How micro-turbulence breaks magnetic surfaces

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Important collaborators …

- From IPP Garching:
  - Frank Jenko,
  - M.J. Pueschel,
  - F. Merz

- Formerly U of Wisconsin (and now IPP Garching):
  - David Hatch

- PPPL
  - Walter Guttenfelder
Equilibrium field structure and rational surfaces

- **q-profile across flux-tube**
  \[ q(r) \approx q(r_0) + q'(r - r_0) \]
  \[ \approx q_0 \left[ 1 + \left( \frac{\rho_i}{r_0} \right) \widehat{s} \left( \frac{r - r_0}{\rho_i} \right) \right] \]

- **At rational surface** \( q = m/n \)
  \[ \sim \frac{1}{n} \quad \sim \rho^* \]
  \[ \left( \frac{m}{n} - q_0 \right) = \left( \frac{\rho_i}{r_0} \right) \widehat{s} \left( \frac{r - r_0}{\rho_i} \right) \]

⇒ Flux-tubes have (high order) rational surfaces
Field-line trajectory in flux coordinates

- Equilibrium flux coordinates:
  - \((x,y)\) are field-line labels
  - \(s\) measures distance along \(B\)

\[
\frac{ds}{B} = \frac{dx}{\vec{B} \cdot \nabla x} = \frac{dy}{\vec{B} \cdot \nabla y}
\]

- Naturally contravariant representation of \(\delta B_\perp\)

\[
\delta \vec{B}_\perp = \nabla \times \delta A_\parallel \hat{b} \approx \nabla \delta A_\parallel \times \hat{b}
\]

- Field-lines trajectories:

\[
\frac{\partial x}{\partial s} = \frac{1}{B} \frac{\partial \delta A_\parallel}{\partial y}, \quad \frac{\partial y}{\partial s} = -\frac{1}{B} \frac{\partial \delta A_\parallel}{\partial x}
\]

Analysis by F. Merz using data from GENE
\[ \left\langle \delta A_{||} \right\rangle_\theta \] generates the one-turn field-line mapping

- Integrate field-line trajectory along \( B \) for one poloidal cycle

\[
x_n(\pi) = x_n(-\pi) + \int_{-\pi}^{\pi} \frac{ds}{B} \frac{\partial \delta A_{||}}{\partial y}
\]

\[
y_n(\pi) = y_n(-\pi) - \int_{-\pi}^{\pi} \frac{ds}{B} \frac{\partial \delta A_{||}}{\partial x}
\]

- Flux-tube periodicity:

\[
x_{n+1}(-\pi) = x_n(\pi)
\]

\[
y_{n+1}(-\pi) = y_n(\pi) + 2\pi \hat{s} x_{n+1}
\]

- A one-turn map:

\[
x_{n+1} = x_n + \frac{\partial}{\partial y} \left\langle \delta A_{||} \right\rangle_\theta
\]

\[
y_{n+1} = y_n - \frac{\partial}{\partial x} \left\langle \delta A_{||} \right\rangle_\theta + 2\pi \hat{s} x_{n+1}
\]
ITG turbulence drives magnetic reconnection

- GYRO uses flux coordinates:
  - \((r, \zeta)\) are field line labels
  - \(\theta\) (poloidal angle) measures position along \(B\)

- Project out resonant \(\delta A_{||}\) component by taking 1-turn \(\theta\)-average, \(\langle \delta A_{||} \rangle_{\theta}\)

- Magnetic reconnection occurs when resonant intensity,
  \(\langle \delta A_{||} \rangle_{\theta}^2(k_{\perp 0}, t)\)
  is finite at rational surface

⇒ These simulations exhibit turbulence-driven reconnection

Data from GYRO simulation at \(\beta_e=0.1\%\)
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Mode parity and magnetic surface integrity

- Ballooning modes (like ITG)
  - $\delta \Phi$ is even in $s$
  - $\delta A_\parallel$ is odd in $s$

  $$\Rightarrow \int \frac{ds}{B} \delta A_\parallel = 0$$

  - Ballooning modes don’t cause magnetic reconnection

- Micro-tearing modes
  - $\delta A_\parallel$ is even in $s$
  - $\delta \Phi$ is odd in $s$
  - Micro-tearing modes do cause magnetic reconnection

  $$\Rightarrow \text{No reconnection if micro-tearing modes are stable???}$$

  - Even electrostatic modes have implied magnetic parity!
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from D. Hatch et al., PRL 106, 115003 (2011)
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**Mode parity and magnetic surface integrity**

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  - $\Rightarrow$ No reconnection if micro-tearing modes are stable???

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FIG. 4: A plot of the amplitudes, time averaged over the nonlinear state, of the 315 (of 8192) least damped eigenmodes (orthogonalized) of the linear gyrokinetic operator for Fourier mode $k_\phi = 0.3, k_x = 0.0$.

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How odd $\varphi(s)$ generates even $A_{||}(s)$

- $j_{||}$ from non-adiabatic electron response

$$\delta f_{e} = \frac{e\varphi}{T_{e}} f_{0} + h_{e}$$

$$\left( \frac{\partial}{\partial t} + v_{||} \frac{\partial}{\partial s} \right) h_{e} = -\frac{e}{T_{e}} f_{0} \frac{\partial \varphi}{\partial t} + \frac{\nabla \varphi \times B}{B^{2}} \cdot \nabla f_{0}$$
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  ... but weakly non-adiabatic, $\omega \ll k_{\parallel}v_{te}$

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$$v_{\parallel} h_e = \int ds' \left( - \frac{e}{T_e} f_0 \frac{\partial \varphi}{\partial t} + \frac{\nabla \varphi \times B}{B^2} \cdot \nabla f_0 \right)$$
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- Integrate to get $j_{\parallel}$ and put it into Ampere’s law

\[
\delta f_e = \frac{eq}{T_e} f_0 + h_e
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\[
\frac{\partial^2 \tilde{A}_{\parallel}}{\partial \tilde{x}^2} - k^2 y \rho_s^2 \tilde{A}_{\parallel} \approx i \frac{\beta_e}{2} \left( \omega - \omega_e \right) \int_s ds \tilde{\varphi}(s)
\]
How odd $\varphi(s)$ generates even $A_{\|}(s)$

- $j_{\|}$ from non-adiabatic electron response … but weakly non-adiabatic, $\omega << k_{\|}v_{te}$

- Integrate to get $j_{\|}$ and put it into Ampere’s law

- And $A_{\|}$ is even … but also small for $\beta_e << 1$

\[ \delta f_e = \frac{e\varphi}{T_e} f_0 + h_e \]

\[ \left( \frac{\partial}{\partial t} + v_{\parallel} \frac{\partial}{\partial s} \right) h_e = -\frac{e}{T_e} f_0 \frac{\partial \varphi}{\partial t} + \frac{\nabla \varphi \times B}{B^2} \cdot \nabla f_0 \]

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\[ \frac{\partial^2 \tilde{A}_{\|}}{\partial \tilde{x}^2} - k_y^2 \rho_s^2 \tilde{A}_{\|} \approx i \frac{\beta_e}{2} (\omega - \omega_{*e}) \int^s ds \tilde{\varphi}(s) \]
Poincaré surface-of-section and microstructure of $B$-field

- Integrate magnetic field-line trajectories
  - Many poloidal cycles (3000)
  - Many initial positions (100)
- Poincare surface-of-section formed by putting one • in $(x,y)$-plane each time a field line crosses the outboard mid-plane (3000 crossing/field line, so •’s merge into lines …).
- Possible outcomes:
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Surface-of-section plot with isolating surfaces
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- Possible outcomes:
  - Island formation about rational surfaces
  - Stochastic regions with isolating surfaces
  - Destruction of (almost) all magnetic surfaces
Cyclone base case, Beta=0.2%
Why is micro-field stochastic? Do islands of like order overlap?

- Separation between rational surfaces of given order \((k_y)\):
  \[
  \Delta x(k_y) = \frac{1}{k_y \hat{s}}
  \]

- Island width is given by:
  \[
  w(k_y) = 4 \sqrt{\frac{qR}{\hat{s}B_T}} \left| \langle \delta A_{||}(k_y) \rangle \right|^{1/2}
  \]

- High-order islands overlap if \(S_{\delta A}(k_y) \approx |\delta A_{||}|^2\) falls off slower than \(k_y^{-4}\)

- \(S_{\delta A}(k_y) \sim k_y^{-4}\) for all GYRO simulations in this \(\beta_e\)-scan
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What about islands of neighboring orders?

- Minimum rational surface separation for neighboring $k_y$

  $$\Delta x \approx \frac{\Delta k_y}{k_y^2 \hat{s}}$$

  $\Rightarrow$ As $k_y$ increases, island separation falls off faster than width

  $\Rightarrow$ Islands will always overlap at high $k_y$
Can overlap of neighboring $k_y$ island produce global magnetic stochasticity?

- A single pair of neighboring $k_y$-modes produces bands of stochasticity separated by isolating surfaces
- Many $k_y$-pairs can occur, producing stochastic bands at different radii
- Generally, these bands do not overlap (so stochastic almost everywhere)
- Except near low order surfaces
  - WE expect general stochasticity with islands of stability near O-points of low-order islands

**GENE data retaining 2 highest $k_y$-modes**

(with thanks to M.J. Püschell)
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which is just what we see!
Is micro-scale magnetic stochasticity ubiquitous?

CON:
- Data shown is from CYCLONE base case
  - Heat transport larger than typical exp’t by 50x
    - $|\varphi^2|$, $|A_\parallel^2|$ also about 50x too large (!)

PRO:
- $<A_\parallel^2(k_y)> \sim k_y^{-4}$ is a consequence of non-linear physics, so this spectral index is (probably) a general result
- The fall-off probably extends out to $k_y \rho_e \sim 1$ (a factor of $\sim 100$ in $k_y$ at $\beta_e = 10^{-3}$, or $10^8$ in $<A_\parallel^2>(k_y)$)
  - More than compensates for 50x. With enough resolution, you will always be stochastic on micro-scale!