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Modeling of Long-Range Frequency Sweeping Phenomena

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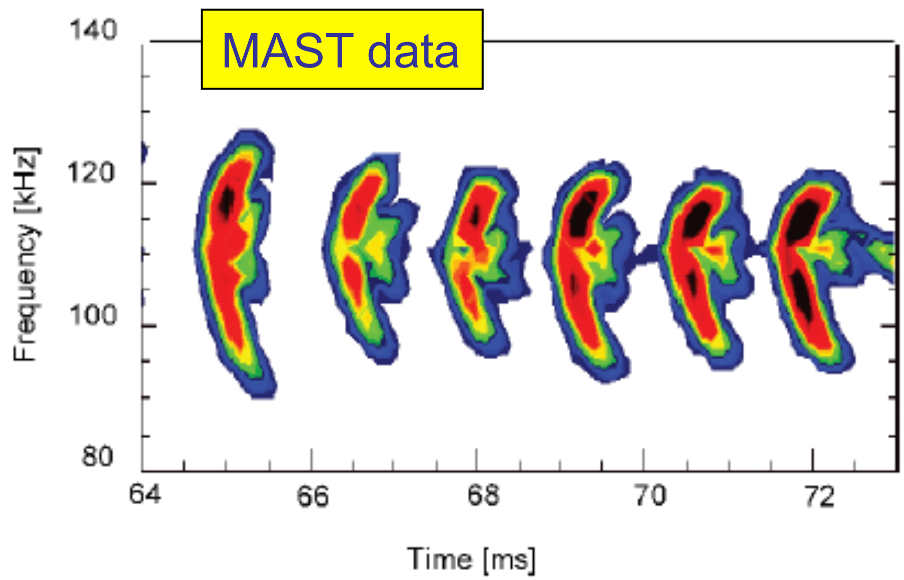
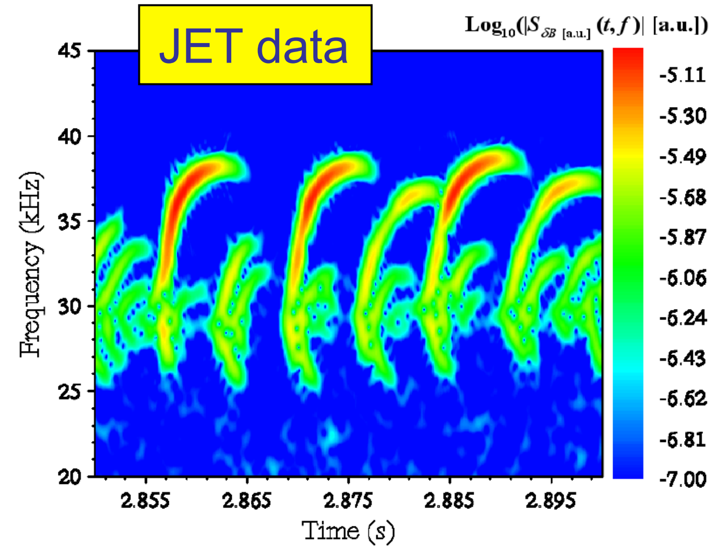
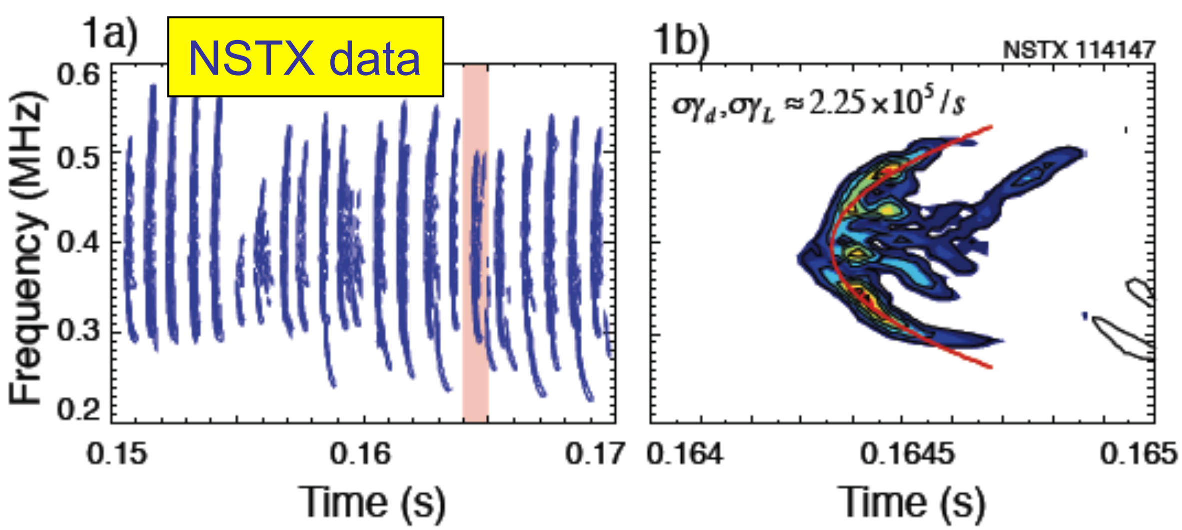
Outline

- Background frequency sweeping events (reminder)
- Bump-on-tail model (reminder)
- Adiabatic numerical model (ABBOT) for long-range sweeping
- Particle trapping into growing separatrix
- Effect of separatrix filling
- Summary

Rapid Frequency Sweeping Events

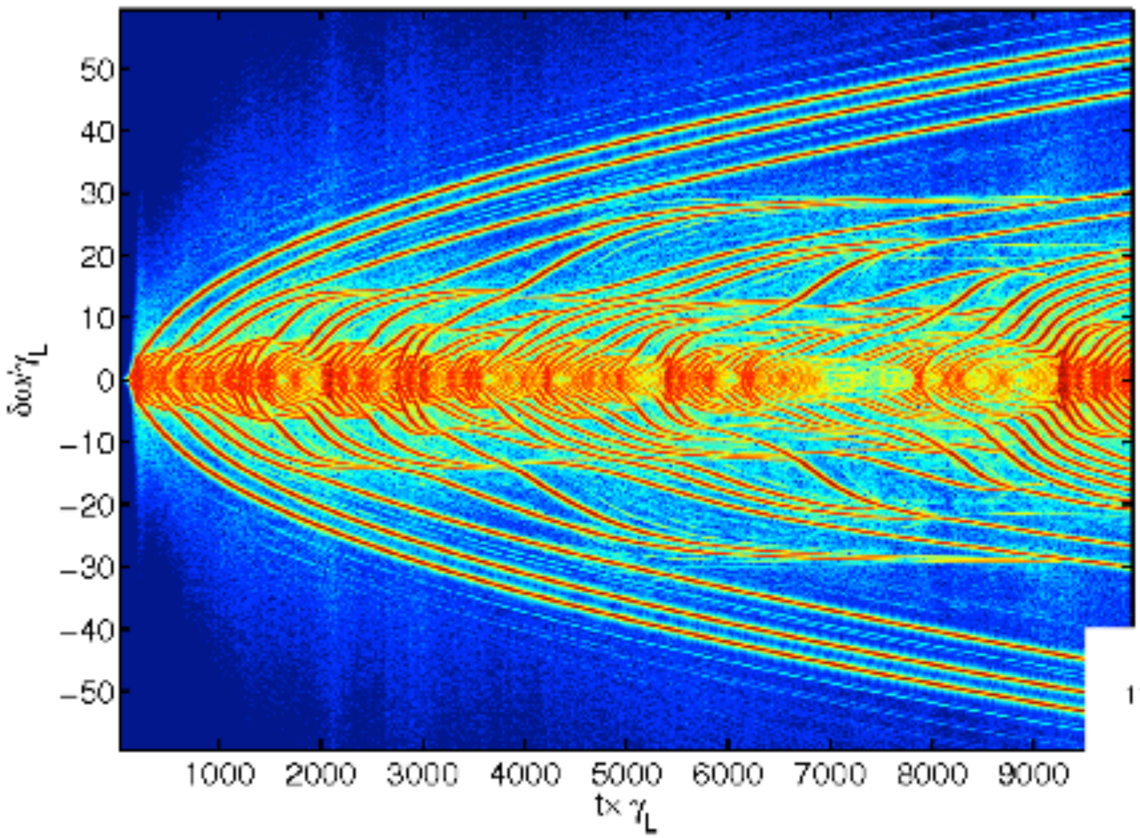






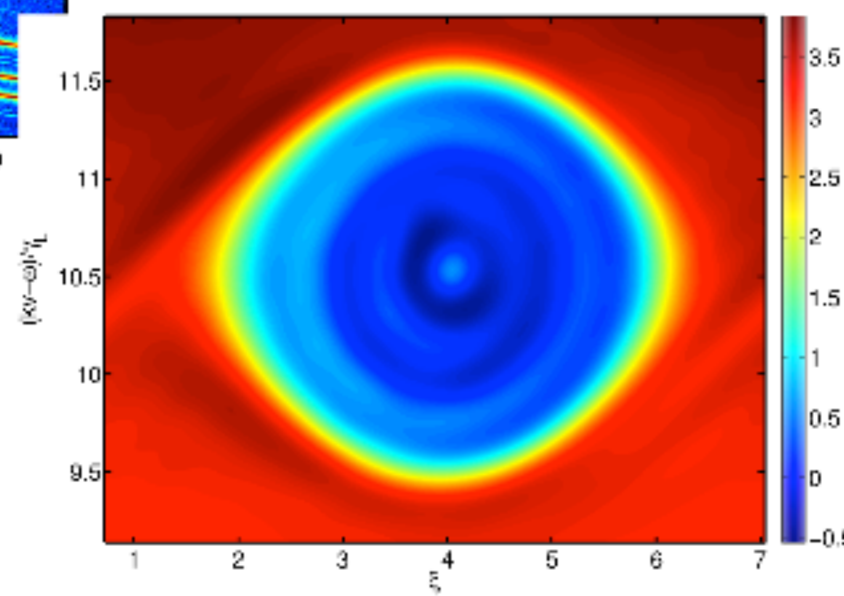
The ms timescale of these events is much shorter than the energy confinement time in the plasma

Collisionless Sweeping Simulation



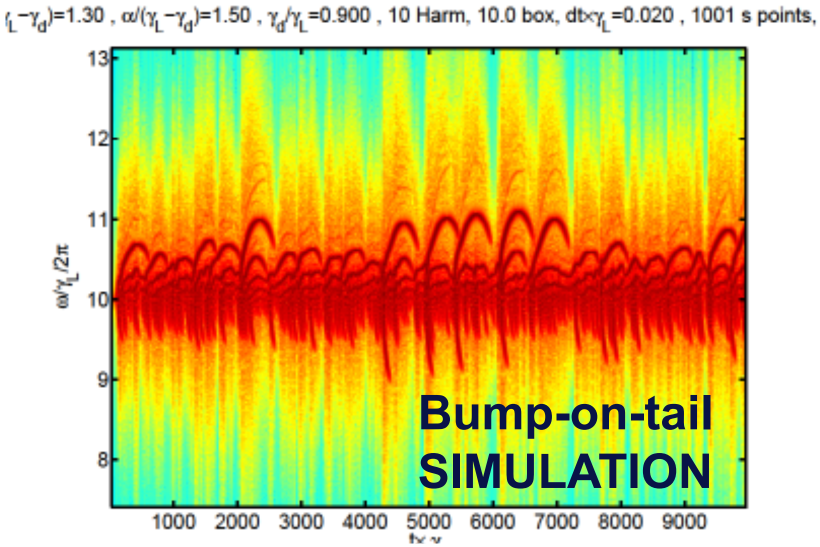
Holes and clumps form and travel in phase space.

The first hole is well phase mixed at $\gamma_L t > 7000$

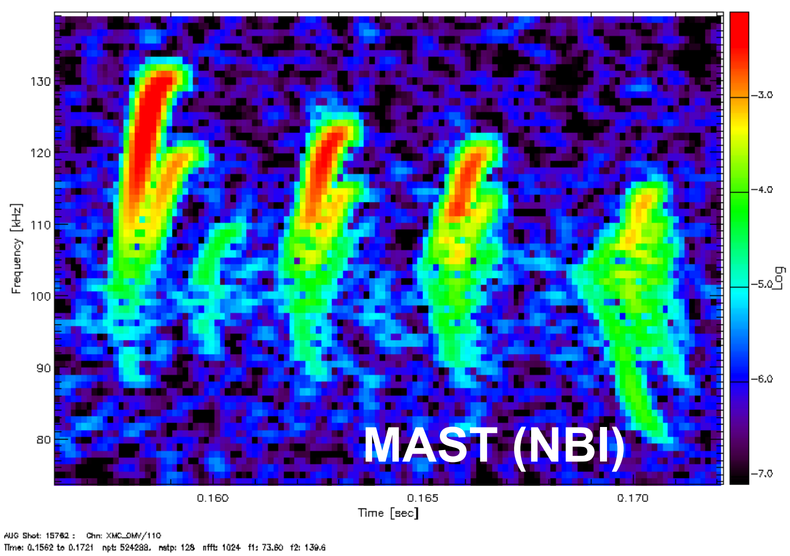


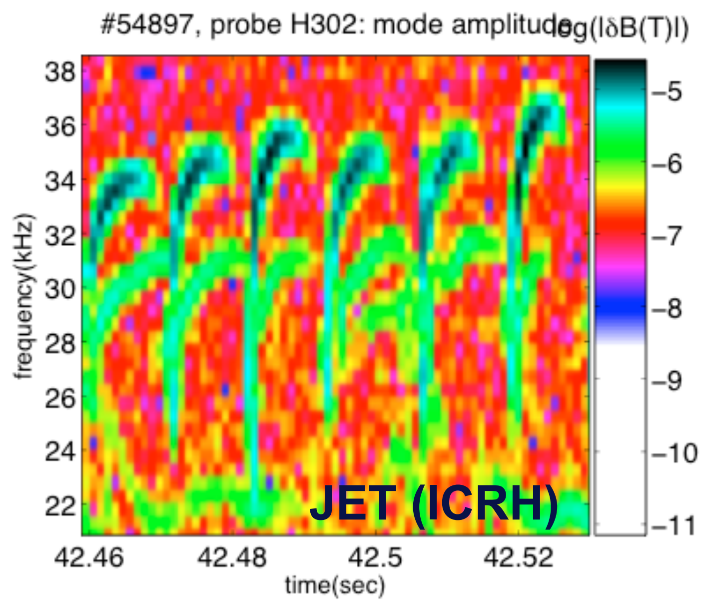
M.K. Lilley , B.N. Breizman, S.E. Sharapov, Phys. Plasmas ***17****, 092305 (2010).*

Drag+Diffusion Give Hooked Frequency Pattern



- Hooked frequency chirp seen in BOT simulations.
- Also seen in MAST (NBI) and JET (ICRH)





 *M.K. Lilley , B.N. Breizman, S.E. Sharapov, Phys. Plasmas **17**, 092305 (2010).*

Bump-on-Tail Model For Sweeping Onset

$$\boxed{\frac{\partial F}{\partial t}+u\frac{\partial F}{\partial \zeta}+\frac{ek}{2m}\left[\hat{E}(t)e^{i\zeta}+\text{c.c.}\right]\frac{\partial F}{\partial u}=\left[v^{3}\frac{\partial^{2}}{\partial u^{2}}+\alpha^{2}\frac{\partial}{\partial u}-\beta\right](F-F_{0})}$$

$$\boxed{\frac{\partial \hat{E}}{\partial t}=-4\frac{\omega}{k^{2}}\pi e\int f_{1}\,du-\gamma_{d}\hat{E}}$$

$$u\equiv k\text{v}-\omega$$

$$\zeta\equiv kx-\omega t$$

$$F=F_{0}+f_{0}+\sum_{n=1}^{\infty}\left[f_{n}\exp(in\zeta)+c.c.\right]$$

$$E=\frac{1}{2}\left[\hat{E}(t)e^{i\zeta}+\text{c.c.}\right]$$

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Time Scales and Mode Evolution Phases

γ_L - linear growth rate due to resonant particles

γ_d - background damping rate due to bulk plasma

ω_b - trapped particle bounce frequency

1. Linear near-threshold instability (excitation of a plasma eigenmode)

$$\tau_1 \sim 1/(\gamma_L - \gamma_d) << 1/\omega_b$$

2. Explosive nonlinear growth of the mode
Formation of phase-space holes and clumps with trapped particles
Initiation of frequency sweeping

$$\tau_2 \sim 1/\gamma_L \sim 1/\omega_b$$

3. Slow (adiabatic) evolution of phase-space holes and clumps
Significant frequency sweeping
Transition from the bulk plasma eigenmode to a beam mode

$$\tau_3 >> 1/\gamma_L \sim 1/\omega_b$$

How to Treat Long-range Frequency Sweeping

Observation:

Experiments exhibit signals with large
(order of unity) frequency sweeping.

Issue:

How can a small group of particles
produce a large change in the mode frequency?

Physics mechanism:

Initial instability leads creates a modulated beam of resonant particles (BGK-type structure).

As the beam particles slow down significantly , they can produce a signal that deviates considerably from the initial mode frequency.

Slowly Varying Periodic Electrostatic Wave

- **Wave electrostatic potential (with a spatial period λ):**

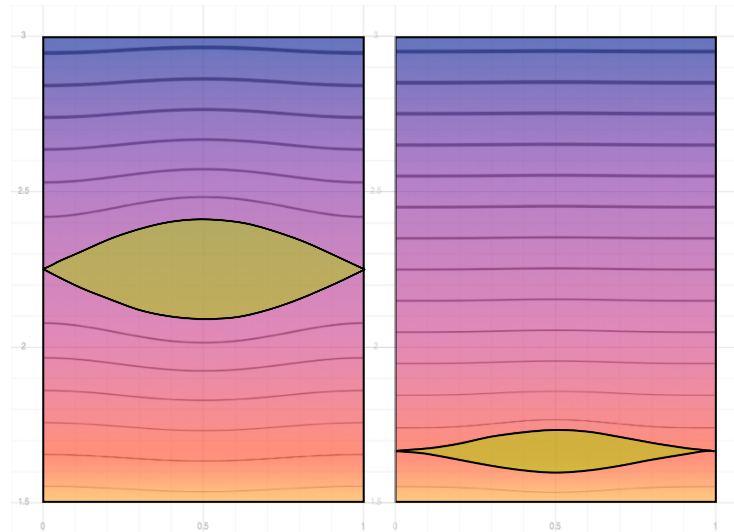
$$\varphi[z-s(t);t]$$

- **Lab-frame Hamiltonian:**

$$H(p;z;t)=\frac{p^2}{2m}-|e|\varphi[z-s(t);t]$$

- **Wave-frame Hamiltonian:**

$$H(p;x;t)=\frac{(p-m\dot{s})^2}{2m}-|e|\varphi(x;t) \qquad x=z-s(t)$$



Downward Drift of Phase Space Clump

- **Adiabatic invariants**
  - **Passing particles:** $J_{\pm}=\int_0^{\lambda}\left\{m\dot{s}\pm\sqrt{2m\left[H+|e|\varphi(x;t)\right]}\right\}dx$
  - **Trapped particles:** $J_{trapped}=\oint\sqrt{2m\left[H+|e|\varphi(x;t)\right]}dx$

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Wave Structure

- Electron potential energy U has a given spatial period λ, a slowly varying shape, and a slowly varying phase velocity $\dot{s}$:

$$U\left[z-s(t);t\right] \equiv -|e|\varphi$$

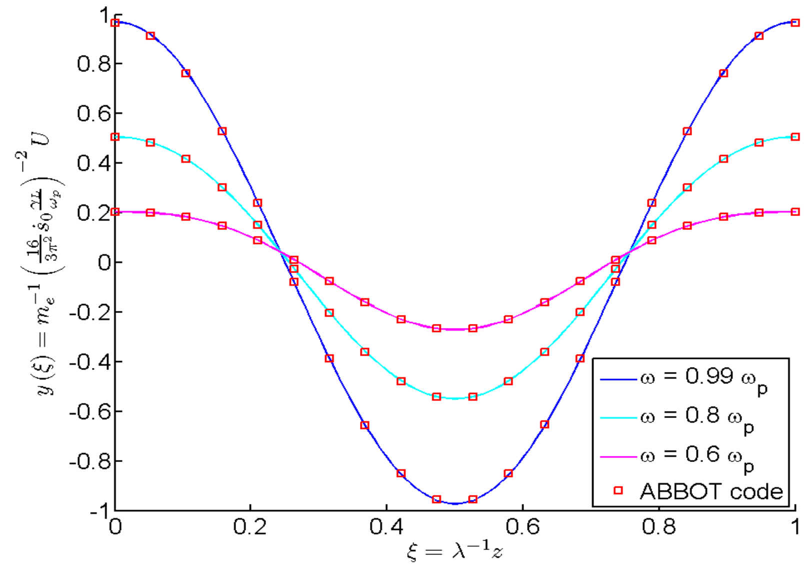
- Perturbed density of plasma electrons: $\delta n = n_0 U / m\dot{s}^2$

- Perturbed density of energetic electrons:

$$\delta n_{fast} = \sqrt{\frac{2}{m}}\int_{trapped}\frac{F(J)-F_0(\dot{s})}{\sqrt{H-U(x;t)}}\frac{dH}{dJ}dJ - \left\langle \sqrt{\frac{2}{m}}\int_{trapped}\frac{F(J)-F_0(\dot{s})}{\sqrt{H-U(x;t)}}\frac{dH}{dJ}dJ \right\rangle_x$$

- Nonlinear Poisson equation is used in the ABBOT code to find the potential:

$$\frac{\partial^2 U}{\partial x^2} = -k^2 U - 4\pi e^2 \delta n_{fast}$$
$$k^2 \equiv \omega_p^2 / \dot{s}^2$$



Perturbative Treatment of Mode Equation

- Consider small deviation of the mode phase velocity from that of the initially unstable linear mode: $\delta\dot{s} \equiv \dot{s} - \dot{s}_0 << \dot{s}_0$

- Perturbed Poisson equation: $\frac{\partial^2 U}{\partial x^2} + k_0^2 U = 2\frac{\delta\dot{s}}{\dot{s}_0}k_0^2 U - 4\pi e^2 \delta n_{fast}$

- Lowest order solution is a sinusoidal wave: $U = U_0 \cos k_0 x$

- Use the lowest order solution to calculate the small (red) terms:

$$\delta n_{fast} = \sqrt{\frac{2}{m}} \int_{trapped} \frac{F(J) - F_0(\dot{s})}{\sqrt{H - U_0 \cos(k_0 x)}} \frac{dH}{dJ} dJ - \left\langle \sqrt{\frac{2}{m}} \int_{trapped} \frac{F(J) - F_0(\dot{s})}{\sqrt{H - U_0 \cos(k_0 x)}} \frac{dH}{dJ} dJ \right\rangle_x$$

- Solvability condition requires the red terms to be orthogonal to the lowest order solution, which relates the velocity shift to the mode amplitude:

$$\frac{\delta\dot{s}}{\dot{s}_0} = \frac{2e^2}{k_0 U_0} \int_{-\pi/k_0}^{\pi/k_0} \delta n_{fast} \cos(k_0 x) dx$$

Power Balance

- Energy released by the fast particles goes into Ohmic heating of the bulk plasma.

- Collisional dissipation in the bulk:

$$Q = 2\gamma_d n_0 \frac{1}{m\dot{s}^2} \int_0^{\lambda} U^2 \, dx$$

- Power release via sweeping and drag (only trapped particle area is essential !):

$$P = -m\dot{s}\left(\frac{d\dot{s}}{dt} + \frac{\alpha^2}{k}\right) \int_{trapped} \left[F(J) - F_0(\dot{s})\right] dJ$$

- **Power balance condition ($Q = P$) determines sweeping rate.**

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Bounce-averaged Kinetic Equation

- Distribution function of passing particles remains close to their equilibrium distribution $F_0(\dot{s})$ in the vicinity of a hole or clump.

- Trapped particle equation:

$$\frac{\partial F}{\partial t} = m\frac{v^3}{k_0^2}\frac{\partial}{\partial J}\left(J\frac{\partial J}{\partial H}\frac{\partial F}{\partial J}\right) - \frac{\alpha^2}{k_0}\left(\frac{\partial F_0}{\partial V}\right)_{V=\dot{s}} - \beta\left(F - F_0(\dot{s})\right)$$

- The trapped particle domain is $0 < J < J_s(t)$, where $J_s(t)$ is the value of the adiabatic invariant at the separatrix.

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ABBOT Model: Separatrix Motion

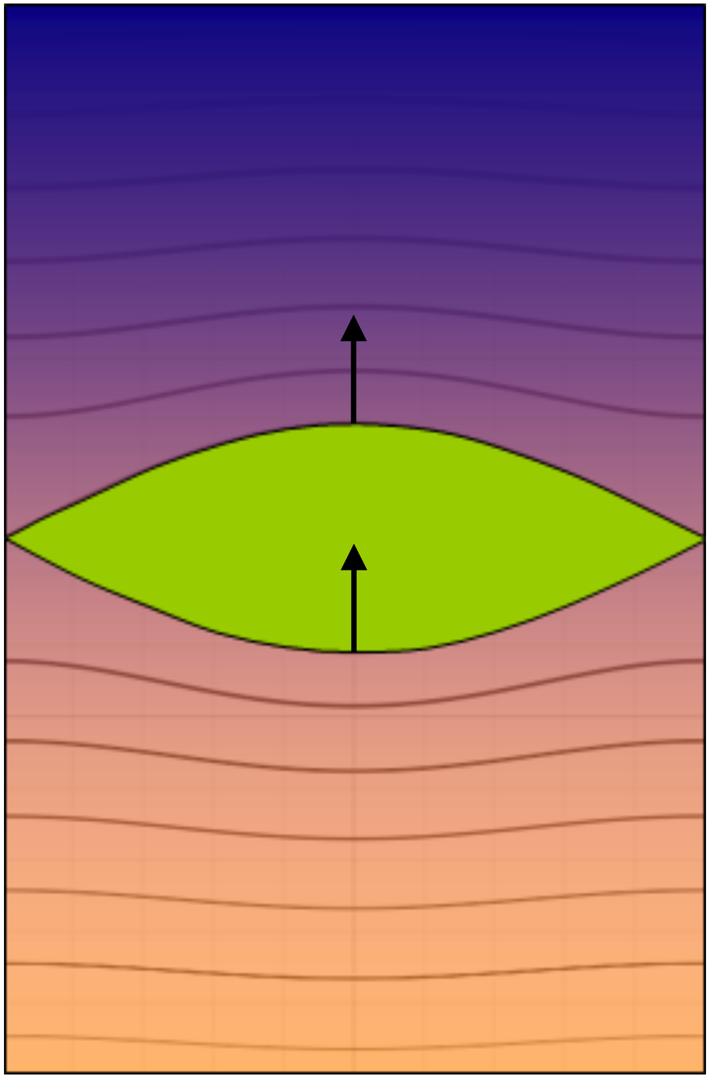
- ❑ Separatrix moves in phase space. Passing particles flow around the trapped area.

- ❑ Frequency sweeping and/or drag tend to create a discontinuity at the separatrix.

- ❑ Diffusive collisions broaden the step-like distribution function.

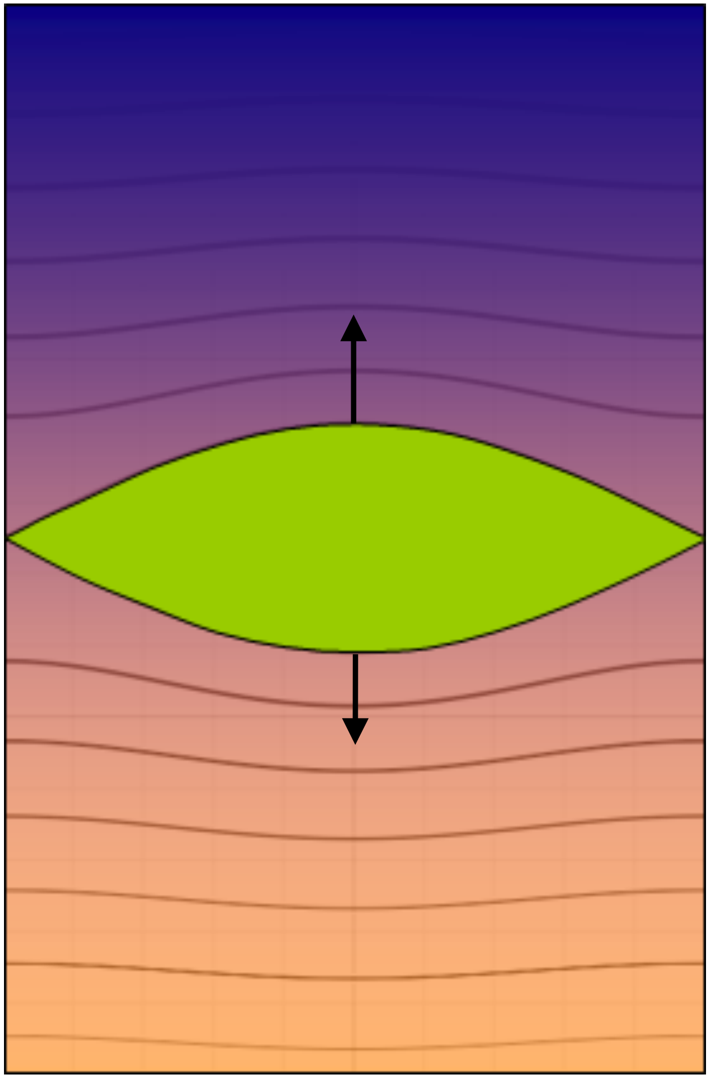
- ❑ Kinetic equation needs to be solved for trapped particles only.

- ❑ Passing particles provide a boundary condition at the separatrix : $F = F_0(\dot{s})$



ABBOT Model: Particle Trapping

- ❑ Separatrix expands when the wave field grows.
- ❑ Passing particles are captured and become trapped.
- ❑ Conservation of phase space area determines the distribution of captured particles.
- ❑ Diffusive collisions provide additional flux through the separatrix.



ABBOT Code Algorithm

1. Choose initial distribution function of trapped particles and calculate initial amplitude and phase velocity of a sinusoidal BGK wave perturbatively.

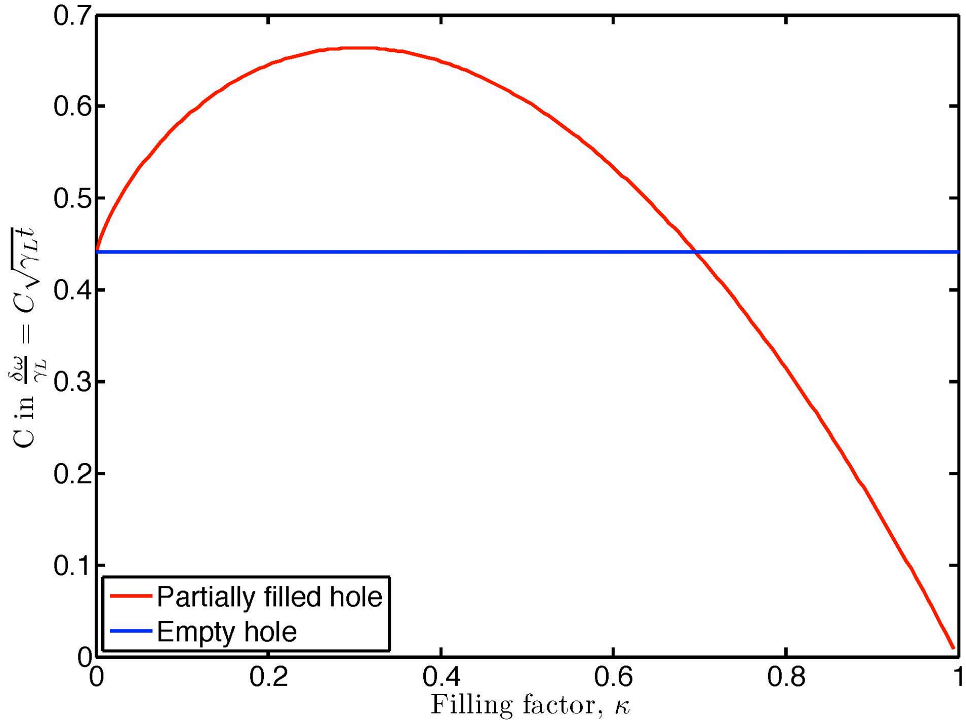
2. Advance trapped particle distribution function (kinetic equation)

3. Advance phase velocity of the wave (energy balance equation)

4. Advance wave structure and wave amplitude (Poisson equation)

5. Return to step 2.

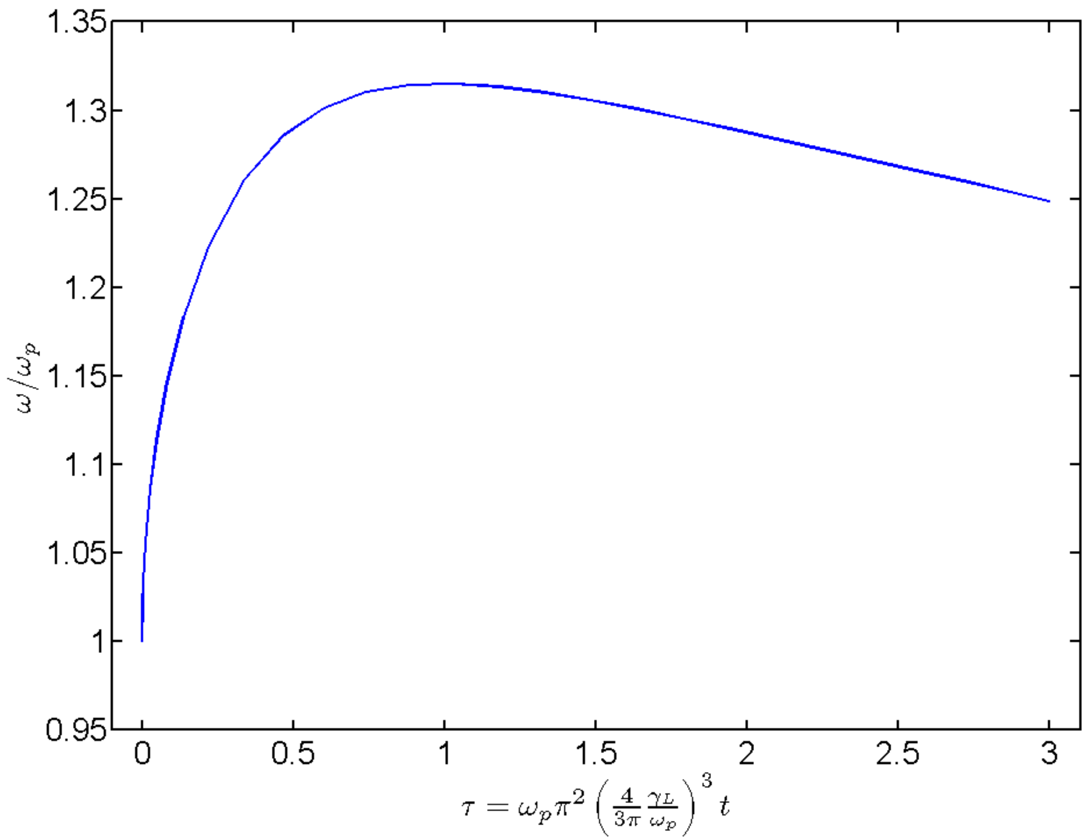
Effect of Separatrix Filling on Sweeping Rate



- Partial filling of phase space hole (near the separatrix) increases the wave amplitude and enhances collisional dissipation.
- Filling the hole at the bottom has an opposite effect.

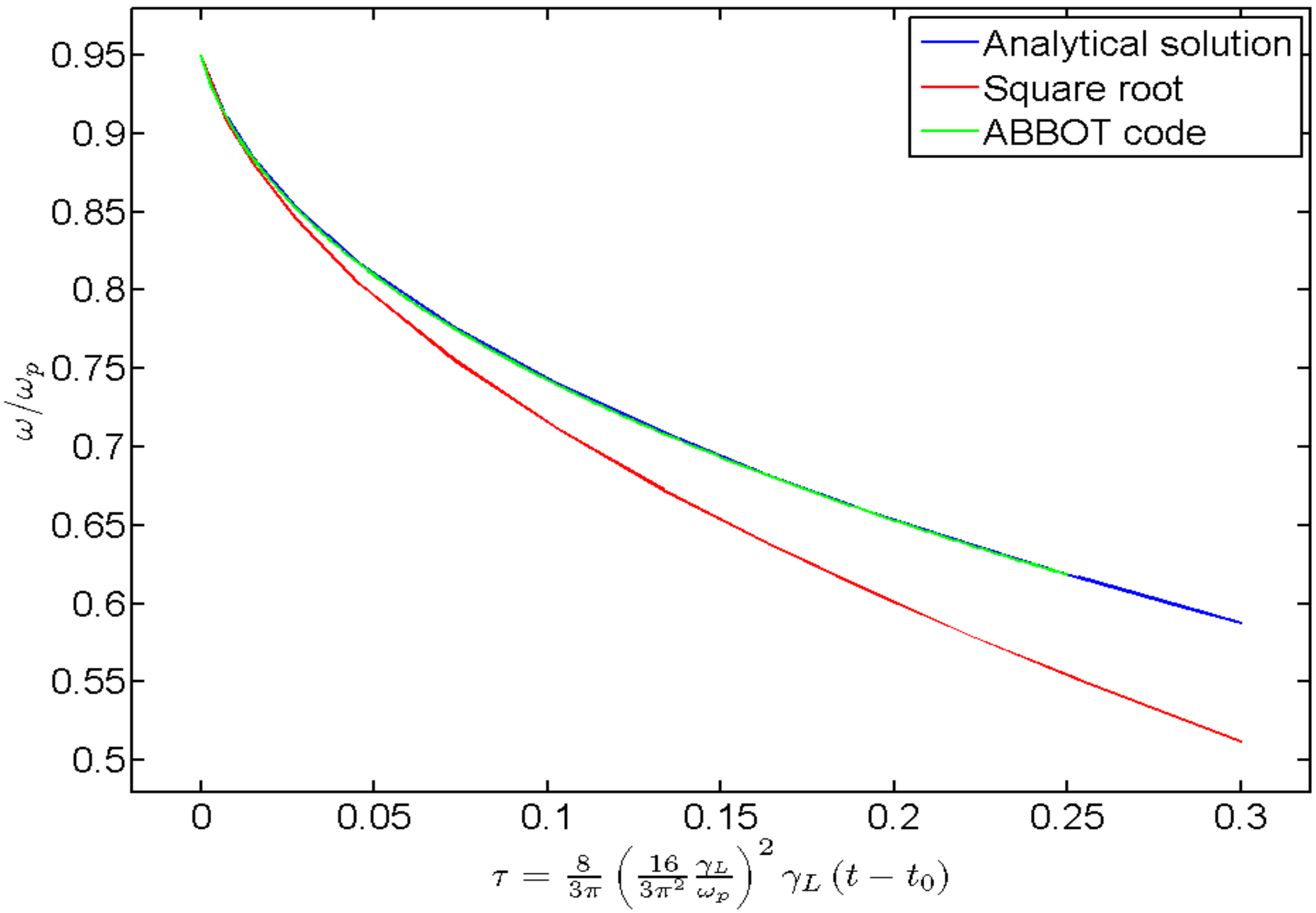
Sweep Reversal (Hook)

- Analytic solution for long-range sweeping of a hole
- Separatrix shrinks during frequency sweeping
- Drag and Krook-type collisions lead to sweep reversal



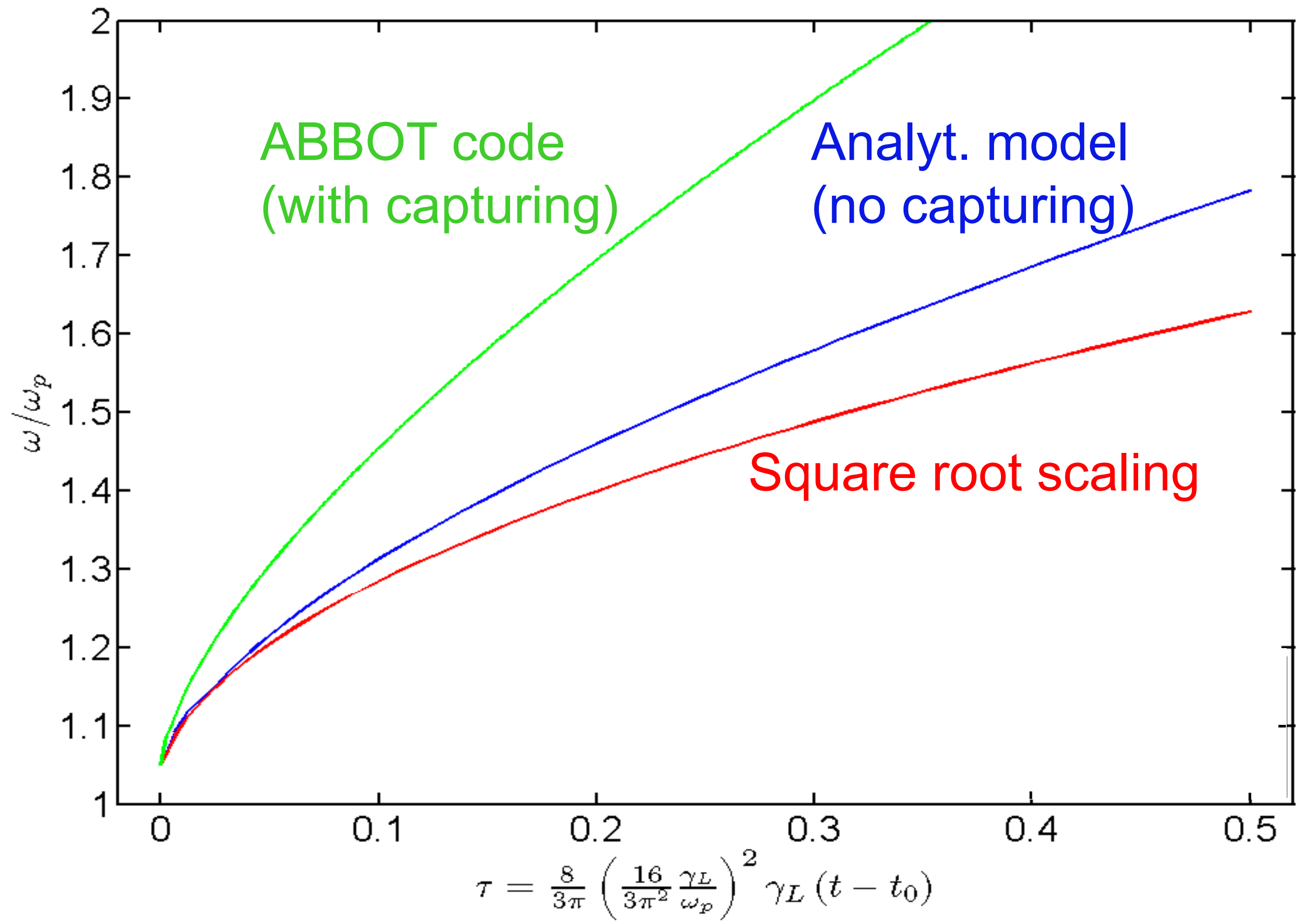
ABBOT Model: Code Benchmarking

- Simulation reproduces analytical solution for a clump



ABBOT Model: Collisionless Sweeping

- Hole sweeps faster due to electron capturing!



Summary

- New adiabatic code for long-range sweeping incorporates collisional processes and particle trapping/detrapping.
- Only trapped particles are essential in the simulations.
- Hole-clump asymmetry and evolution of the wave structure described.
- Particle trapping into phase space hole can enhance sweeping rate.