M3D-K simulation of beam-driven Alfvén modes in DIII-D

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Outline

• Motivation

• Benchmark M3D-K with NOVA using model equilibrium in mhd limit

• Comparison of mode structure between M3D-K and NOVA with DIIIID equilibrium in mhd limit

• The effect of energetic particles on mode structure and frequency in M3D-K simulations

• Comparison with DIIIID measurement
Main equations for M3D-K code

- Momentum equation: \[ \rho \frac{dv}{dt} = -\nabla p - \nabla \cdot p_h + J \times B \]

- Ohm’s law: \[ E + v \times B = \eta J \]

- Continuity equation for plasma mass density: \[ \frac{d\rho}{dt} = -\rho \nabla \cdot v \]

- Pressure equation for thermal plasmas: \[ \frac{dP}{dt} = -\gamma P \nabla \cdot v \]

- The particle pressure is calculated from particle distribution:

  \[ P_{\parallel}(x) = \int M v_{\parallel}^2 \delta(x - X - \rho_h) F(X, v_{\parallel}, \mu) d^3X d\nu_{\parallel} d\mu d\theta \]

  \[ P_{\perp}(x) = \int M v_{\perp}^2 \delta(x - X - \rho_h) F(X, v_{\parallel}, \mu) d^3X d\nu_{\parallel} d\mu d\theta \]
Introduction to M3D-K (continued)

- Drift kinetic model is used to describe energetic particles and single fluid model is used to describe thermal plasmas. The model is fully nonlinear.

- The code uses numerically-calculated equilibrium including finite beta, finite aspect ratio and shaping.

- Energetic particle collision, source and sink are included.

- Turbulence-induced energetic particle radial diffusion is included.
Model equilibrium for $n=2$, $m=4$ RSAE

$\beta=0$ and uniform plasma density are applied in this model.
n=2, m=4 RSAE model equilibrium is used to benchmark M3D-K and NOVA with analytical theory.

At low $q_{\text{min}}$, simulation shows mode transits from RSAE to TAE.
Mode structure of $n=2$, $m=4$ RSAE agrees well between NOVA and M3D-K with model equilibrium

Simulation results at $q_{\text{min}}=1.84$
DIII-D discharge #142111
Equilibrium of DIII-D discharge #142111 at time slice T~ 540ms

$\beta_{\text{tot}} = 1.84\%$, $\beta_{\text{hot}} = 0.4\beta_{\text{tot}}$, and uniform plasma density are applied in this model.
Mode structure of $n=2$ at $q_{\text{min}}=3.86$ agrees well between NOVA and M3D-K with mhd limit.
Energetic particles affect $n=2$ mode structure

- Mode rotates along poloidal direction
- Mode peak shifts from $q_{\text{min}}$ location to outside
- Mode width becomes broader
Linear mode frequency of n=2 mode is not sensitive to $q_{\text{min}}$ or fast particle pressure.

$\beta_{\text{hot}}/\beta_{\text{total}}$ increases from 4% to 40%.

$q_{\text{min}} = 3.86$ and $q_{\text{min}} = 3.92$.
Linear mode frequency of n=3 mode is not sensitive to $q_{\text{min}}$

For $T \sim 540$ ms DIII-D equilibrium

- $\beta_{\text{hot}}/\beta_{\text{total}}$ increases from 4% to 40%
- $q_{\text{min}}=3.83$
- $q_{\text{min}}=3.97$
Nonlinear evolution of n=3 mode energy at $q_{\text{min}}=3.97$
Poloidal harmonics of n=3 mode at $q_{\text{min}}=3.97$

\begin{align*}
T &= 250/\omega_a \\
T &= 310/\omega_a \\
T &= 350/\omega_a \\
T &= 400/\omega_a \\
T &= 480/\omega_a \\
T &= 620/\omega_a
\end{align*}
Middle plane mode amplitude of n=3 mode at $q_{\text{min}}=3.97$ of DIII-D equilibrium

- $T=250/\omega_A$
- $T=310/\omega_A$
- $T=350/\omega_A$
- $T=400/\omega_A$
- $T=480/\omega_A$
- $T=620/\omega_A$
Nonlinear evolution of n=3 mode frequency at \( q_{\text{min}} = 3.97 \) at different location
DIII-D discharge #142111

- n=6 RSAEs
- n=5 RSAEs
- n=4 RSAEs
- n=3 RSAEs
- n=4 RSAEs
- n=5 RSAEs

Frequency (kHz)
Time (ms)
DIII-D equilibrium at time slice around $T \sim 725\text{ms}$

$\beta_{\text{tot}} = 1.84\%$, $\beta_{\text{hot}} = 0.4\beta_{\text{tot}}$

Theory indicates dominant RSAE being $n=3$, $m=10$. 
Linear mode frequency of n=3 mode is not sensitive to $q_{\text{min}}$
Mode structure of $n=3$ at $q_{\text{min}}=3.25$ shifts outside in the presence of energetic particles compared to NOVA result.

NOVA result

M3D-K result

Plasma displacement in the middle plane

DIII-D equilibrium around time slide $T=725$ ms
Nonlinear evolution of mode kinetic energy and mode frequency at $q_{\text{min}}=3.25$

Mode frequency is measured near $q_{\text{min}}$ location.

Frequency shifts in the nonlinear stage indicates the linear mode transits to a different one.

No source/sink
Poloidal harmonics of $n=3$ mode at $q_{\text{min}}=3.25$ of DIII-D equilibrium

$T = \frac{456}{\omega_A}$

$T = \frac{600}{\omega_A}$

$T = \frac{726}{\omega_A}$

$T = \frac{852}{\omega_A}$
Middle plane mode amplitude of $n=3$ mode at $q_{\text{min}}=3.25$ of DIII-D equilibrium

\[ T = \frac{456}{\omega_A} \quad T = \frac{600}{\omega_A} \quad T = \frac{726}{\omega_A} \quad T = \frac{852}{\omega_A} \]
Conclusion

• M3d-K simulation results agree very well with NOVA in the MHD limit

• In the presence of energetic particles, both mode structure and mode frequency are different from the results in MHD limits

• The frequency of excited mode does not sweep as qmin varies, which indicates it is TAE-like mode

• Both mode structure and mode frequency change during nonlinear evolution
Future work

- Code benchmark with GEM
- To explore the effects from energetic particle profile including realistic beam distribution function
- Nonlinear evolution with source and sink
n=3, m=12 at q_min =3.83 from M3D-K for the same DIIIID equilibrium

Linear mode structure

Nonlinear mode structure
The time evolution of mode kinetic energy and mode frequency at $q_{\text{min}}=3.20$

Mode frequency is measured near $q_{\text{min}}$ location.

Frequency shifts in the nonlinear stage indicates the linear mode transits to a different one.
Poloidal harmonics of $n=3$, $m=10$ modes at $q_{\text{min}}=3.20$ of DIII-D equilibrium

$T = \frac{456}{\omega_A}$  

$T = \frac{600}{\omega_A}$  

$T = \frac{726}{\omega_A}$  

$T = \frac{832}{\omega_A}$
Middle plane mode amplitude of n=3, m=10 modes at $q_{\text{min}}=3.20$ of DIII-D equilibrium

\[
\begin{align*}
T &= 456/\omega_A \\
T &= 600/\omega_A \\
T &= 726/\omega_A \\
T &= 832/\omega_A
\end{align*}
\]