NSTX

Supported by

U.S. DEPARTMENT OF ENERGY

Office of Science

Millimeter Wave Polarimetry Modeling on NSTX

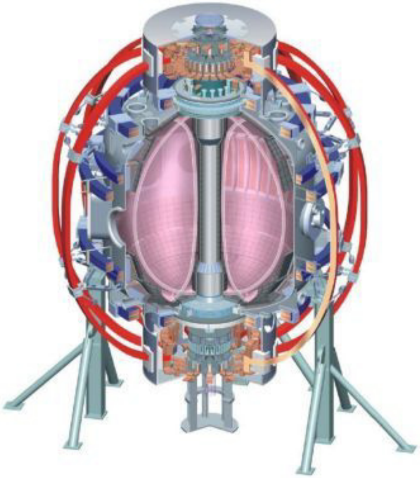
Jie Zhang

UCLA

N. A. Crocker, W. A. Peebles, T. A. Carter, S. Kubota (UCLA)
W. Guttenfelder (PPPL)
and the NSTX Research Team

Joint EU-US Transport Task Force Workshop
San Diego, CA
April 6-9, 2011

College W&M
Colorado Sch Mines
Columbia U
CompX
General Atomics
INL
Johns Hopkins U
LANL
LLNL
Lodestar
MIT
Nova Photonics
New York U
Old Dominion U
ORNL
PPPL
PSI
Princeton U
Purdue U
SNL
Think Tank, Inc.
UC Davis
UC Irvine
UCLA
UCSD
U Colorado
U Illinois
U Maryland
U Rochester
U Washington
U Wisconsin



Culham Sci Ctr
U St. Andrews
York U
Chubu U
Fukui U
Hiroshima U
Hyogo U
Kyoto U
Kyushu U
Kyushu Tokai U
NIFS
Niigata U
U Tokyo
JAEA
Hebrew U
Ioffe Inst
RRC Kurchatov Inst
TRINITI
KBSI
KAIST
POSTECH
ASIPP
ENEA, Frascati
CEA, Cadarache
IPP, Jülich
IPP, Garching
ASCR, Czech Rep
U Quebec

Results of polarimetry modeling on NSTX

- An interaction between Faraday rotation and Cotton-Mouton effect is clearly identified
  - Faraday rotation modifies elliptization (Cotton-Mouton effect)
    - elliptization is most sensitive to angle between “polarization direction” and “perpendicular magnetic field” in high-field region
    - Faraday rotation modifies the “polarization direction”, so it MUST affect elliptization
  - Cotton-Mouton effect intrinsically causes polarization rotation

- Polarimetry calculations for Neoclassical Tearing Modes (NTM) and MicroTearing modes (MT) indicate direct $\tilde{\boldsymbol{B}}$ measurements possible
  - Calculations based on simple magnetic island model show ~0.4° phase response caused by 0.1% $\tilde{\boldsymbol{B}}_r$ of NTM
  - Calculations based on MT data from GYRO simulation show polarimeter sensitive to primarily $\tilde{\boldsymbol{B}}_r$ for chords near plasma center

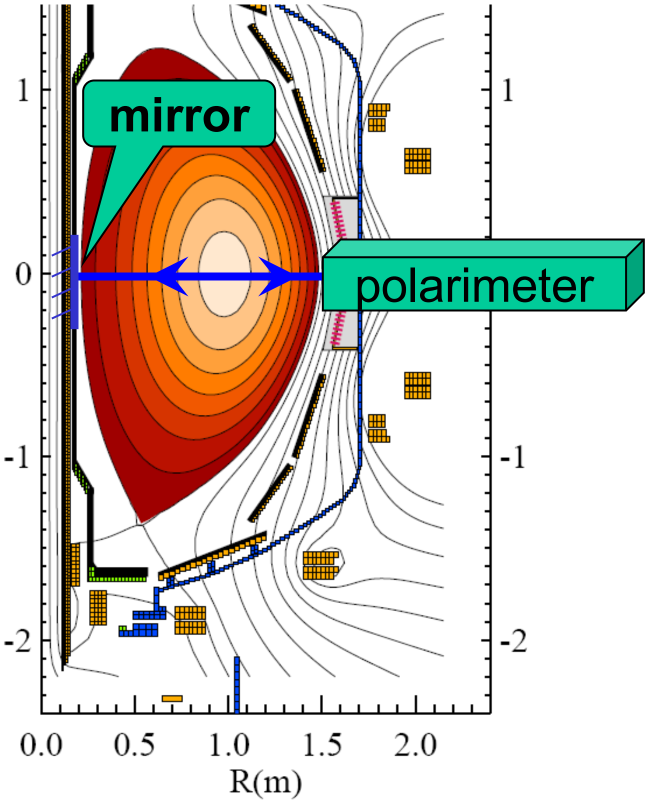
Planned polarimeter for NSTX aims to measure $\tilde{B}$

- Polarimetry measures change of wave polarization caused by magnetized plasma
- Polarimetry on NSTX can investigate $\tilde{B}$ of various modes
  - Microtearing modes
  - (Neoclassical) Tearing modes
  - Alfvén eigenmodes
- *288 GHz* polarimeter planned for NSTX
  - Horizontal retroreflection from Center Stack
  - Polarimetry modeling calculates phase response for equilibrium and MHD modes
  - Laboratory testing underway

Faraday rotation:

$$\psi \propto \int n\underline{B}\cdot d\underline{l}$$

$$\tilde{\psi} \propto \int \bar{n}\underline{\tilde{B}}\cdot d\underline{l} + \int \tilde{n}\underline{\bar{B}}\cdot d\underline{l}$$



Model of Polarimetry Calculations

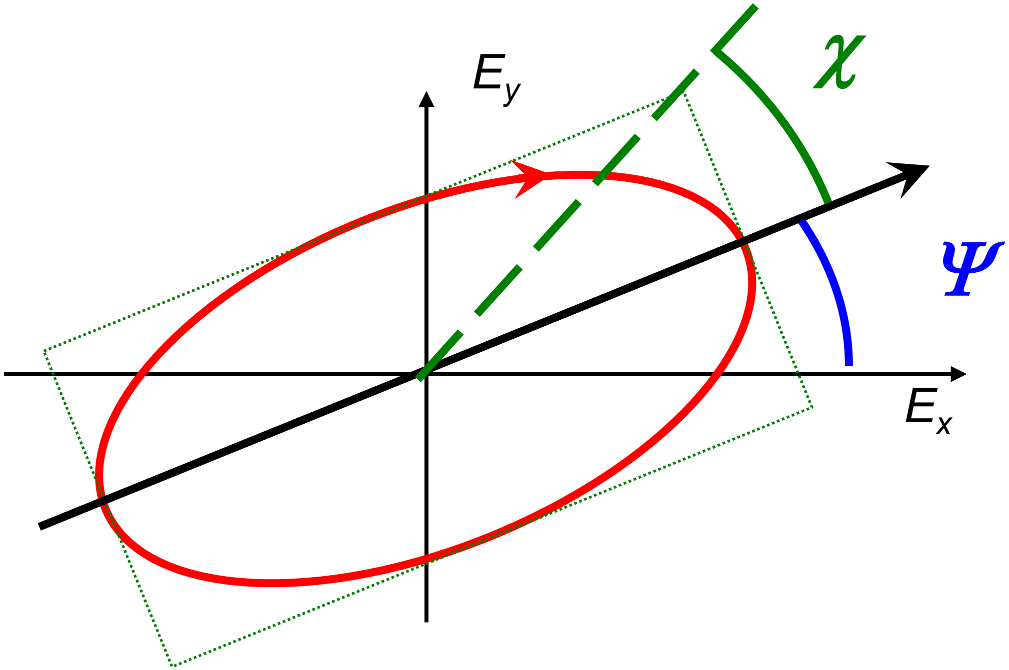
Assumptions are made to simplify modeling of polarization evolution along wave path

- Cold collisionless plasma model
  - no dissipation of beam
  - relativistic finite temperature polarimetry effects are neglected
- WKB (G. Wentzel, H.A. Kramers, and L. Brillouin) approximation used
  - plasma parameters is slowly varying

$$\left|\vec{B}\right| \gg \left|\frac{1}{k}\frac{d\vec{B}}{dz}\right|, n \gg \left|\frac{1}{k}\frac{dn}{dz}\right|$$

- Ion motion ignored $\omega_{pi}, \omega_{ci} \ll \omega_{pe}, \omega_{ce} \ll \omega$
- Refraction neglected

Polarization properties are characterized by "polarization direction" & "elliptization angle"



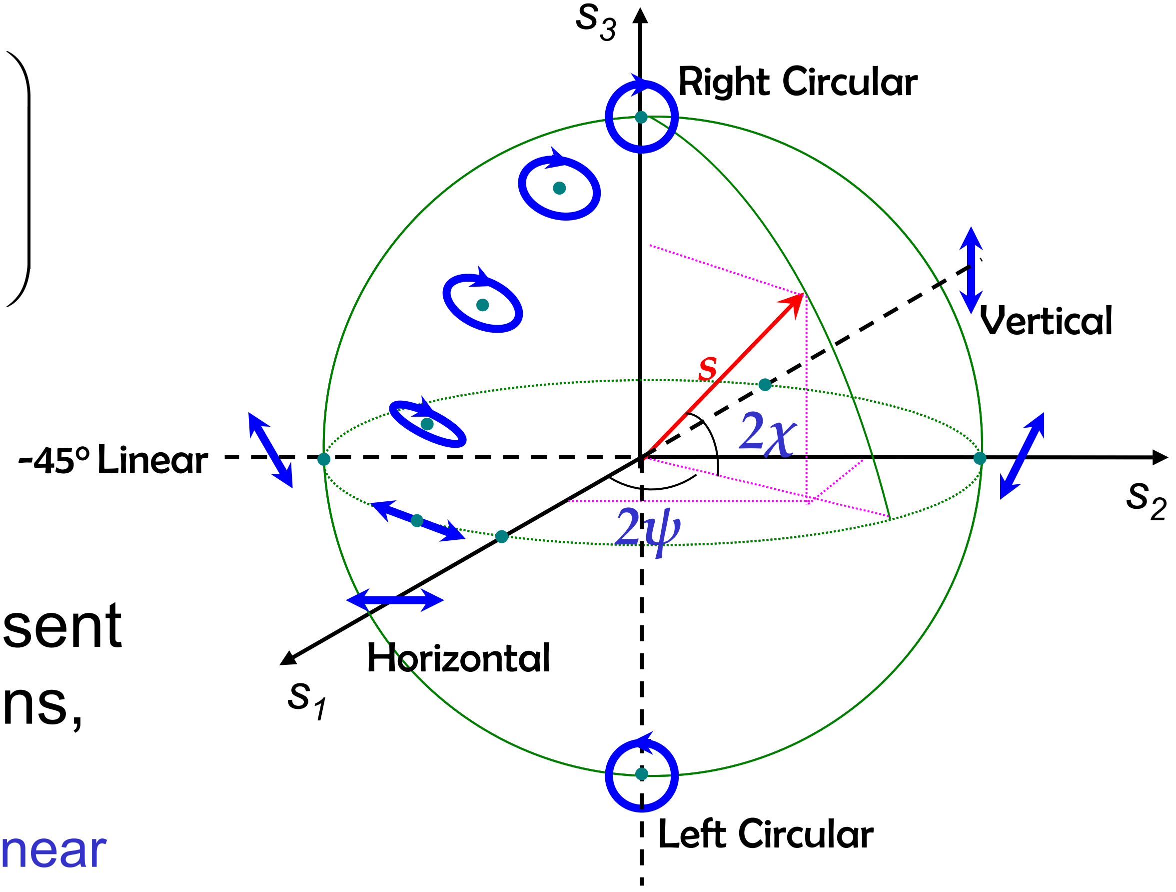
Ψ: polarization direction angle

χ: elliptization angle (+/- sign means Right/Left handedness)

Modeling treats polarization state as point on Poincaré sphere

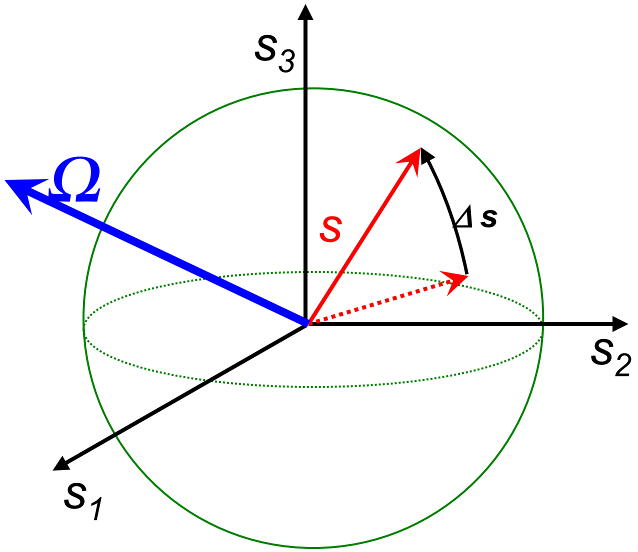
$$\vec{s} = \begin{pmatrix} s_1 \\ s_2 \\ s_3 \end{pmatrix} = \begin{pmatrix} \cos 2\chi \cos 2\psi \\ \cos 2\chi \sin 2\psi \\ \sin 2\chi \end{pmatrix}$$

- Opposing poles represent orthogonal polarizations, e.g.
  - –**Horizontal & Vertical** linear polarization (s_1 poles)
  - –**Left- & Right-handed** circular polarization(s_3 poles)



Poincaré sphere

Evolution of polarization can be described by a trajectory on Poincaré sphere



$$\frac{d\vec{s}(z)}{dz} = \vec{\Omega}(z)\times\vec{s}(z)$$

$$\vec{\Omega} = \frac{\omega}{2c}\left(\frac{\omega_{pe}}{\omega}\right)^2 \frac{1}{\left(\frac{\omega_{ce}}{\omega}\right)^{-2} - 1} \begin{pmatrix} (B_x^2 - B_y^2)/B^2 \\ 2B_xB_y/B^2 \\ 2\left(\frac{\omega_{ce}}{\omega}\right)^{-1} B_z/B \end{pmatrix}$$

Cotton-Mouton effect

Faraday Rotation

- Each small step along trajectory is a small rotation of ***s*** around *Ω axis*
- *Ω* depends on local plasma parameters *(electron density, B-field)*
- Using realistic NSTX equilibria (Thomson Scattering, EFIT) indicates interaction between Faraday rotation & Cotton-Mouton effect is important to polarization evolution

Interaction between Faraday Rotation & Cotton-Mouton Effects

INTERACTION—theoretical approach

- Interaction seen clearly from the geometrical description of polarization evolution
  - e.g. how s_3 (relates to elliptization) changes depends on direction of $\boldsymbol{s}$ relative to $\boldsymbol{\Omega}$
- Interaction can also be seen from differential expressions
  - elliptization is sensitive to ψ
  - Faraday rotation (FR) modifies ψ, so it affects elliptization
  - Cotton-Mouton effect (CM) also intrinsically causes polarization rotation

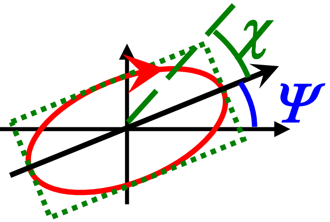
$$\begin{cases} d\chi = \frac{1}{2}\sin 2\psi \, d\delta\big|_{CM} \\ d\psi = d\psi\big|_{FR} - \frac{1}{2}\tan 2\chi \cos 2\psi \, d\delta\big|_{CM} \end{cases}$$

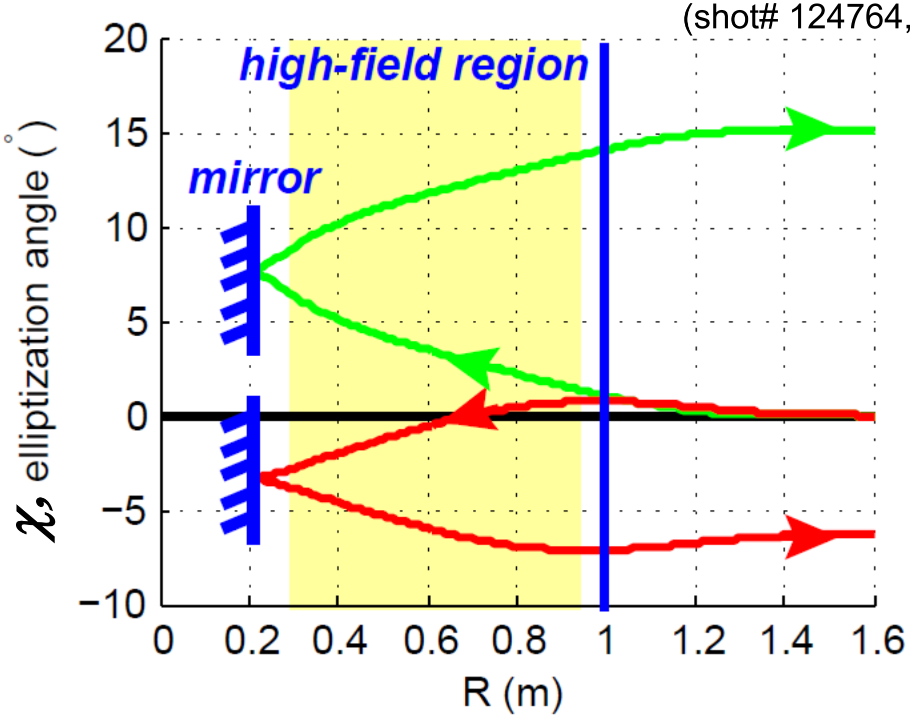
$$\begin{cases} d\psi\big|_{FR} \propto \lambda^2 n_e B_{\|} dz \\ d\delta\big|_{CM} \propto \lambda^3 n_e B_{\perp}{}^2 dz \end{cases}$$

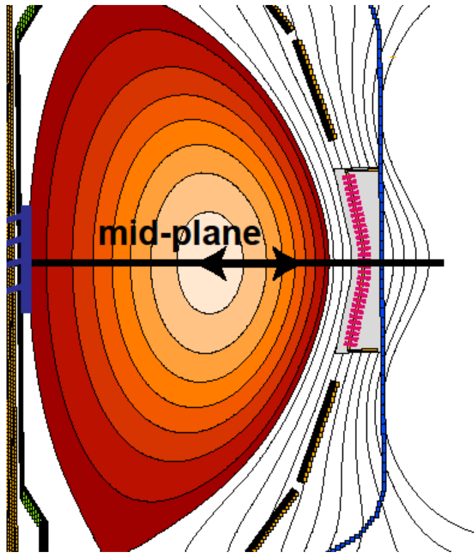
Elliptization most sensitive to polarization direction in high-field region

horizontal ($\psi=0^\circ$) and $\psi=45^\circ$ linear launch (mid-plane)

(shot# 124764, EFIT01, 0.325 s)



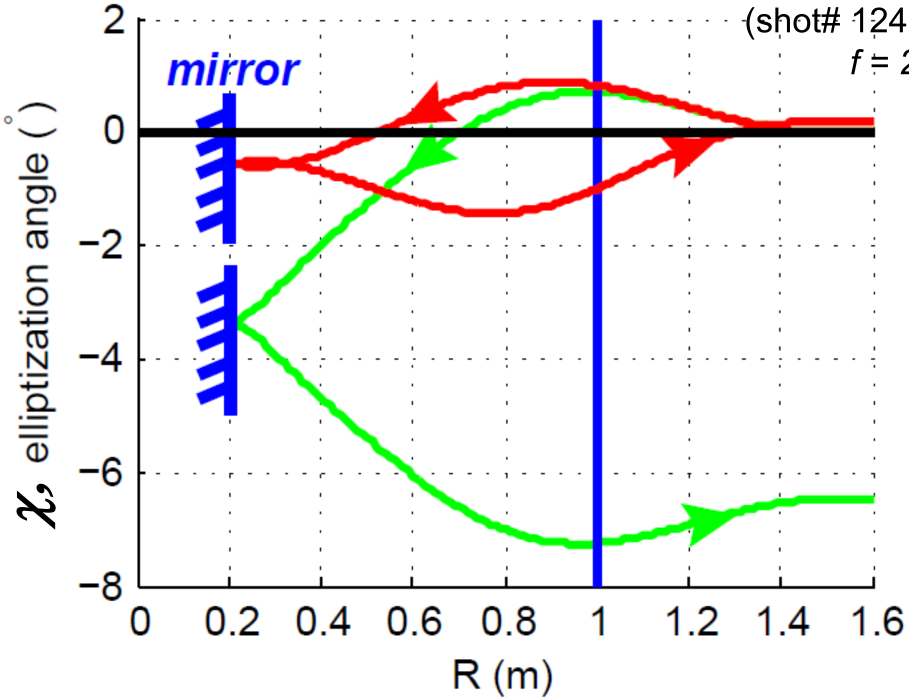




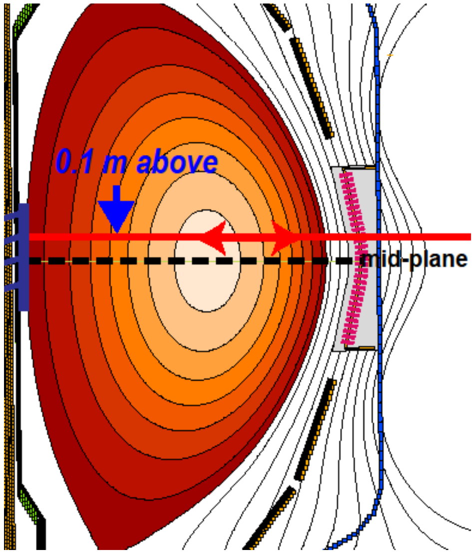
- Initial polarization direction determines direction in high-field region for mid-plane chord
  - –Faraday rotation very weak in mid-plane
- Figure shows elliptization is sensitive to initial polarization direction

Faraday rotation modifies elliptization

horizontal linear launch with/without Faraday rotation



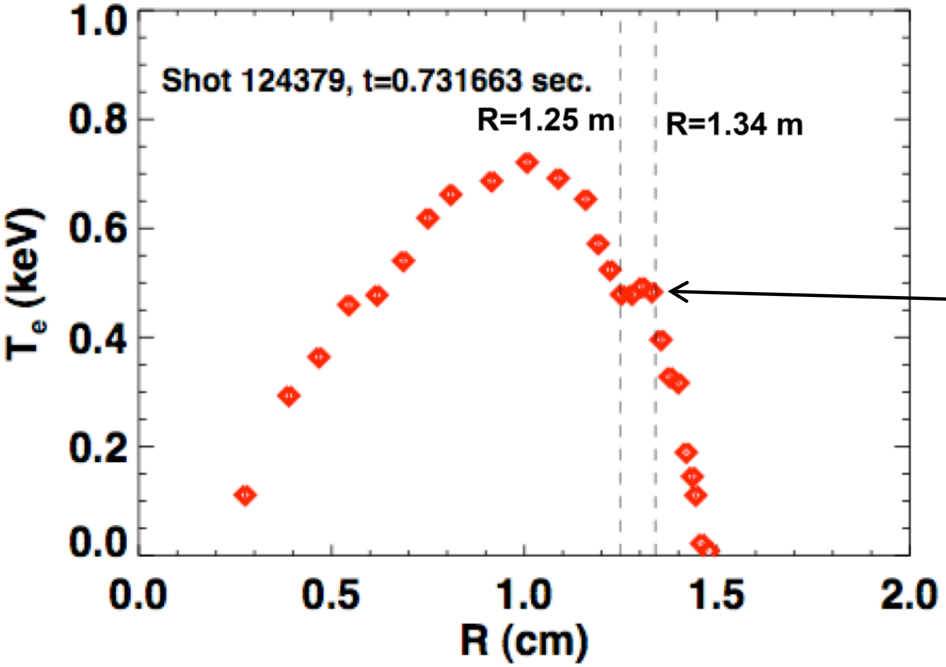
(shot# 124764, EFIT01, 0.325 s)
f = 288 GHz, $\lambda = 1.04$ mm

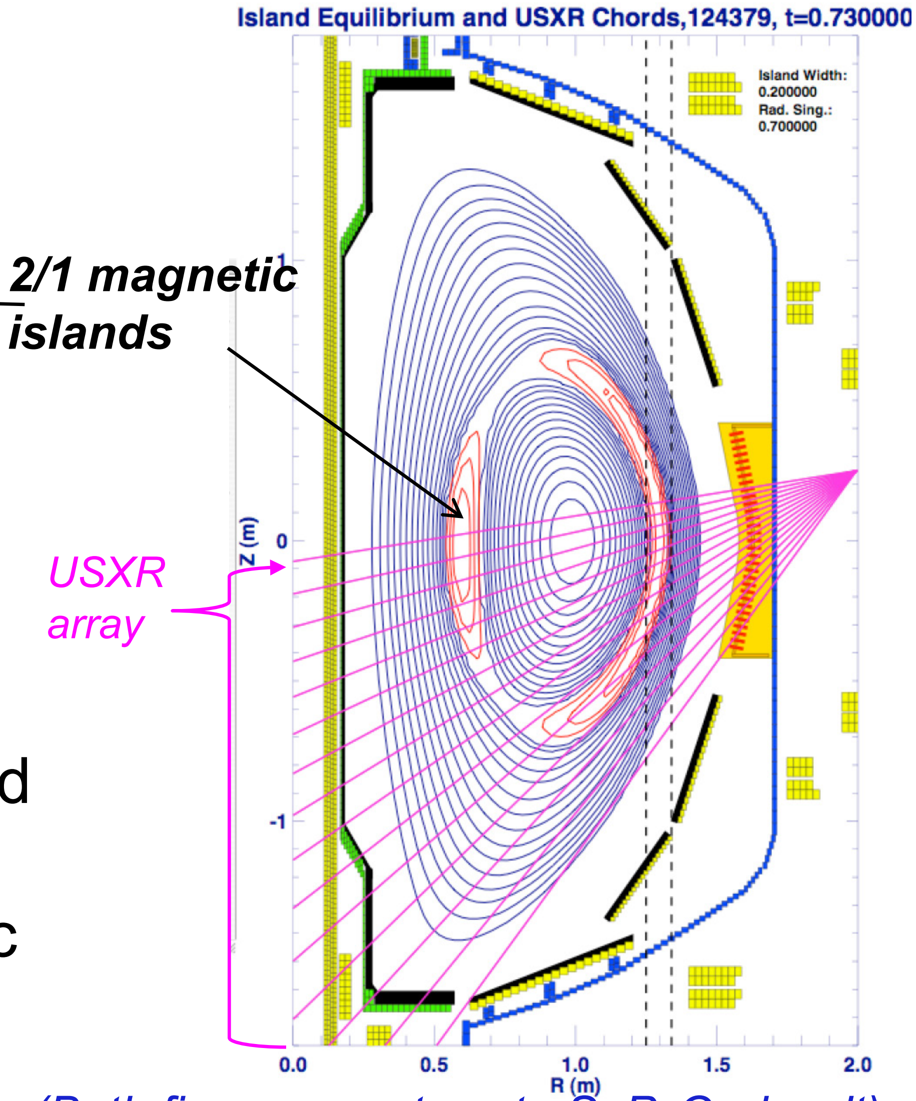


- In general, Faraday rotation & initial polarization direction determine the polarization direction in high-field region
- Figure shows weak elliptization along chord 0.1 m above mid-plane with Faraday rotation, but strong elliptization without
  - –Faraday rotation is counter-clockwise; turned off by setting $\boldsymbol{B}_{//}=0$

Polarimetry Modeling of NTM & MT

NTM—Realistic magnetic islands structure used in polarimetry modeling



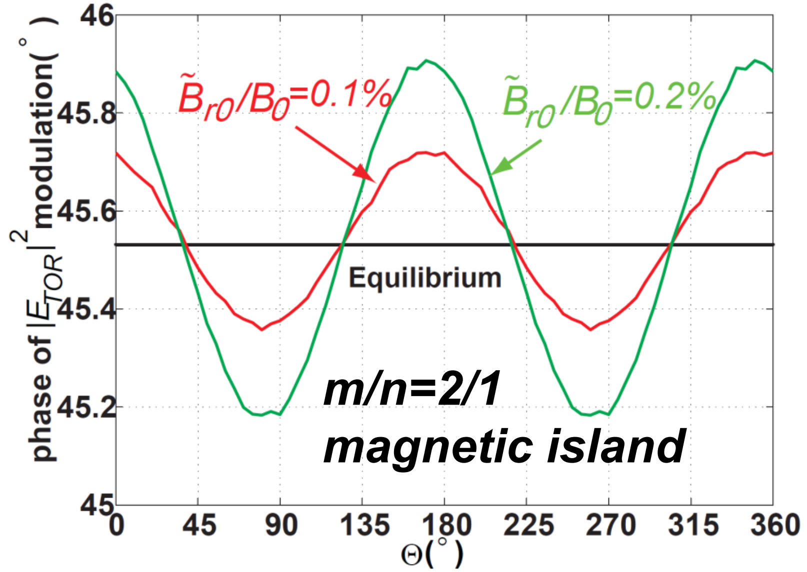


(Both figures courtesy to S. P. Gerhardt)

- Magnetic island structure determined using T_e profile and USXR
- On NSTX, typical 2/1 magnetic island
  - island width w ~ 0.1 m
  - radial location $\hat{r}_{2,1} \sim 0.15$

Polarimetry modeling shows ~0.4° phase response caused by 0.1% $\tilde{B}_r$

Equilibrium from shot #133959, t=0.882s

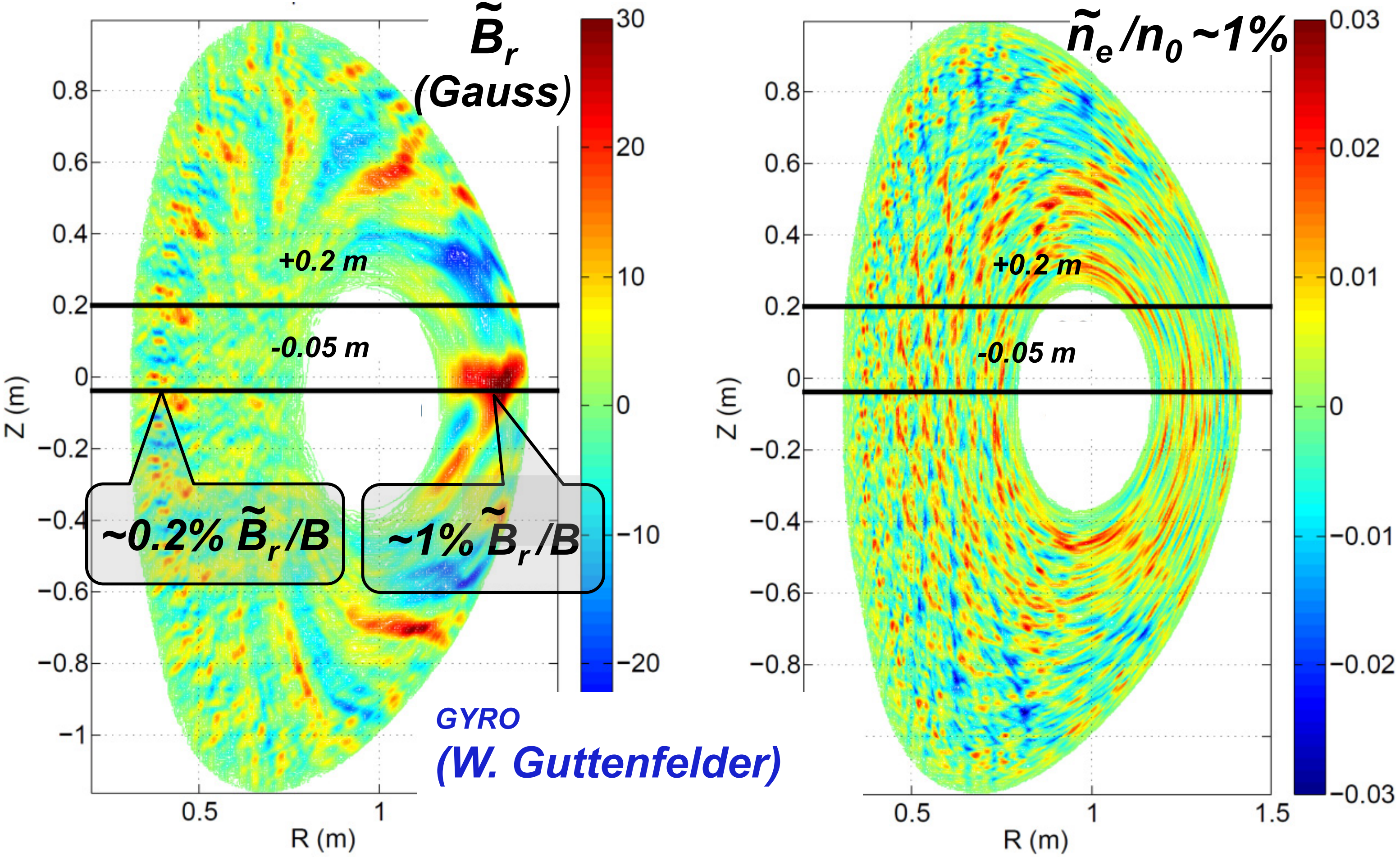


- Model assumes helically perturbed B-field around q=m/n rational surface

$$\tilde{B}_r = \tilde{B}_{r0} e^{-\frac{(\hat{r}-\hat{r}_{m,n})^2}{(w/a)^2}} \cos(m\Theta - n\phi)$$

- *m/n=2/1* island is modeled: (*w~0.1 m*, $\hat{r}_{2,1} \sim 0.15$)
  - beam propagates along chord *0.1 m* below midplane
- Phase change of $|E_{TOR}|$ modulation is *~0.4°* with *0.1%* $\tilde{B}_{r0} / B_0$
- Phase change dominated by Faraday rotation
  - amount of phase change approximately proportional to fluctuation amplitude

MT—Modeling used to evaluate polarimetry sensitivity to microtearing modes



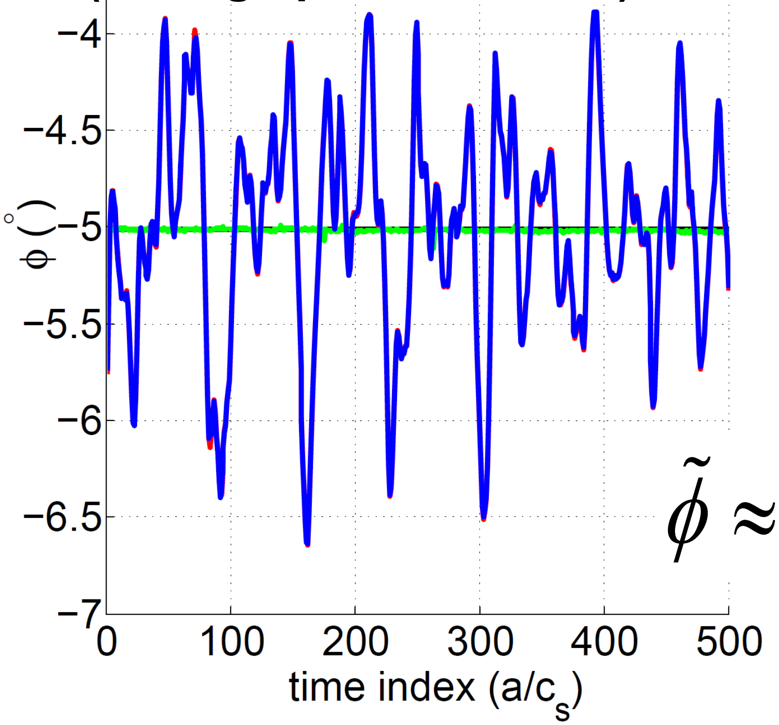
GYRO

(W. Guttenfelder)

- $\tilde{B}_r$ and $\tilde{n}_e$ of microtearing modes from GYRO simulations
- Chord heights varied to evaluate polarimetry sensitivity to $\tilde{B}_r$, $\tilde{n}_e$ respectively

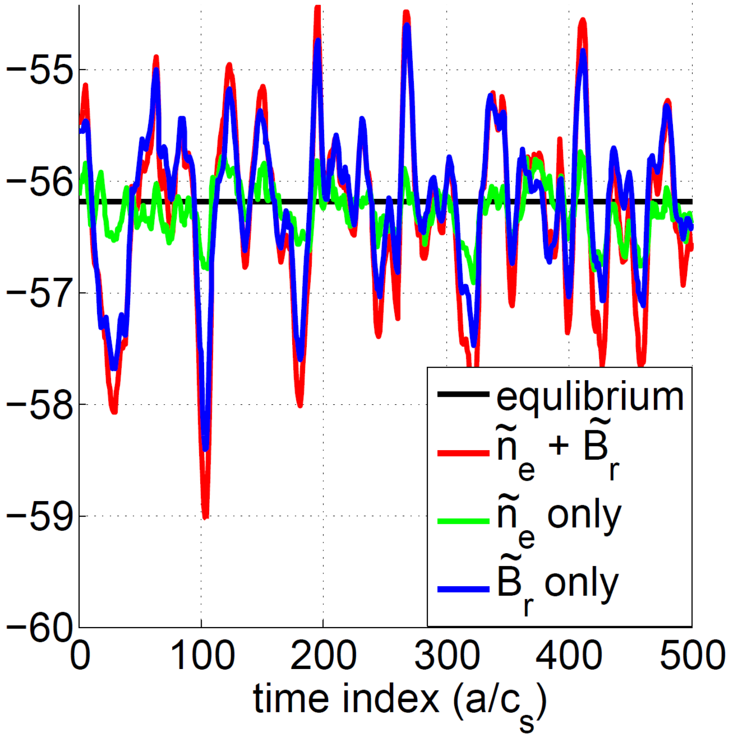
Polarimeter sensitive to primarily $\tilde{B}_r$ for chords near plasma center

0.05 m below midplane (through plasma center)



$$\tilde{\phi} \approx 2\tilde{\psi}$$

0.2 m above midplane



- $\tilde{\phi}$~1—2°, expected to be detectable

$$\tilde{\psi} \propto \int \bar{n}\underline{\tilde{B}} \cdot d\underline{l} + \cancelto{0}{\int \tilde{n}\underline{\bar{B}} \cdot d\underline{l}} \qquad \underline{\bar{B}} \cdot d\underline{l} \sim 0$$

for chords near plasma center

Summary—Results of polarimetry modeling on NSTX

- An interaction between Faraday rotation and Cotton-Mouton effect is clearly identified
  - Faraday rotation modifies elliptization (Cotton-Mouton effect)
    - elliptization is most sensitive to angle between “polarization direction” and “perpendicular magnetic field” in high-field region
    - Faraday rotation modifies the “polarization direction”, so it MUST affect elliptization
  - Cotton-Mouton effect intrinsically causes polarization rotation

- Polarimetry calculations for Neoclassical Tearing Modes (NTM) and MicroTearing modes (MT) indicate direct $\tilde{\boldsymbol{B}}$ measurements possible
  - Calculations based on simple magnetic island model show ~0.4° phase response caused by 0.1% $\tilde{\boldsymbol{B}}_r$ of NTM
  - Calculations based on MT data from GYRO simulation show polarimeter sensitive to primarily $\tilde{\boldsymbol{B}}_r$ for chords near plasma center

Requests for electronic copy