Inter-Species Energy Transfer and Turbulent Heating in Drift Wave Turbulence

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Outline

- We reconsider the classic problems of calculating “net turbulent heating” and the inter-species transfer of energy in drift wave turbulence

- Motivation: Transfer vs Transport → "Roles" in energy budget
  - Consider
    - Net volumetric heating → Does turbulence heat a given volume of plasma?
    - Physics of Electron → ion collisionless energy transfer channels

- Calculate and Estimate Energy Transfer Channels
  - Electron cooling: quasilinear
  - Ion heating: quasilinear, nonlinear, Ion Pol & Dia → Zonal flow

- Implication for ITER
  - Turbulent vs collisional transfer
  - Turbulent transport vs Turbulent transfer

- Results and Discussion
Motivation

• **Transfer vs Transport**

\[ n \frac{\partial T}{\partial t} + \nabla \cdot Q = \left\langle \tilde{E} \cdot \tilde{J} \right\rangle \mp n \nu \frac{m_e}{m_i} (T_e - T_i) + \ldots \] heat balance; \( \alpha = e, i \)

- Transport collisionless transfer
- Collisional transfer

→ \( \dot{Q} \) heat flux, energy loss by turbulent transport
→ \( \left\langle \tilde{E} \cdot \tilde{J} \right\rangle \) electron-ion collisionless energy transfer

→ **ITER**: low collisionality, electron heated plasma

• **Issues with** \( \left\langle \tilde{E} \cdot \tilde{J} \right\rangle = \sum_{\alpha=e,i} \left\langle \tilde{E} \cdot \tilde{J}_\alpha \right\rangle \)

• **Is the net heating zero?** (Manheimer 77)

• Periodical boundary condition, no boundary term exist

\[ \left\langle \tilde{E} \cdot \tilde{J} \right\rangle = 0 \]

\[ \text{But} \quad \int dr \left\langle \tilde{E} \cdot \tilde{J} \right\rangle = -\bar{\varphi} \tilde{J}_r \bigg|_{r_2}^{r_1} + \int dr (\nabla \cdot \tilde{J} \bar{\varphi}) \neq 0 \]

Boundary effects in a finite annular region

Surface term survives! \( \text{Net heating} \)
• **Another perspective: Poynting theorem**

\[
\frac{\partial W}{\partial t} + \nabla \cdot S + \langle \vec{E} \cdot \vec{J} \rangle = 0
\]

\( W \equiv \) Wave energy density \( S \equiv \) wave energy density flux

• At steady state

\[
\int_{r_1}^{r_2} dr \langle \vec{E} \cdot \vec{J} \rangle = -S_r \bigg|_{r_1}^{r_2}
\]

\[
S_r = V_{gr,r} \varepsilon_\omega = -2 \frac{\rho_s^2 k_r k_\theta \varepsilon_\omega V_* \varepsilon_\omega}{(1 + k_\perp \rho_s^2)^2}
\]

⇒ wave Energy flux differential  ⇒  **net heating**

\[
\langle \vec{V}_r \vec{V}_\theta \rangle = \sum -k_r k_\theta |\vec{\phi}|^2
\]

🌟 We need reconsiderr both the turbulent heating and energy transfer chanels in an annular region!
• Collisionless, inter-species energy transfer
  – Where does the net energy transfer go?
  – How is energy transferred from electrons to ions
    (turbulent transfer channels)?
  – How reconcile with saturation mechanisms?
  – Role of ZF in heating?

  – **ZF is important to saturation, so must enter energy transfer as well!**?
    ➢ Zonal flow frictional damping is another energy transfer channel
    ➢ **Nonlinear damping (considered in future) is another possibility**
Turbulent Energy flow Channels

\[ \langle \tilde{E} \cdot \tilde{J} \rangle \]

\[ \langle \tilde{E}_{\parallel} \tilde{J}_{\parallel} \rangle^{(2)} < 0 \]

\[ \langle \tilde{E}_{\parallel} \tilde{J}_{\parallel} \rangle^{(2)} > 0 \]

\[ \langle \tilde{E}_{\parallel} \tilde{J}_{\parallel} \rangle^{(4)} > 0 \]

\[ \langle \tilde{E}_{\perp} \cdot \tilde{J}_{\perp pol} \rangle^{(2)} \]

ST & ZF>0

\[ \langle \tilde{E}_{\perp} \cdot \tilde{J}_{\perp dia} \rangle^{(2)} \]

ST

b: Bounce average

ST: Surface Term

Ion Landau Damping

Nonlinear Landau damping

Primary mode resonance

Beat Mode Resonance

Collisional Zonal Flow Damping

Nolinear damping
**Necessary Correspondence:** Nonlinear Saturation and Energy Transfer

- Nonlinear saturation in turbulent state implies energy transfer from source $(\nabla T_e, \nabla n)$ to sink
- Schematically, saturation implies some balance condition must be satisfied
  
i.e. $0 = \gamma = \gamma_{Linear \ electron} + \gamma_{Linear \ ion} + \gamma_{Zonal \ Flow} + \gamma_{NLLD} + \ldots$
  
  $>0 <0 <0 <0$

- Channels for electron $\rightarrow$ ion energy transfer **must** be consistent with saturation balance

  **In particular:**
  - If zonal flows control saturation, they **must** contribute to energy transfer
  - As zonal flows are nonlinerly generated (Reynolds stress), we should consider other nonlinear heating channels, as well, for completeness
Quasilinear Turbulent Heating in Drift Wave

• Calculate \( \langle \tilde{E}_{\|} \tilde{J}_{\|e} \rangle^{(2)} \) in quasilinear theory
  
  ➢ DKE for electron
  
  ➢ Take non-adiabatic electron distribution function
    
    \[
    \tilde{g}_k = \frac{(\omega_s - \omega)}{\omega - k_z v_z} \frac{e \tilde{\phi}_k}{T_e} \langle f_e \rangle, \quad \omega_s = \frac{k_i \rho_s c_s}{L_n}, \quad \langle f_e \rangle \text{ is Maxwellian}
    \]
    
    ➢ \( \omega = \frac{\omega_s}{1 + \frac{k_{\perp}^2 \rho_s^2}} \), \( \langle \tilde{E}_{\|} \tilde{J}_{\|e} \rangle^{(2)} < 0 \) the electrons cool via inverse electron Landau damping

• Similarly, calculate \( \langle \tilde{E}_{\|} \tilde{J}_{\|i} \rangle^{(2)} \) for ion
  
  ➢ \( \langle \tilde{E}_{\|} \tilde{J}_{\|i} \rangle^{(2)} = \sum_k \pi n T_i \left| \frac{e \tilde{\phi}_k}{T_i} \right|^2 \frac{\omega}{k_z V_{th}} \left( \omega + \frac{T_i}{T_e} \omega_s \right) \langle f_i \rangle \omega \rangle_{k_z} \)
    
    ➢ \( \langle \tilde{E}_{\|} \tilde{J}_{\|i} \rangle^{(2)} > 0 \), the ions gain energy via ion Landau damping
Perpendicular Current Induced Turbulent Heating

- The turbulent heating induced by ion polarization current
  \[ \langle \vec{E}_\perp \cdot \vec{J}_{\perp i}^{pol} \rangle = -\langle \vec{\nabla}_\perp \cdot (\phi \vec{J}_{\perp i}^{pol}) \rangle + \langle \phi \vec{\nabla}_\perp \cdot \vec{J}_{\perp i}^{pol} \rangle \]
  Defining a annular region
  \[ \langle...\rangle = \int_0^{2\pi} \int_0^{\pi} \int_{r_0-\Delta}^{r_0+\Delta} (...)dr \]

- Net turbulent heating
  \[ \langle \vec{E}_\perp \cdot \vec{J}_{\perp i}^{pol} \rangle = n m_i A \left( \langle V_\theta \rangle \langle \vec{V}_r \vec{V}_\theta \rangle \right)_{r=+\Delta} - \int_{r=-\Delta}^{r=+\Delta} dr \langle V_\theta \rangle \frac{\partial}{\partial r} \langle \vec{V}_r \vec{V}_\theta \rangle \]
  Surface term
  - Reynolds work on mean flow in annular
  - Directly linked to zonal flow drive

- At steady state
  \[ \langle \vec{E}_\perp \cdot \vec{J}_{\perp i}^{pol} \rangle = \int_{r=-\Delta}^{r=+\Delta} d\nu_{col} \langle V_\theta \rangle^2 > 0, \quad \text{Zonal flow frictional damping is the fate of net electron-ion energy transfer} \]

- Diamagnetic current induced turbulent heating
  \[ \langle \vec{E}_\perp \cdot \vec{J}_{\perp i}^{D} \rangle = -n c \vec{\phi} \frac{B \times \nabla \vec{p}}{B^2} \right|_{r=+\Delta} \quad \text{Heat flux differential} \quad \text{Zonal flow} \]
Nonlinear Turbulent Heating

- ZF coupling to $o\left(\frac{e\phi}{T}\right)^4$, so need calculate parallel heating to $o\left(\frac{e\phi}{T}\right)^4$
- Nonlinear turbulent heating $\rightarrow$ perturbation theory (Dupree 68)

$$\langle \vec{E} \cdot \vec{J} \rangle^{(4)} = -\int dV m V D_4 \frac{\partial}{\partial V} \langle f \rangle$$

$D_4$ fourth order diffusion coefficient

- The nonlinear turbulent heating for ions

$$\langle \vec{E}_i \cdot \vec{J}_i \rangle^{(4)} = \sum_{k,k} \pi n_i \left( \frac{e\phi_k}{T_i} \right)^2 \left( \frac{e\phi_k}{T_i} \right)^2 \frac{k_z^2 k_z'^2 V_{thi}^2}{k_z''^2} \omega''^2 \left( \frac{k - k'}{k' - \omega}(k' - \omega') \right)^2 \langle f_i \rangle_{v = \frac{\omega''}{k''}} > 0$$

- The beat mode resonance

$$\omega'' = \omega \pm \omega', k'' = k \pm k'$$

- Nonlinear beat Landau resonance is a strong nonlinear effect!

$$\frac{\langle \vec{E}_i \cdot \vec{J}_i \rangle^{(4)}}{\langle \vec{E}_i \cdot \vec{J}_i \rangle^{(2)}} \sim \frac{e\phi}{T}^2 \exp\left(\frac{\omega^2}{k_z'^2 V_{thi}^2}\right)$$
# Overview of Results

## Estimation of the turbulent heating

<table>
<thead>
<tr>
<th>Turbulent heating</th>
<th>analytical</th>
<th>Mixing length approximation for fluctuation levels $\frac{e\phi}{T_e} \sim \rho_*$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\left\langle \tilde{E} \cdot \tilde{J} \right\rangle^{(2)}_{e}$</td>
<td>$\left</td>
<td>e\phi \right</td>
</tr>
<tr>
<td>$\left\langle \tilde{E} \cdot \tilde{J} \right\rangle^{(2)}_{i}$</td>
<td>$\left</td>
<td>\tilde{e}\phi \right</td>
</tr>
<tr>
<td>$\left\langle \tilde{E} \cdot \tilde{J} \right\rangle^{(4)}_{i}$</td>
<td>$\left</td>
<td>\tilde{e}\phi \right</td>
</tr>
<tr>
<td>$\left\langle \tilde{E} \cdot \tilde{J} \right\rangle^{(2)}_{pol}$</td>
<td>$m_v c_{pol} \left\langle V_\theta \right\rangle^2$</td>
<td>$\rho_<em>^2 v_</em> \varepsilon^{3/2} m_i C_s^2 \frac{V_{thi}}{Rq}$</td>
</tr>
</tbody>
</table>
Basic comparison of channels

**ITER Parameters** $R=6.2\,m$, $a=2\,m$, $q=2$

<table>
<thead>
<tr>
<th>Ratios</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{\langle \tilde{E}\cdot \tilde{J}_i \rangle}{\langle \tilde{E}\cdot \tilde{J}_e \rangle}$</td>
<td>Short wavelength $k_\perp \rho_s \sim 1$</td>
</tr>
<tr>
<td>$\langle \tilde{E}\cdot \tilde{J} \rangle_{i}^{(2)}$</td>
<td>1.56$\times 10^{-2}$</td>
</tr>
<tr>
<td>$\langle \tilde{E}\cdot \tilde{J} \rangle_{pol}^{(2)}$</td>
<td>0.8$v_*$</td>
</tr>
<tr>
<td>$\langle \tilde{E}\cdot \tilde{J} \rangle_{i}^{(4)}$</td>
<td>$0.08\times 10^4 \rho_*^2$</td>
</tr>
</tbody>
</table>

**Ratios of energy dissipation channels at different collisionality**

- Landau Damping
- Zonal flow frictional
- Nonlinear LD

$\nu_* = 10^{-3}$ $\rho_* = 10^{-3}$ $\nu_* = 10^{-1}$ $\rho_* = 10^{-3}$

**Zonal flow frictional damping can be a significant dissipation channel**

"Collisionless drift wave" $\omega \gg \nu_* > 0"
Implication ➞ Bottom Line

- Electron turbulent energy transport

\[ \frac{\partial T_e}{\partial t} + \nabla \cdot Q_e = \left\langle \widetilde{E} \cdot \widetilde{J}_e \right\rangle - n \nu \frac{m_e}{m_i} (T_e - T_i) \] ➞ Electron heat balance

Collisional  
Collisionless

- Comparsion of the collisional and collisionless energy transfer.
  - Collisionless can be dominant at low collisionalitiy

- Comparsion of energy transfer in collisionless and energy transport by heat flux \( Q \)
  - Can be of the same order
Collisionality

• Collisonality $\nu_*$ in ITER
  – dimensionless
    $$\nu_* = \frac{\varepsilon^{-3/2} R q \nu_e}{V_{th}}$$
    $\nu_* \sim 10^{-3}$

• Collisonality at crossover of collisional and collisionless coupling
  – Energy transfer in collision:
    $$Q_i \approx \frac{nm_e \nu_e \tilde{T}_e}{m_i}$$

  – Quasilinear trapped electron cooling in CTEM
    $$\left\langle \tilde{E} \cdot \tilde{J}_e \right\rangle_{b}^{(2)} \approx 4\pi^{1/2} \varepsilon^{1/2} nT_e \left( \frac{R}{2a} \right)^{3/2} \rho_*^2 (\omega - \omega_n) \left\langle J_{fe} \right\rangle$$
    $$E = \frac{\omega RT_e}{k_0 \rho \omega_s C_s}$$

  – At crossover $Q_i \approx \left\langle \tilde{E} \cdot \tilde{J}_e \right\rangle_{b}^{(2)}$
    $\nu_* \sim 10^{-2}$

The collisionless turbulent energy transfer then dominates inter-species coupling process

★
Transfer vs Transport

- The transfer and transport energy loss in CTEM
  - Compare the volume integral of the electron cooling to the surface integrated of the electron heat flux
    \[ A\tilde{Q}_e \right|_{\text{boundary}} = \int d^3 r \left\langle \tilde{E} \cdot \tilde{J} \right\rangle \]
  - The heat flux for electrons: \[ \tilde{Q}_e = \left\langle \tilde{v}_r \tilde{P}_e \right\rangle = -\frac{c}{B} \sum_k k_\theta \text{Im} \tilde{P}_e^{(1)} \tilde{\phi} \]
  - The pressure fluctuation
    \[ \tilde{P}_e^{(1)} = \int d^3 r \frac{1}{2} m v^2 \tilde{g}_b \]
    \[ \tilde{Q}_e = \sum \frac{4\pi^2 \epsilon^2}{2} \left( \frac{R}{L_n} \right)^2 \frac{\epsilon \tilde{\phi}}{T_e} \frac{V^2_{\text{th}} k_\theta n T_e}{\Omega_e} (\omega - \omega_n) \left\langle f_e \right\rangle \right] \]

The ratio

\[ \frac{\Delta r \left\langle \tilde{E} \cdot \tilde{J} \right\rangle}{\tilde{Q}_e \right|_{\text{boundary}}} \approx 2 \frac{a}{R} \sim o(1) \]

The rate of energy lost by collisionless energy transfer is comparable as turbulent transport
Result and Discussion

• Net heating
  – Quasilinear turbulent energy transfer in drift wave
    • electron cooling and ion heating
  – Nonlinear ion heating by beat wave resonance
  – Energy flux differential gives rise to the net heating ➡️ zonal flow

• Energy transfer channels
  – Identify a important energy transfer channels
  – Zonal flow frictional damping can be comparable to LD damping

• For low collisionality ITER plasma, collisionless energy transfer can be a critical element of transport model analysis
  – Collisionless energy transfer has same order as transport