ELM modeling using the 2DX eigenvalue code

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Edge-localized modes (ELM's) are studied with the 2DX code, which is a flexible eigenmode solver designed for toroidal plasma configurations with an x-point topology.

Both high resolution and short run times are readily achievable with 2DX, which employs state-of-the-art eigensolving techniques through the SLEPc sparse matrix package[1]. In addition, its use of a specialized equation parser allows for rapid customization and alteration of model equations. This equation parser also permits implementation of gyrofluid models and iterative approximation of kinetic effects.

These capabilities make 2DX a useful tool for applications to experimental situations where the linear physics is important, in particular the linear stability of the peeling-ballooning (PB) mode generally believed to be the cause of ELM's.

We consider a PB mode case in a shifted circle geometry, using the physics model of high-beta ideal reduced MHD equations. Growth rates and eigenmodes from 2DX calculations are compared with the previously benchmarked results from ELITE and BOUT++[2].

[1] http://www.grycap.upv.es/slepc/
[2] B. D. Dudson *et al*, Comp. Phys. Comm. 180 (2009) 1467.

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Outline

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Introduction to 2DX

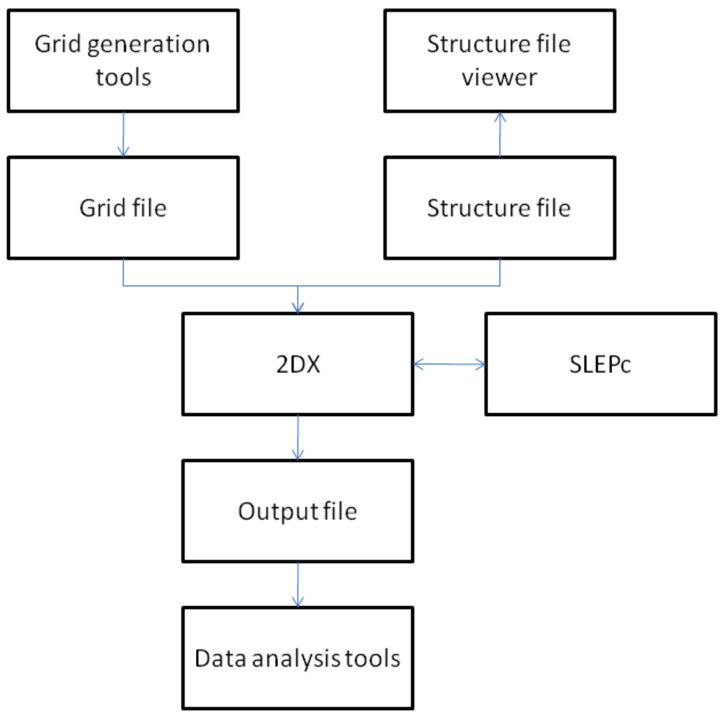
- 2DX is a linear eigensystem solver for edge instabilities in an axisymmetric toroidal geometry.
  - Differential equations converted to finite difference equations in space domain.
  - Time domain represented by eigenvalues.
  - Difference equations represented in matrix form.
  - End result is generalized eigenvalue problem:

$$Ax=\lambda Bx$$

- Equations are quasi-2D.
  - Toroidal direction represented by mode number.
  - Other directions use field line following coordinates.
- 2DX currently has the capacity to model single X-point divertor geometries.
  - Periodic and sheath boundary conditions can exist on a single grid.
- Eigenvalues solved using SLEPc sparse matrix solver.
  - Moderate sized problems solved in a few CPU-minutes.

2DX architecture

- Separates physics model, geometry, and numerical method.



ELM background

- Edge-localized modes (ELM's) are important for understanding edge confinement in tokamaks.
  - Sudden bursts of particles and energy can damage wall materials.
- Type I ELM's are believed to be due to peeling-ballooning (PB) instability.
- Linear onset of PB instability has been studied with numerous codes.
  - ELITE
  - GATO
  - MISHKA
  - BOUT++
- These studies form a basis for benchmarking other edge plasma codes.
  - Useful verification for 2DX.

ELM model

- Begin with three-field reduced MHD model containing peeling and ballooning terms:

$$\gamma \nabla_\perp{}^2 \delta\phi = \frac{2B}{n} C_r \, \mathrm{n}\, \delta T_i - \frac{B^2}{n} \partial_\parallel \nabla_\perp{}^2 \delta \mathrm{A} + \mathrm{i} \frac{B^2}{n} \frac{k_b}{B} \delta \mathrm{A} \, \partial_r \frac{J_\parallel}{B}$$

$$\gamma \, \delta \mathrm{T}_i = -\,\mathrm{i} \frac{k_b}{B} \delta\phi \, \partial_r T_i$$

$$\gamma \left(\frac{n}{\delta_{\mathrm{er}}{}^2} \right) \delta \mathrm{A} = -\mathrm{n}\mu \nabla_\parallel \delta\phi$$

- Equivalent to one-field model:

$$\gamma^2 \nabla_\perp{}^2 \delta\phi = -\frac{2B}{n} C_r \left(-\,\mathrm{i} \frac{k_b}{B} \delta\phi \, \partial_r T_i \right) + \frac{B^2}{n} \delta_{\mathrm{er}}{}^2 \mu \, \partial_\parallel \nabla_\perp{}^2 \nabla_\parallel \delta\phi + \mathrm{i} \frac{B^2}{n} \frac{k_b}{B} \partial_r \frac{J_\parallel}{B} \left(\frac{-\mu}{\delta_{\mathrm{er}}{}^2} \right) \nabla_\parallel \delta\phi$$

  – Avoids numerical complications due to grid-scale modes.

Benchmark case

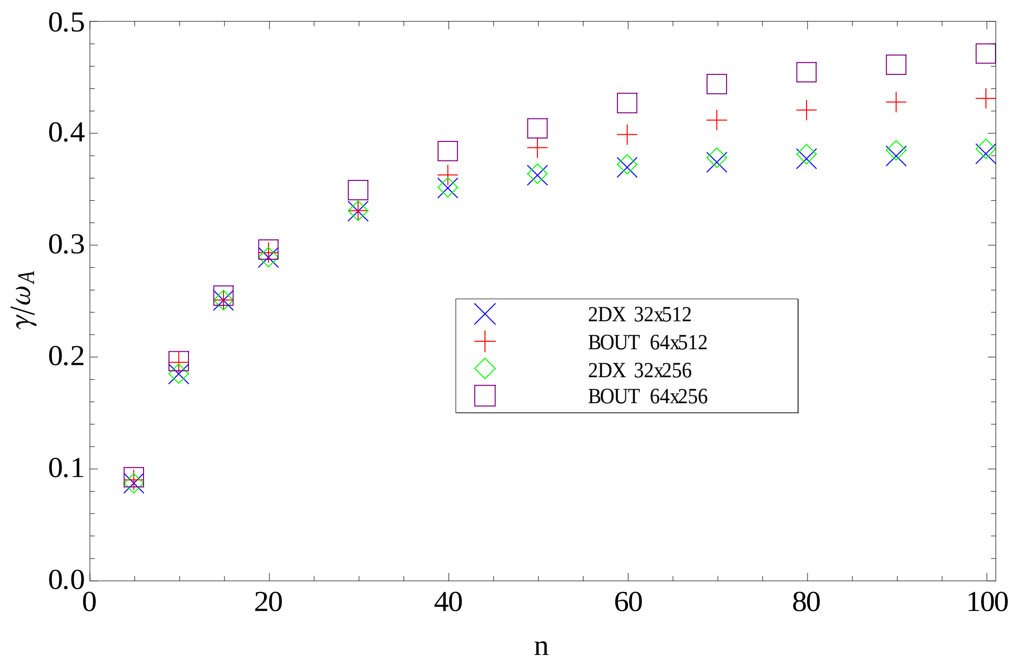
- Simplified toroidal geometry.
  – Shifted annulus.
  – Temperature gradient provides ballooning drive.
  – Current gradient near edge provides kink/peeling drive.
- Comparison with BOUT++.
  – Both codes run on equivalent geometry and profile functions.
  – Growth rates compared.
  – Variation in growth rate with resolution compared.

Benchmark results

- BOUT++ and 2DX results agree for low mode numbers.
- BOUT++ and 2DX results show slight disagreement for high mode numbers.
- However, BOUT++ results are not converged at high mode numbers.
  - Increasing resolution causes significant changes in BOUT++ results.
  - 2DX does not show such noticeable changes with resolution.
    - Suggests 2DX results are converged.
- Trend with resolution suggests that 2DX results are close to converged BOUT++ results.
  - Increasing resolution reduces discrepancy between codes.

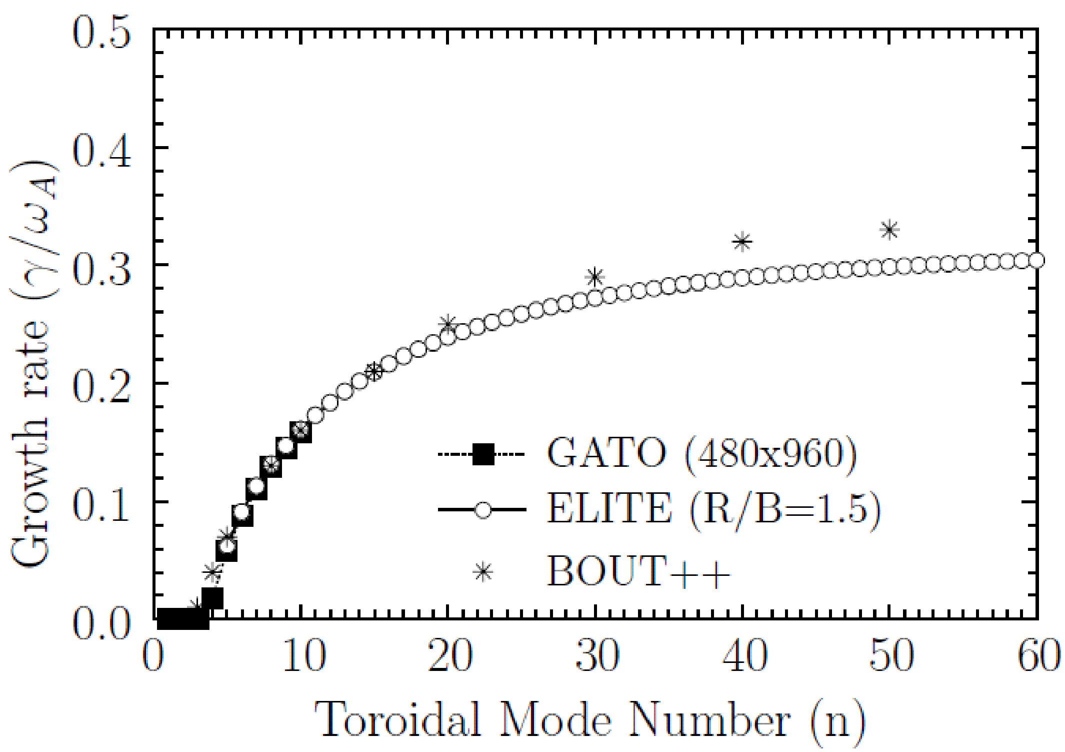
Benchmark results

Growth rates as a function of toroidal mode number for test of 2DX vs. BOUT



Prior benchmark results

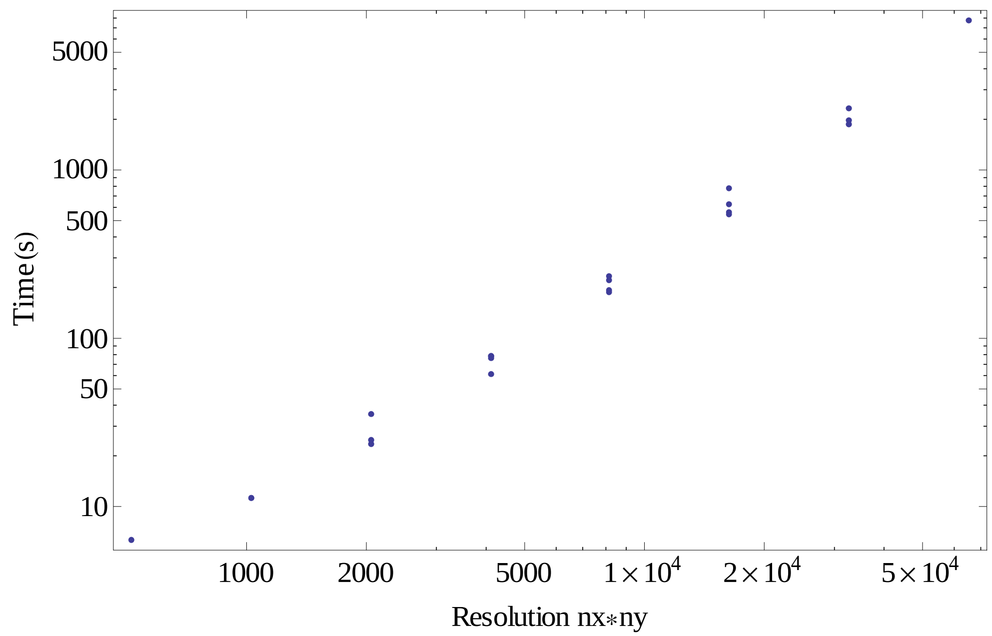
- Similar resolution issues in BOUT++ were noted in previous benchmark tests vs. ELITE (Dudson *et. al.*).
- Suggests discrepancy between BOUT++ and 2DX places 2DX results close to previous benchmark tests of ELITE.
- Direct comparison of 2DX vs. ELITE unavailable because different profiles were used.



Computational scaling

- Benchmark case run at different resolutions to observe effect on error, run time.
  - Provides a test of 2DX computational capabilities on a realistic physics application.

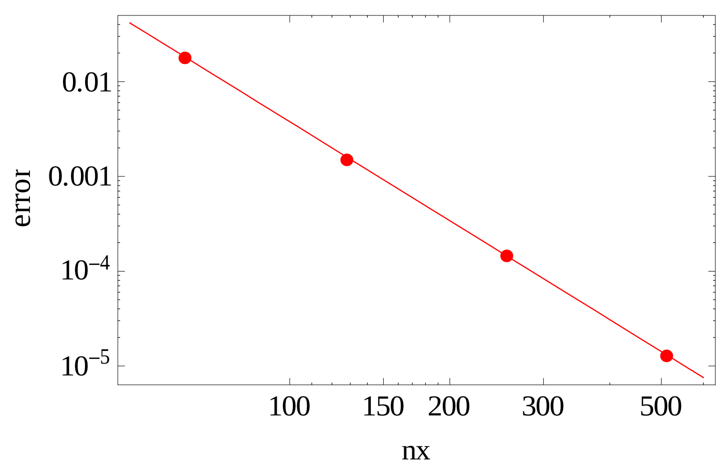
Run time as a function of resolution for ELM test case using 2DX.



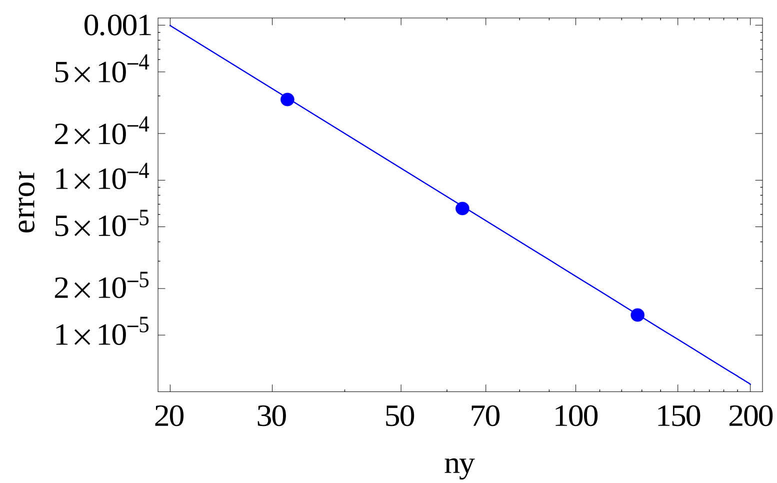
Computational scaling

- Convergence measured by comparing eigenvalues at different resolutions.
  - Runs at nx=32-512 and ny=16-128.
  - Correct eigenvalue estimated by extrapolation in nx, then in ny.
- Comparing difference from correct value as a function of resolution gives scaling law.
  - Scaling law of 3.46 in nx, 2.31 in ny.
- Overall error values suggest code is well-converged for cases of interest.

Error scaling as a function of nx at ny=64.



Error scaling of asymptotic limits as a function of ny.



Conclusions

- 2DX shows good agreement with BOUT++ at low mode numbers.
- Discrepancy at high mode numbers consistent with resolution scaling of BOUT++
- Further benchmarking requires different profile functions.
  - Direct comparison of 2DX vs. other codes requires cases run with same profiles as published results.
- Lays groundwork for further studies in ELM physics.
  - PB mode in full x-point geometry.
  - non-ideal MHD physics.