Staircases in Tokamaks

—on Rotation & Transport near Criticality—

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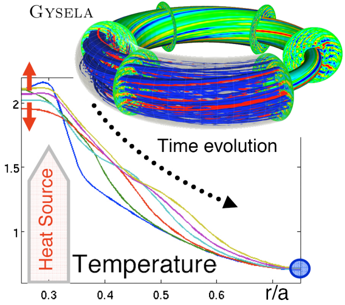
Full–f gyrokinetics, global & sources: free profile evolution

$$\partial_t(\bar{f} + \delta f) - [H, \bar{f} + \delta f] = \mathcal{C}(\bar{f} + \delta f) + \mathcal{S}(\bar{f} + \delta f) \quad \& \quad \delta n_i / n_i = \delta n_e / n_e$$

- **collisional gyrokinetic** system
- **global** geometry: large scale transport events
- **full–*f***: free profile evolution
  - ☛ non-Maxwellian ; faster than diffusive ;
  - ☛ local/non-local ; avalanching ; . . .
- internal dynamics + **flux-driven**: steady-state
  - ☛ open system ; transport "bifurcations"



NB: momentum and vorticity sources
electrostatic & adiab. electrons (*full–f* kinetic e^- under way)

Questions: What self-organised state —flow structures, gradients,. . . ?
What mechanisms for transport & rotation ?

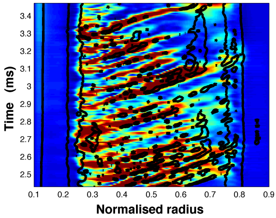
What kind of self-organisation can we expect ?

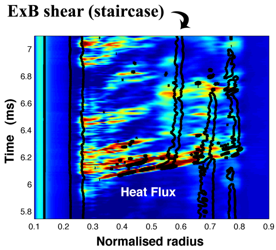
{Full–f GK + Source} ➠ T_i evolves ➠ redistribution of stored energy ?

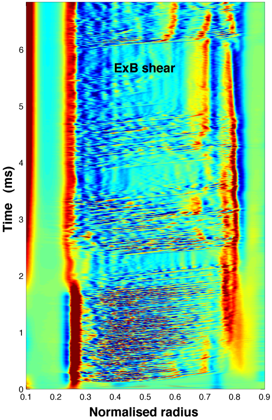
- metastable states: vortices, flows, stresses
- “order” parameters: B, q, . . .

❶ criticality = a perturbation does not propagate throughout the whole system

➠ **clustering behaviour**







❷ dynamic convergence: “staircase” of flows [Dif-Pradalier, Phys. Rev. E 2010]

❸ depending on the source: { staircase / staircase / quiescent / staircase / "dithering" ➠ pre-ITB ? } ⇓ $\mathcal{S}\nearrow$

How does criticality look like in steady-state ?

from **{Dimits upshift+weak turb.+local}** paradigm...

- intrinsically a *profile*
- "Dimits upshift" : not much sense

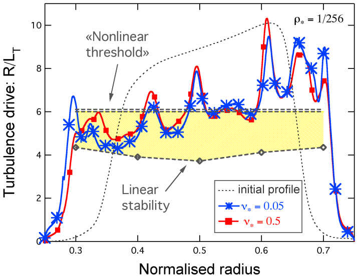
...to **"criticality"** as a dynamic convergence

➠ intrinsically a self-organising process

δT_i ➠ **I** ➠ V_E ➠ $\langle v_x v_y \rangle$

staircase nucleates from large avalanches

$$\frac{R/L_T - \langle R/L_T \rangle}{\langle R/L_T \rangle} \quad \begin{cases} \text{staircase phase} \geq 300\% \\ \text{quiescent phase} \geq 100\% \end{cases}$$

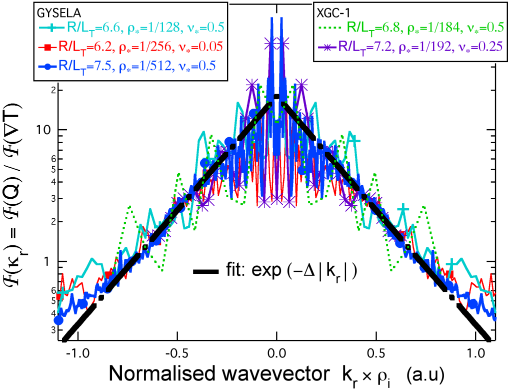


[full-f description]

$\delta T_i \Rightarrow$ I $\Rightarrow V_E \Rightarrow \langle v_x v_y \rangle$

Heat transport: non-local

$$Q = -n\chi(r)\nabla T \Rightarrow Q = -\int \mathcal{K}(r,r')\nabla T(r')\,\mathrm{d}r'$$



$$\mathcal{F}\left[f\cdot g\right](r) = \int \mathrm{d}r'\,\mathcal{F}\left[f\right](r-r')\mathcal{F}\left[g\right](r')$$

$(i)\quad \rho_\star = \frac{1}{128} \rightarrow \frac{1}{512}$

$(ii)\quad R/L_T = 6 \rightarrow 7.5$

$(iii)\quad \nu_\star = 0.05 \rightarrow 0.5$

Kernel is Lorentzian :

$$\mathcal{K}(r,r') = \frac{1}{\pi}\frac{\Delta/2}{(\Delta/2)^2+(r-r')^2}$$

δT_i ➠ I ➠ V_E ➠ $\langle v_x v_y \rangle$

Heat transport: non-local

What sets the 'influence length' Δ ?

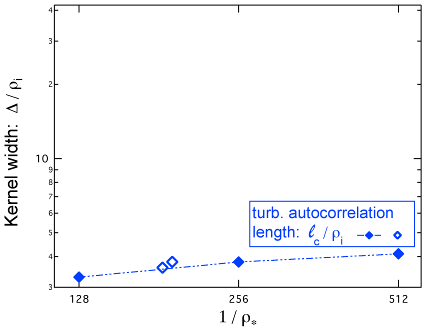
$\Delta \sim$ tail autocorrelation

$\Delta \sim$ avalanche size

$\Delta \sim$ '$\mathbf{E} \times \mathbf{B}$ staircase' width

$$\mathcal{K}(r, r') = \frac{1}{\pi} \frac{\Delta/2}{(\Delta/2)^2 + (r - r')^2}$$

For all $\rho_\star$, $\nu_\star$, R/L_T



- dynamics is scale invariant within Δ *i.e.* $\int (r - r')^2 \mathcal{K}(r, r')$ **divergent**
- heat conductivity is non-local

Connection between stochastic avalanches & mean flow pattern step

$\delta T_i \Rightarrow \mathbf{I} \Rightarrow V_E \Rightarrow \langle v_x v_y \rangle$

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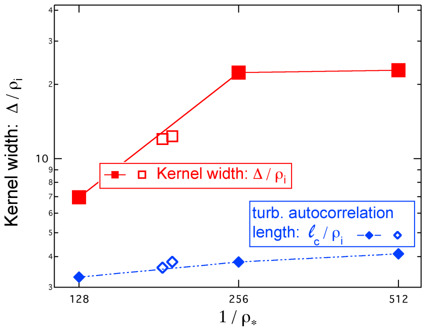

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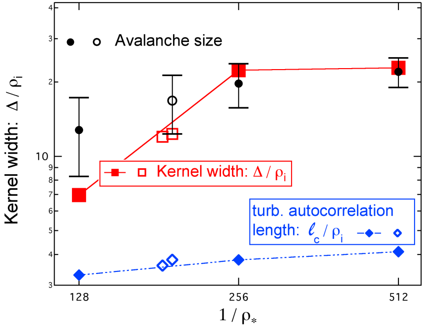

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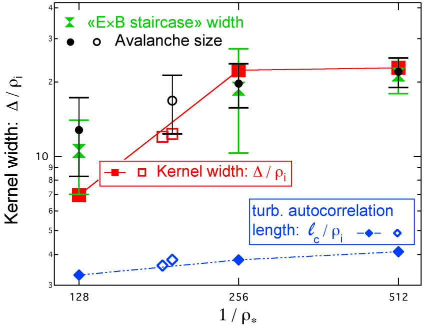

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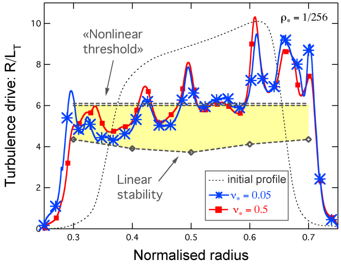
Connection between stochastic avalanches & mean flow pattern step

$\delta T_i \Rrightarrow I \Rrightarrow V_E \Rrightarrow \langle v_x v_y \rangle$

A new hierarchy of shears ?

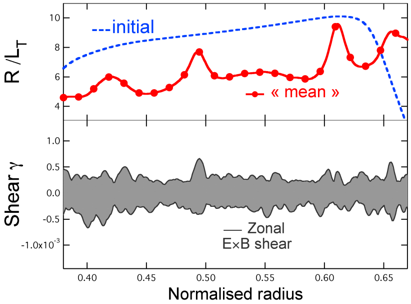


- $\gamma^{ZF} \Rrightarrow$ fluctuations around 'fixed gradient'

- during one collision time
- compute the shear :

$$\gamma_E = r\partial_r(E_r/rB)$$

[Dif-Pradalier, Phys. Rev. Lett. 2009]



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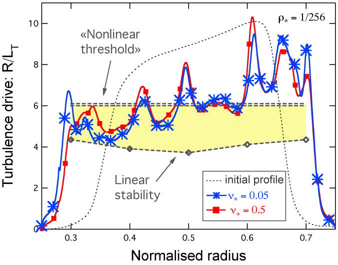

- γ^{ZF} ➠ fluctuations around 'fixed gradient'
- γ^{MF} ➠ steady-state shear due to **mean profile corrugation**

Equilibrium E × B flows

➠ **"mean shear" ≫ "zonal shear"**

- during one collision time
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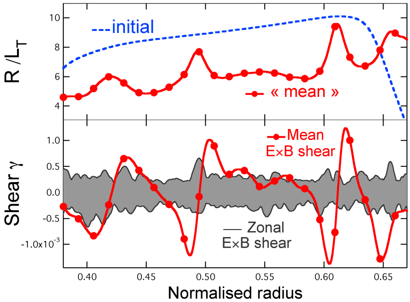
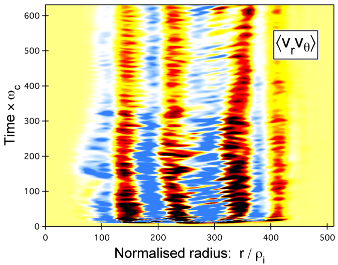


$\delta T_i \Rightarrow I \Rightarrow V_E \Rightarrow \langle v_x v_y \rangle$

From turbulent stresses. . .



Generation of Reynolds stresses

- connection to drive/dissipation profile of turb.
- relation to potential vorticity flux

[McDevitt, Phys. Plasmas 2010]

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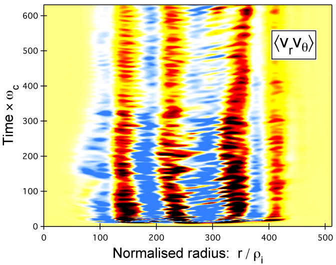

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. . . to poloidal flows

$$\langle v_\theta \rangle = \langle v_\theta^{NC} \rangle - \frac{1}{\mu_{ii}} \frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \langle \tilde{v}_\theta \tilde{v}_r \rangle \right)$$

- precursor for L–H bifurcation [Diamond '94]
- contribution to E_r [Burrell '94, McDermott '09]
- $v_\theta \neq v_\theta^{NC}$ [Koide '94, Bell '98, Crombé '05]

w/o turb : $v_\theta = v_\theta^{NC}$

with turb : $v_\theta \neq v_\theta^{NC}$ ➠ v_θ^{turb} (?)

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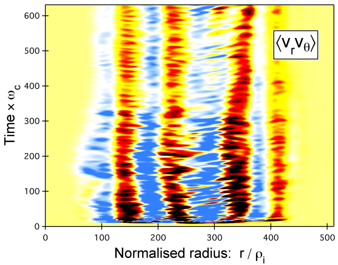

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Trends for v_θ^{turb} :

- $\mu_{ii} \propto \nu_{ii}$
  ➠ **low collisionality**
- **barrier regime**

Turbulent poloidal flows

« Worst case » :

L-mode parameters

[Dif-Pradalier, Phys. Rev. Lett. 2009]

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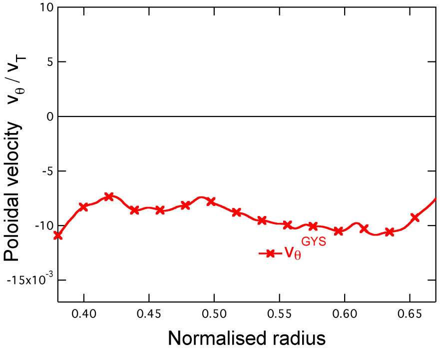

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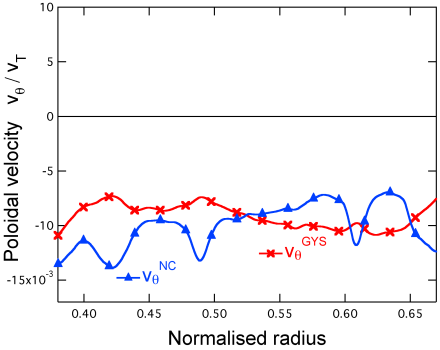

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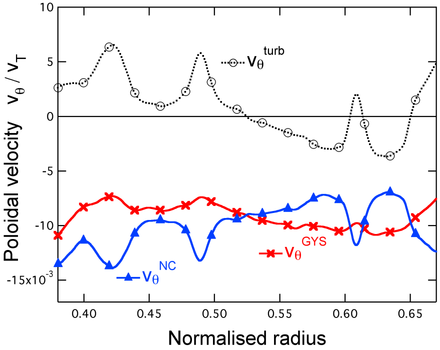

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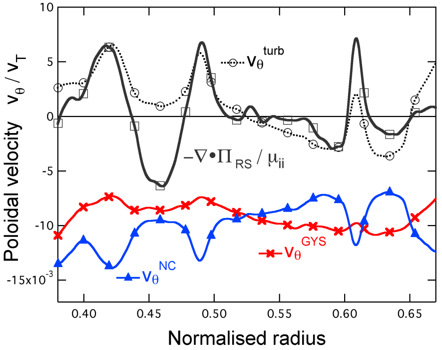

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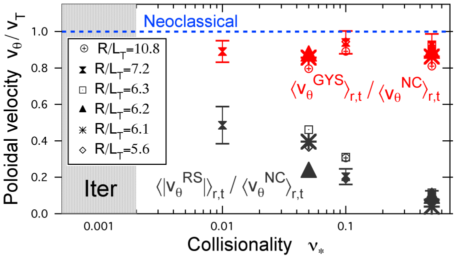

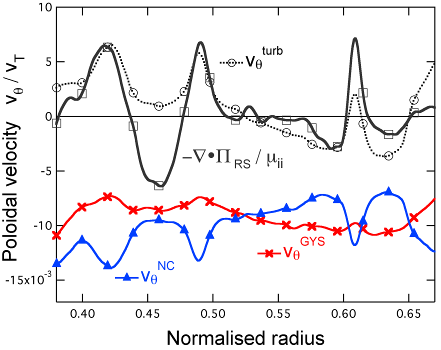

Low collisionality : $\left| v_\theta^{turb} \right| \nearrow$

Take-home ideas:

Free profile evolution (full–f) + sources ➠ a renewed way to think about criticality

- beyond {Dimits upshift+weak turb.+local} paradigm
- clustering behaviour ➠ staircase as a dynamic convergence
- emphasis on physics at mesoscale ➠ novel flow & stress patterns
- rich dynamics: "dithering" staircase/quiescent ➠ ITB ?

Practical consequences:

- heat transport is non-local, no ad-hoc assumptions needed
- additional mean $\mathbf{E} \times \mathbf{B}$ shear
- generation of poloidal rotation from turbulence
- poloidal rotation & {ZF + MF}: symmetry breakers for toroidal rotation [Kwon, this morning]