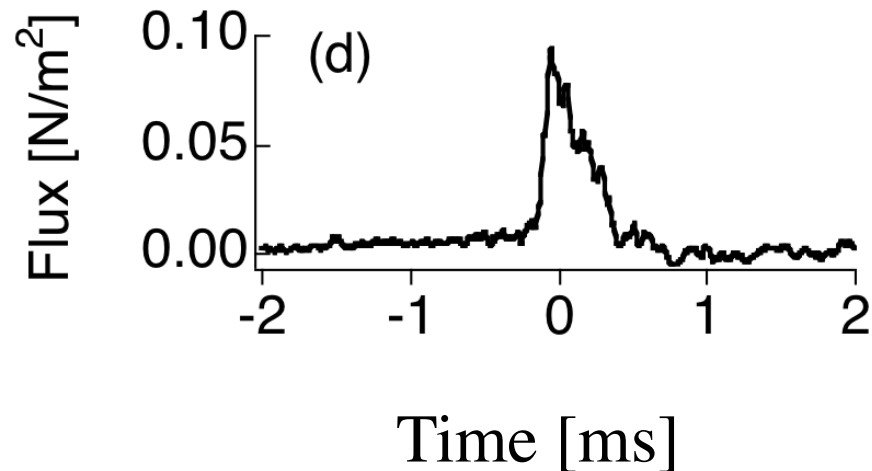
## Phase between $\delta n$ and $\delta b$



## Magnetic Fluctuation



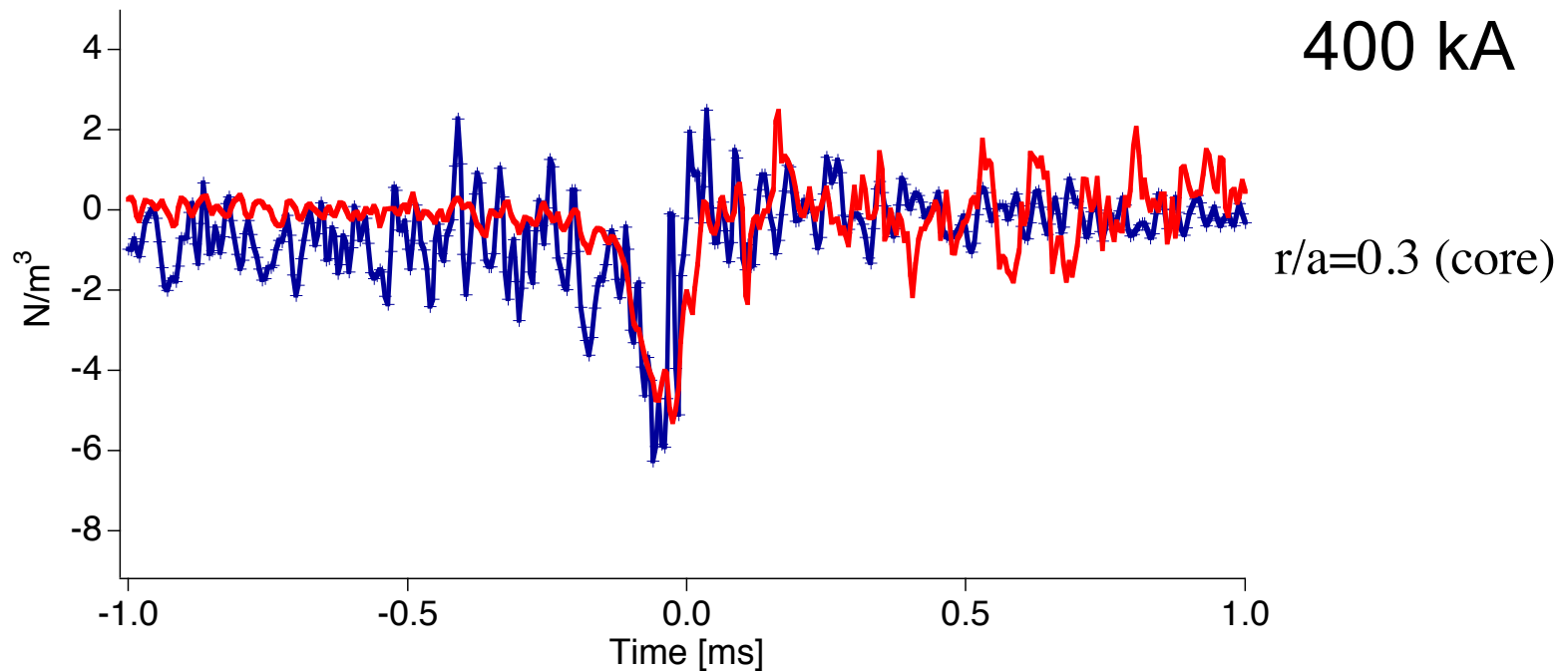
## Momentum Flux



*Momentum flux surges at sawtooth crash*



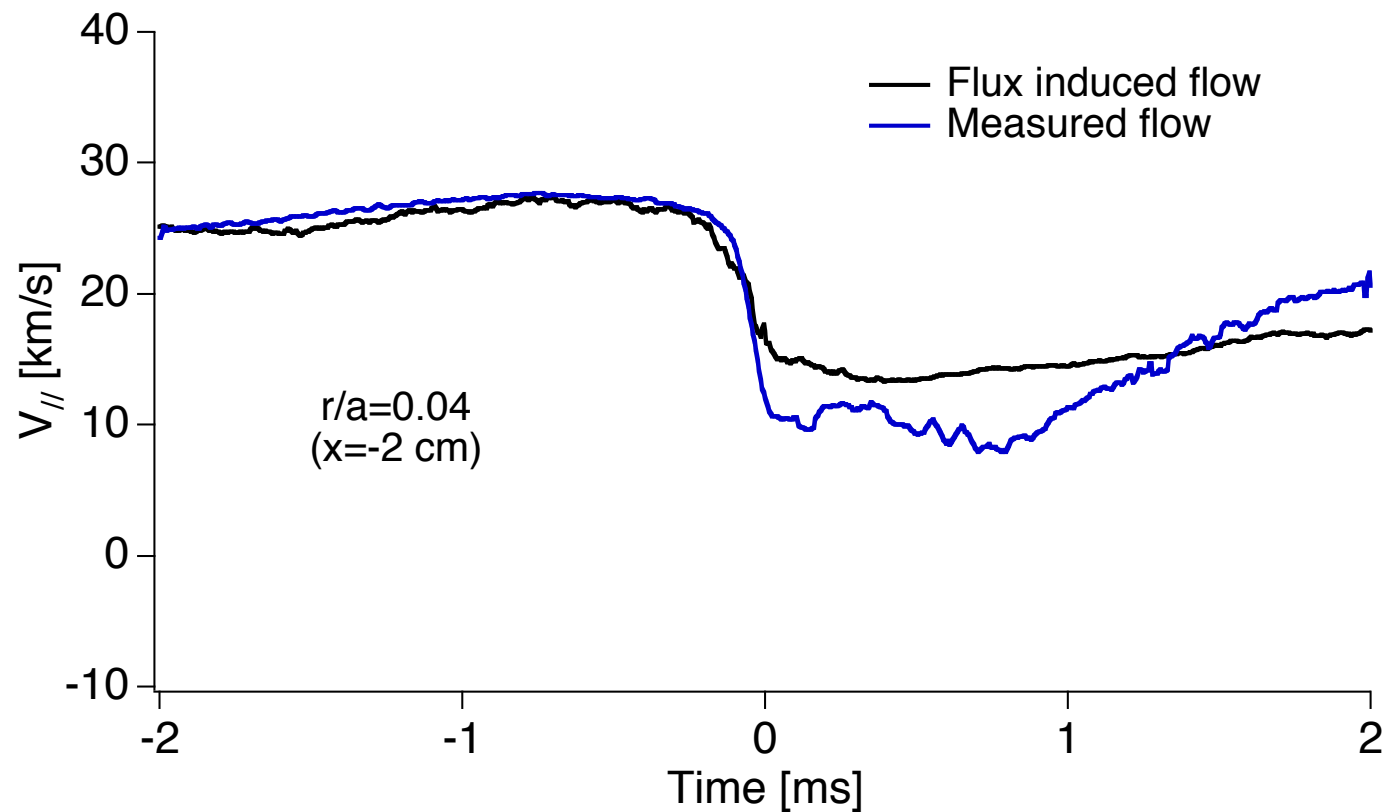
# Density fluctuation-induced force is comparable to momentum change



$$\rho \frac{\partial}{\partial t} \langle V_{\parallel} \rangle = \langle \delta \vec{J} \times \delta \vec{B} \rangle_{\parallel} - \rho \langle \delta \vec{V} \cdot \nabla \delta \vec{V} \rangle_{\parallel} - \nabla \cdot \frac{\langle \delta p_{\parallel} \delta b_r \rangle}{B} \vec{e}_r$$

# Comparison between Density Fluctuation-induced Flow and Plasma Flow

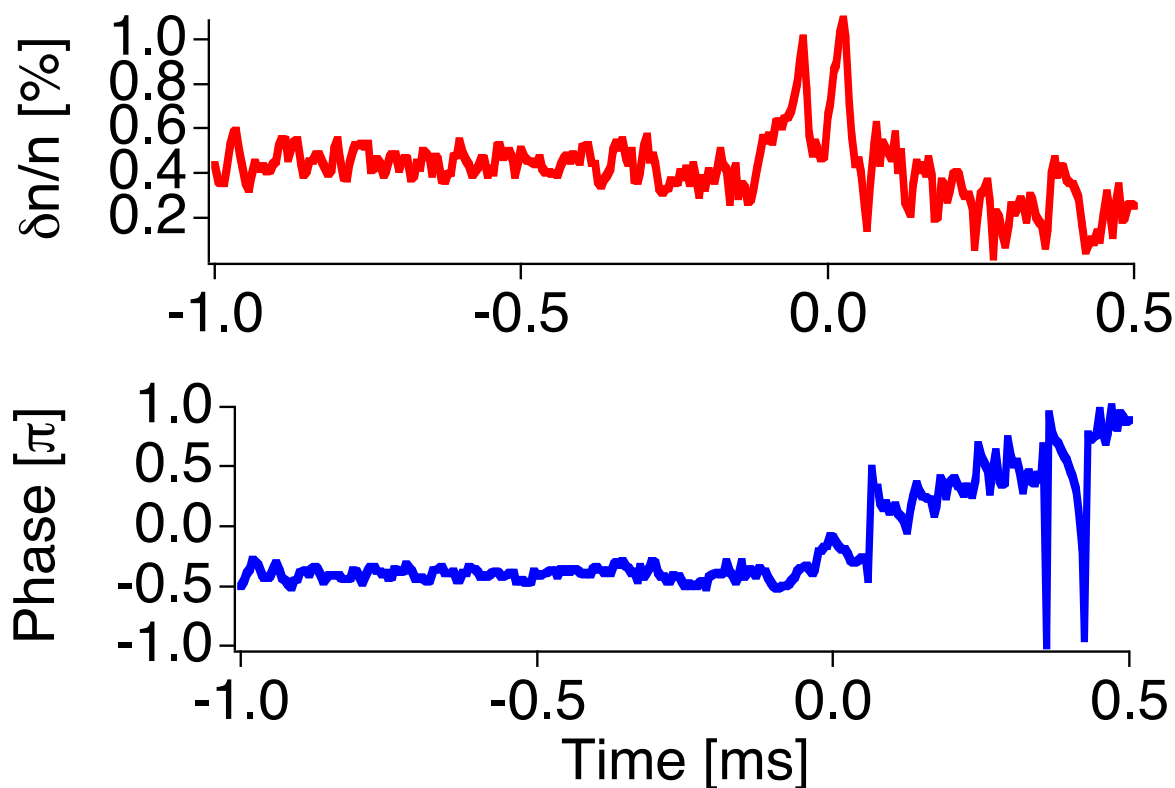
$$V_{\parallel}(t) = V_{\parallel,-2} - \frac{1}{nM} \int_{-2}^t \left[ \nabla \cdot \frac{\langle \delta p_{\parallel} \delta b_r \rangle}{B} \vec{e}_r \right] dt$$



*Direction of density fluctuation-induced force is consistent with flow change*

# The Effect of Phase between Fluctuations on Momentum Flux

$$Flux = T \frac{\langle \delta n \delta b_r \rangle}{B} \sim |\delta n| |\delta b_r| \cos \delta_{n,b}$$



*Why does phase change during a sawtooth cycle?*

# Coupling between Density Fluctuations and Magnetic Fluctuations in Plasmas

$$\frac{\partial n_e}{\partial t} + \nabla \cdot (n_e \vec{V}_e) = 0$$

[Fluctuation Equation]

$$\frac{\partial \delta n}{\partial t} = -\delta v_r \nabla n_0 - \delta v_r \nabla \delta n - \dots \quad [1]$$

[Magnetic field line equation]

$$\frac{dr}{b_r} = \frac{dl}{B} \longrightarrow \delta v_r = \delta b_r \frac{V_{\parallel,e}}{B} \quad [2]$$

$\times \delta n \longrightarrow$

$$\underbrace{\left(\frac{B}{2V_{\parallel,e}}\right) \frac{\partial}{\partial t} \langle \delta n^2 \rangle}_{\text{Fluct. Energy}} = \underbrace{- \langle \delta n \delta b_r \rangle \nabla n_0}_{\text{Quasi-linear}} - \underbrace{\sum \langle \delta n \delta b_r \nabla \delta n \rangle}_{\text{Nonlinear terms}} - \dots \quad [3]$$

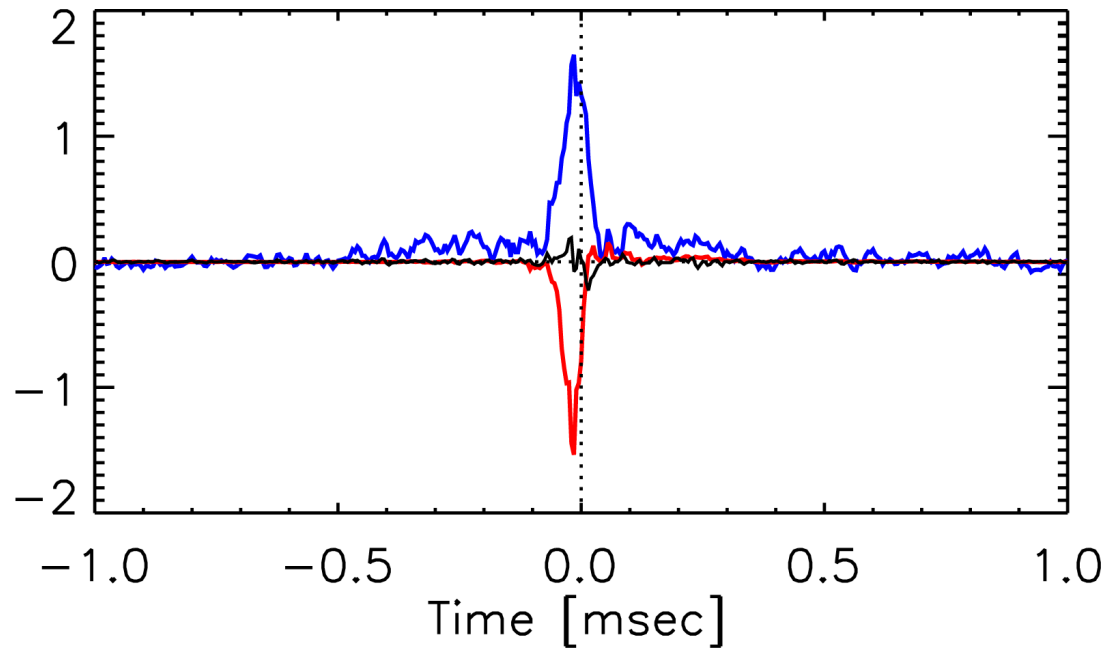
Fluct. Energy

Quasi-linear

Nonlinear terms

$\langle \dots \rangle$  Flux Surface Average

# Nonlinear Coupling between Density and Magnetic Fluctuations is Significant over Sawtooth



$$\underbrace{\left(\frac{B}{2V_{||}}\right) \frac{\partial}{\partial t} \langle \delta n^2 \rangle}_{\text{black}} = - \underbrace{\langle \delta n \delta b_r \rangle}_{\text{blue}} \nabla n_0 - \underbrace{\sum \langle \delta n \delta b_r \nabla \delta n \rangle}_{\text{red}} - \dots$$

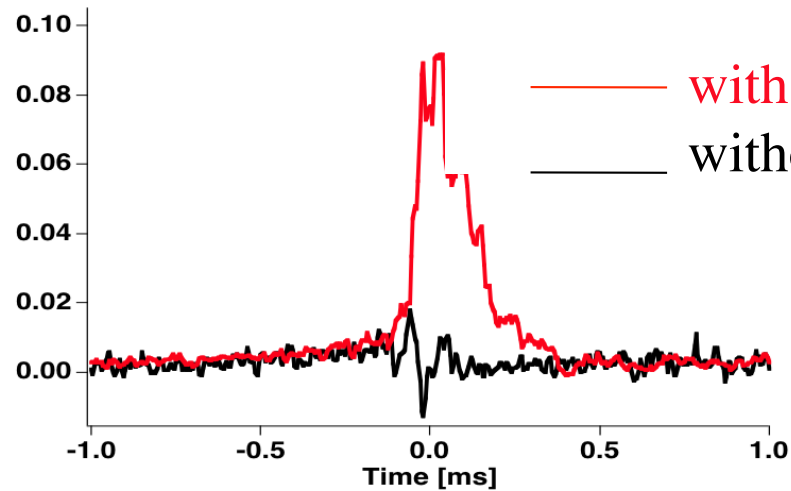
$$\langle \delta n \delta b_r \rangle = - \frac{1}{\nabla n_0} \sum \langle \delta n \delta b_r \nabla \delta n \rangle - \dots$$

# Removal of (0,1) Edge Mode Reduces Nonlinear Coupling

$$\langle \delta n_k \delta b_{r,k} \rangle = -\frac{1}{\nabla n_0} \sum_{k=k_1 \pm k_2} \langle \delta n_k \delta b_{r,k_1} \nabla \delta n_{k_2} \rangle - \dots \quad \begin{pmatrix} 1 \\ 7 \end{pmatrix} = \begin{pmatrix} 1 \\ 6 \end{pmatrix} + \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

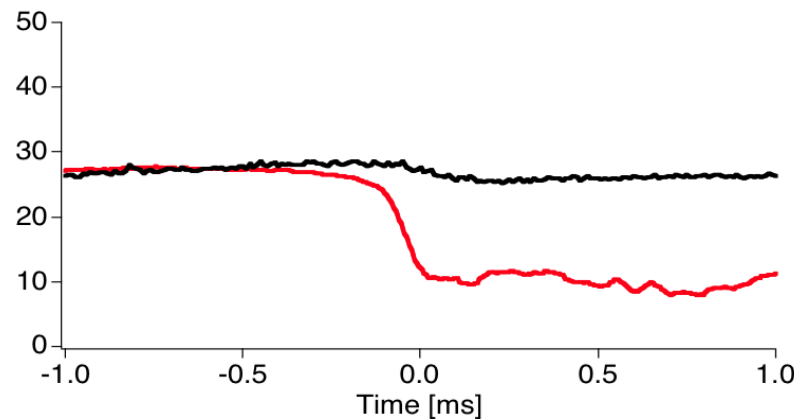
$$\vec{k} = \vec{k}_1 \pm \vec{k}_2$$

Flux [N/m<sup>2</sup>]



Momentum Flux

V<sub>//</sub> [km/s]



Plasma Flow

*This confirms the importance of nonlinear coupling on momentum transport on MST*

# Summary

- (1) Kinetic stress, Maxwell stress and Reynolds Stress are measured to be important on momentum transport for MST plasmas.
- (2) Nonlinear coupling can alter the phase between fluctuations, leading to finite momentum transport.

*Pressure fluctuation is important for momentum transport at fusion-relevant beta*

# Open Issues

- What is the role of temperature fluctuation on Momentum Transport? (*Code Prediction*)

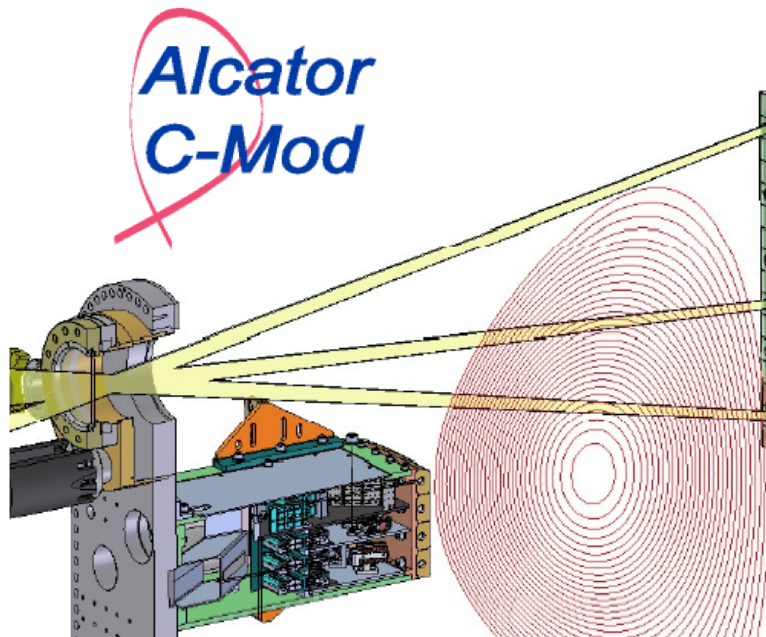
$$\frac{\langle \delta p_{//} \delta b_r \rangle}{B} = T \frac{\langle \delta n \delta b_r \rangle}{B} + n \frac{\langle \delta T_{//} \delta b_r \rangle}{B}$$

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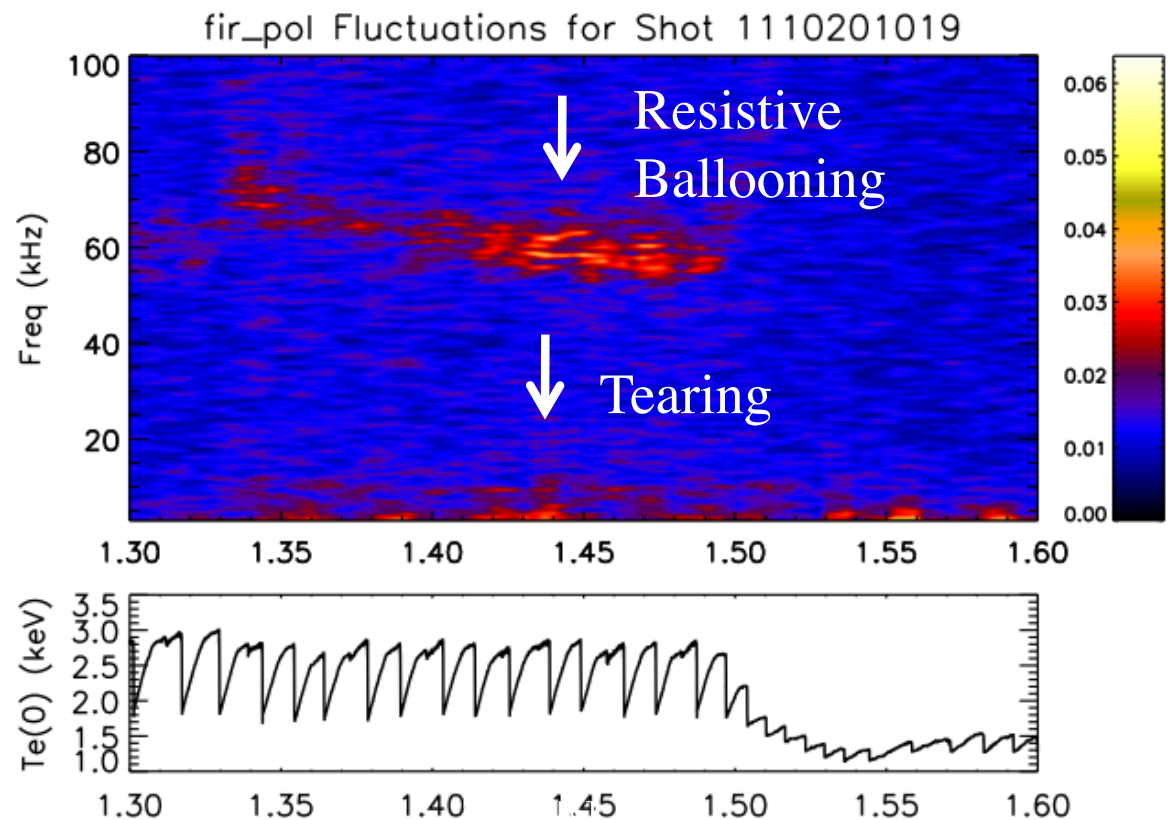
- (2) Fluid Stress Measurements in the core. (*Code Validation*)



# Advanced Faraday Rotation System on C-Mod



*Multiple chords with 4MHz bandwidth*



*by Irby, Xu, Bergerson, Brower, Ding, etc*

*Faraday Rotation Fluctuations are Measured on C-Mod , providing new opportunity to study transport on Tokamak.*