Intrinsic Rotation Generation as a Plasma Flow Engine Process

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Motivation

"We must attribute to heat the great movements that we observe all about us on the Earth. Heat is the cause of currents in the atmosphere, of the rising motion of clouds, of the falling of rain and of other atmospheric phenomena ..."

S. Carnot, 1824 $\rightarrow$ alumnus of Ecole Polytechnique

$\Rightarrow$ Role of heat in driving fluid motions

$\Rightarrow$ Is there a simple physics concept underlying intrinsic rotation in tokamaks?

$\Rightarrow$ How and Why does intrinsic rotation scale with plasma parameters?

Outline

- Background $\Rightarrow$ A paradigm: Intrinsic Rotation as Heat Engine
- Entropy: setting-up "engine" calculation
  - relation to δf^2 balance $\rightarrow$ production and nonlinear coupling
  - Stationary state with $\langle V_E' \rangle$ $\rightarrow$ akin to improved confinement
- Plasma Flow Engine
  - Intrinsic Rotation (IR) as a consequence of engine process

    $\rightarrow$ Conversion of Heat (∇T) to Flow ($\langle V_\parallel \rangle$) by residual stress
  - Comparison to experiment, C-Mod data:

$$\frac{\langle V_\parallel \rangle}{v_{thi}} \cong \frac{\rho_*}{2} \frac{\chi_i}{\chi_\phi} \sqrt{\frac{T_i}{T_e}} \frac{L_s}{L_T} \sim 0.18$$

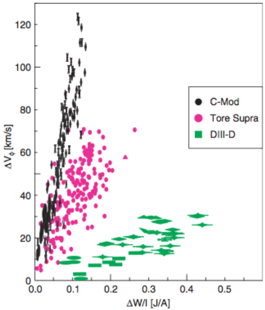
- Conclusion

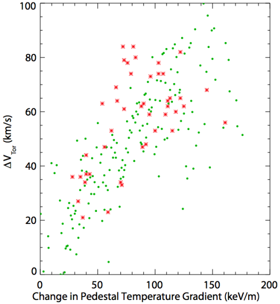
Background

- Tokamak plasmas observed to rotate via input of heat, in the absence of obvious momentum input $\rightarrow$ Intrinsic Rotation
- Scalings in improved confinement:

	H-mode	I-mode
Global	$\Delta v_\phi \sim \Delta W_p / I_p$	$\Delta v_\phi \sim \Delta W_p / I_p$
Local	$\Delta v_\phi \sim \nabla p \sim \nabla T$	$\Delta v_\phi \sim \nabla T$

$\Rightarrow \Delta v_\phi \sim \nabla T$ both in H- and I-mode





Background, Engine

- a la Carnot and $\Delta v_\phi \sim \nabla T \rightarrow$ a consequence of "engine" process

$Q \Rightarrow \nabla T \Rightarrow$ Turbulence →
- $\int \chi_i (\nabla T)^2$, Thermal Relaxation
- $\langle V_E \rangle' \sim v_{thi}/L_T$, Zonal Flow
- $\Delta v_\phi(0) \sim \Delta W_p / I_p$, Intrinsic Flow

Y.K. et al '10

- How quantify plasma as engine?
  - Entropy budget of plasma $\rightarrow$ evolution via heat input, flow generation, spatial flux of entropy, ...
  - Based on the entropy budget, modeling of $\Pi^{res}_{r\|}(\nabla T, V'_E, \ldots)$

Entropy (Coarse grained) $\Rightarrow \langle \delta f^2 \rangle$ evolution

- Entropy evolution: $\partial_t \int d\Gamma \langle \delta f^2 \rangle / \langle f \rangle \sim \partial_t S_0$, $S_0 \equiv -\int d\Gamma \langle f \rangle \ln \langle f \rangle$: Entropy from coarse grained f, $d\Gamma \equiv d^3v d^3x$
- $\langle \delta f^2 \rangle \rightarrow$ Drift kinetic equation for ions + adiabatic electrons

$$\partial_t f + v_\parallel \nabla_\parallel f + \frac{c}{B} \hat{\mathbf{z}} \times \nabla \tilde{\phi} \cdot \nabla f + \frac{|e|}{m_i} \tilde{E}_\parallel \frac{\partial f}{\partial v_\parallel} = C(f), \quad \frac{\delta n_e}{n_0} = \frac{|e| \tilde{\phi}}{T_e}$$

- Entropy/$\langle \delta f^2 \rangle$ evolution:

$$\int d\Gamma \left\{ \frac{\partial}{\partial t} \frac{\langle \delta f^2 \rangle}{2\langle f \rangle} + \underbrace{\frac{\partial}{\partial x} \frac{\langle \tilde{V}_x \delta f^2 \rangle}{2\langle f \rangle}}_{\text{entropy flux}} + \underbrace{\nu_c \frac{\langle \delta f^2 \rangle}{\langle f \rangle}}_{\text{collision}} \right\} = \underbrace{\int d^3x \mathcal{P}}_{\text{production rate}}$$

where $C(\delta f) = -\nu_c \delta f$

  - RHS: Production rate $\mathcal{P}$

$$\mathcal{P} \equiv \int d^3v \left\{ -\langle \tilde{V}_r \delta f \rangle \frac{\langle f \rangle'}{\langle f \rangle} - \frac{e}{m} \frac{\langle \tilde{E}_\parallel \delta f \rangle}{\langle f \rangle} \frac{\partial \langle f \rangle}{\partial v_\parallel} \right\}$$

  $\rightarrow$ free energy in configuration and velocity space

  - LHS: spatial transport of $\langle \delta f^2 \rangle$/entropy + Collisional dissipation

RHS: Production rate

$$\int d^3x\mathcal{P} = \int d\Gamma \left\{ -\langle \tilde{V}_r \delta f \rangle \frac{\langle f \rangle'}{\langle f \rangle} - \frac{e}{m} \frac{\langle \tilde{E}_\parallel \delta f \rangle}{\langle f \rangle} \frac{\partial \langle f \rangle}{\partial v_\parallel} \right\}$$

$$= \int d^3x \left\{ \underbrace{-\frac{n\langle T_i \rangle'}{T_i^2} \langle \tilde{V}_r \tilde{T} \rangle - \frac{n\langle V_\perp \rangle'}{v_{thi}^2} \langle \tilde{V}_r \tilde{V}_\perp \rangle - \frac{n\langle V_\parallel \rangle'}{v_{thi}^2} \langle \tilde{V}_r \tilde{V}_\parallel \rangle}_{\text{spatial part, } \langle \ldots \rangle' \text{ and } \langle \tilde{V}_r \delta f \rangle \rightarrow \text{ Heat flux, mom. flux } \perp \& \parallel} + \underbrace{\frac{1}{T_i} \langle \tilde{J}_\parallel^i \tilde{E}_\parallel \rangle}_{\text{Parallel heating}} \right\}$$

- Spatial Part: Generic form of entropy production, $\sum_i \mathcal{J}_i \mathcal{X}_i$ where

  Generalized Fluxes: $\mathcal{J}_i = \{\langle \tilde{V}_r \tilde{T} \rangle,\ \langle \tilde{V}_r \tilde{V}_\perp \rangle,\ \langle \tilde{V}_r \tilde{V}_\parallel \rangle\}$

  Thermodynamic Forces: $\mathcal{X}_i = \{-\langle T \rangle',\ -\langle V_\perp \rangle',\ -\langle V_\parallel \rangle'\}$

  $\rightarrow$ Need modeling of $\mathcal{J}_i[\mathcal{X}_i]$, Flux-Gradient relation

- Velocity part: Parallel heating

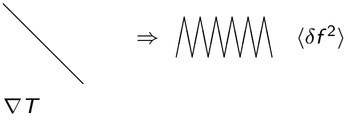
Production, $O(k_\perp)$

$$\int d^3x\mathcal{P} = \int d^3x \left\{ \boxed{-\frac{n\langle T\rangle'}{T_i^2}\langle \tilde{V}_r \tilde{T}\rangle} - \boxed{\frac{n\langle V_\perp\rangle'}{v_{thi}^2}\langle \tilde{V}_r \tilde{V}_\perp\rangle} - \frac{n\langle V_\parallel\rangle'}{v_{thi}^2}\langle \tilde{V}_r \tilde{V}_\parallel\rangle + \frac{1}{T_i}\langle \tilde{J}_\parallel^i \tilde{E}_\parallel\rangle \right\}$$

$\rightarrow$ dominant terms, $O(k_\perp)$: Heat input, Zonal Flow growth

- $\langle \tilde{V}_r \tilde{T}\rangle = -\chi_i \langle T\rangle'$, χ_i turbulent thermal conductivity
- positive definite: $\chi_i \langle T\rangle'^2 > 0$

  $\rightarrow$ production of $\langle \delta f^2\rangle$/entropy



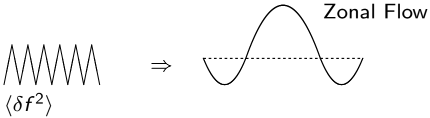
- $\langle \tilde{V}_r \tilde{V}_\theta\rangle \rightarrow$ wave kinetic equation

$$\langle \tilde{V}_r \tilde{V}_\theta\rangle = K\langle V_E'\rangle, \; K \equiv \sum_k \frac{k_\theta^2 \rho_s^2 c_s^2}{\Delta\omega_k}\left(-k_r \frac{\partial\langle \eta_k\rangle}{\partial k_r}\right)$$

$$\langle \eta_k\rangle = (1 + k_\perp^2 \rho_s^2)^2 |\hat{\phi}_k|^2, \; K = \gamma_{ZF} L_l^2$$

- negative definite: $-K\langle V_E'\rangle^2 < 0$ when $\gamma_{ZF} > 0$

  $\rightarrow$ destruction of $\langle \delta f^2\rangle$/entropy



Production, spatial, Mom. Flux ($\|$)

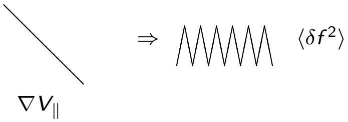
$$\int d^3x\mathcal{P} = \int d^3x \left\{ \frac{n\chi^i_{turb}}{L_T^2} - \frac{nK\langle V_E' \rangle^2}{v_{thi}^2} - \boxed{\frac{n\langle V_\| \rangle'}{v_{thi}^2}\langle \tilde{V}_r \tilde{V}_\| \rangle} + \frac{1}{T_i}\langle \tilde{J}^i_\| \tilde{E}_\| \rangle \right\}$$

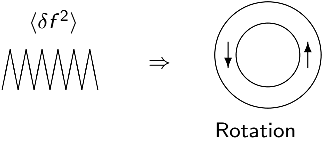
– Mean field model: $\langle \tilde{V}_r \tilde{V}_\| \rangle = -\chi_\phi \langle V_\| \rangle' + \Pi^{res}_{r\|}$ $\rightarrow$ neglect pinch for simplicity

– At a stationary state $\langle \tilde{V}_r \tilde{V}_\| \rangle = 0$; however, it consists of non-trivial competing parts

- Diffusive part
  $\rightarrow$ turbulent viscous heating
- positive definite: $\chi_\phi \langle V_\| \rangle'^2 > 0$
  $\rightarrow$ production of $\langle \delta f^2 \rangle$

- Work done by residual stress
  $\rightarrow$ generation of net rotation
- negative definite:
  $-\langle V_\| \rangle' \Pi^{res}_{r\|} = -(\Pi^{res}_{r\|})^2/\chi_\phi < 0$
  $\rightarrow$ destruction of $\langle \delta f^2 \rangle$





LHS: What competes vs Production?

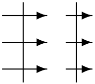
Entropy/$\langle \delta f^2 \rangle$ balance:

$$\int d\Gamma \left\{ \frac{\partial}{\partial t} \frac{\langle \delta f^2 \rangle}{2\langle f \rangle} + \underbrace{\frac{\partial}{\partial x} \frac{\langle \tilde{V}_x \delta f^2 \rangle}{2\langle f \rangle}}_{\text{flux of entropy}} + \underbrace{\nu_c \frac{\langle \delta f^2 \rangle}{\langle f \rangle}}_{\text{collision}} \right\} = \underbrace{\int d^3x \mathcal{P}}_{\text{production rate}}$$

- Spatial transport of $\langle \delta f^2 \rangle$
  - jump in the flux through boundaries

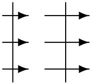
$$\int_{x-\Delta}^{x+\Delta} dx \frac{\partial}{\partial x} \frac{\langle \tilde{V}_x \delta f^2 \rangle}{2\langle f \rangle} = \left. \frac{\langle \tilde{V}_x \delta f^2 \rangle}{2\langle f \rangle} \right|_{x-\Delta}^{x+\Delta}$$

$\rightarrow$ effective local production or destruction of $\langle \delta f^2 \rangle$



$\Rightarrow \langle \delta f^2 \rangle \uparrow$

or



$\Rightarrow \langle \delta f^2 \rangle \downarrow$

- Collisional dissipation $\rightarrow$ local decrement of $\langle \delta f^2 \rangle$

$\langle \delta f^2 \rangle$ balance and stationarity

δf^2 balance (with $O(k_\perp)$ production):

$$\int d\Gamma \left\{ \frac{\partial}{\partial t} \frac{\langle \delta f^2 \rangle}{2\langle f \rangle} + \frac{\partial}{\partial x} \frac{\langle \tilde{V}_x \delta f^2 \rangle}{2\langle f \rangle} + \frac{\nu_c \langle \delta f^2 \rangle}{\langle f \rangle} \right\} = \int d^3x \left(\frac{n\chi_i}{L_T^2} - \frac{nK\langle V_E' \rangle^2}{v_{thi}^2} \right)$$

$\Rightarrow \langle V_E' \rangle^2$ at a stationary state,

$$\overline{\frac{K\langle V_E' \rangle^2}{v_{thi}^2}} = \underbrace{\overline{\left(\frac{\chi_i}{L_T^2} \right)}}_{\text{local production}} - \frac{1}{n} \int d^3v \left\{ \underbrace{\frac{\langle \tilde{V}_x \delta f^2 \rangle}{2\langle f \rangle} \Bigg|_{x-\Delta}^{x+\Delta}}_{\text{jump in } \delta f^2 \text{ flux}} + \underbrace{\frac{\nu_c \overline{\langle \delta f^2 \rangle}}{\langle f \rangle}}_{\text{local decrement}} \right\}$$

where $\overline{(\ldots)} \equiv (2\Delta)^{-1} \int_{x-\Delta}^{x+\Delta} dx(\ldots)$

- $\langle V_E' \rangle^2$ set by:
  - heat input $\rightarrow$ local production of $\langle \delta f^2 \rangle$
  - jump in $\langle \delta f^2 \rangle$-flux: de/increment of $\langle \delta f^2 \rangle \rightarrow$ de/increase $\langle V_E' \rangle^2$
  - local destruction of $\langle \delta f^2 \rangle$ from collision $\rightarrow$ reduce $\langle V_E' \rangle^2$

Residual stress

- $\langle V_E' \rangle \rightarrow$ residual stress $\Pi_{r\|}^{res}(\nabla T_i, \langle V_E' \rangle, ...) \rightarrow$ intrinsic rotation $\langle V_\| \rangle$
- A specific model for $\Pi_{r\|}^{res} \rightarrow$ Gurcan et al '07 and Diamond et al '08

- Net wave momentum

  $\rightarrow k_\|$ symmetry breaking by $\langle V_E' \rangle$ (O.G. et al '07)

$$\left\langle \frac{k_\|}{k_\theta} \right\rangle_{spec} = \frac{\rho_*}{2} \frac{\langle V_E' \rangle}{c_s / L_s}$$

- Transport of wave momentum

  $\rightarrow$ residual stress on flow (P.D. et al '08)

$$\Pi_{r\|}^{res} = \sum_k k_\| v_{g,r} \left\{ -\frac{1}{\Delta\omega_k} \left[v_{g,r} \frac{\partial \langle N_k \rangle}{\partial r} - k_\theta \langle V_E' \rangle \frac{\partial \langle N_k \rangle}{\partial k_r} \right] \right\}$$

$$\simeq \left\langle \frac{k_\|}{k_\theta} \right\rangle_{spec} K \langle V_E' \rangle, \ K \equiv \sum_k k_\theta^2 \Delta\omega_k^{-1} v_{g,r} \frac{\partial \langle N_k \rangle}{\partial k_r}$$

Toroidal Flow

Residual stress, triplet closure + integrating $-\chi_\phi \langle V_\parallel \rangle' + \Pi^{res}_{r\parallel} = 0$ yields...

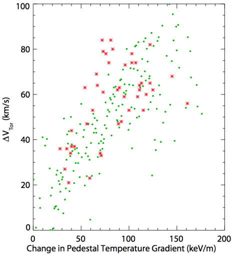
$$\frac{\langle V_\parallel \rangle}{v_{thi}} = \underbrace{\frac{\rho_*}{2}\sqrt{\frac{T_i}{T_e}}\frac{\chi_i}{\chi_\phi}\frac{L_s}{|L_T|}}_{\nabla T \text{ driven}, \propto \nabla T / B_\theta} - \underbrace{\frac{\rho_*}{2}\sqrt{\frac{T_i}{T_e}}\frac{L_s}{2\Delta}\frac{1}{n}\int d^3v \int^x dx' \frac{1}{\chi_\phi}\left(-D_{spr}\frac{\partial}{\partial x}\frac{\langle \delta f^2 \rangle}{2\langle f \rangle}\right)\Big|_{x'-\Delta}^{x'+\Delta}}_{\langle \delta f^2 \rangle\text{-flux driven} \rightarrow \text{jump in } \langle \delta f^2 \rangle \text{ flux}}$$

where $D_{spr} = \sum_p Re(g_p) c_s^2 |\hat{\phi}|_p^2$ and $g_p = (-i\omega + ip_\parallel v_\parallel + ip_\theta \langle V_E \rangle + \Delta\omega_p)^{-1}$

- ∇T driven part $\propto \nabla T / B_\theta$
  - ∇T scaling both in I- and H-mode
  - B_θ^{-1}: Inverse current scaling

    $\rightarrow$ through $L_s \propto q \propto B_\theta^{-1}$
- $\langle \delta f^2 \rangle$-flux driven part
  - $\langle \tilde{V}_x \delta f^2 \rangle \cong -D_{spr} \langle \delta f^2 \rangle'$ after a closure
  - could be large with wave-particle resonance

    $\rightarrow$ phase space density granulation



$\Delta v_\phi(0) \sim \nabla T_{edge}$

An application: Comparison to C-mod data

Typical parameters for C-mod I-/H- plasmas at edge: $\rho_* \sim 0.006$,

$T_e \sim T_i \sim 200eV$, $\chi_i/\chi_\phi \sim 1$, $L_S \sim 0.6m$, $L_T \sim 0.01m$

- Dimensionless #:
  - $\nu_* \sim 1.3 \gtrsim 1 \rightarrow$ trapped electron effects not of prime importance
  - $c \equiv k_\parallel^2 v_{the}^2/(\nu_{ei}\omega) \sim 30 \gg 1$ for $k_\parallel \sim 1/(qR)$, $k_\perp \rho_s \sim 0.2$

    $\rightarrow$ electrons are adiabatic

$\Rightarrow$ model here (drift-ITG) is a reasonably good approximation

- Toroidal Flow Velocity $\langle V_\parallel \rangle / v_{thi}$ (Rice et al '10)

$$\frac{\langle V_\parallel \rangle}{v_{thi}} \cong \frac{\rho_*}{2} \frac{\chi_i}{\chi_\phi} \sqrt{\frac{T_i}{T_e}} \frac{L_s}{L_T} \sim 0.18$$

  $\Rightarrow$ Typical value in experiments: $\langle V_\parallel \rangle / v_{thi} \sim 0.1 - 0.3$

Conclusion

Intrinsic Rotation as a consequence of "plasma flow engine"

- Entropy: setting-up "engine" calculation

$$\int d\Gamma \left\{ \frac{\partial}{\partial t}\frac{\langle \delta f^2 \rangle}{2\langle f \rangle} + \frac{\partial}{\partial x}\frac{\langle \tilde{V}_x \delta f^2 \rangle}{2\langle f \rangle} + \frac{\nu_c \langle \delta f^2 \rangle}{\langle f \rangle} \right\} = \int d^3x \left(\frac{n\chi_i}{L_T^2} - \frac{nK\langle V_E' \rangle^2}{v_{thi}^2} \right)$$

$\Rightarrow$ evolution via heat input, flow generation and spatial flux of entropy

- Plasma Flow Engine: conversion of heat to flow by residual stress

$$\frac{\langle V_\parallel \rangle}{v_{thi}} = \underbrace{\frac{\rho_*}{2}\sqrt{\frac{T_i}{T_e}}\frac{\chi_i}{\chi_\phi}\frac{L_s}{|L_T|}}_{\nabla T \text{ driven, } \propto \nabla T/B_\theta} - \underbrace{\frac{\rho_*}{2}\sqrt{\frac{T_i}{T_e}}\frac{L_s}{2\Delta}\frac{1}{n}\int d^3v \int^x dx' \frac{1}{\chi_\phi}\left(-D_{spr}\frac{\partial}{\partial x}\frac{\langle \delta f^2 \rangle}{2\langle f \rangle} \right)\Bigg|_{x'-\Delta}^{x'+\Delta}}_{\delta f^2\text{-flux driven} \rightarrow \text{jump in } \delta f^2 \text{ flux}}$$

$\Rightarrow \langle V_\parallel \rangle / v_{thi} \sim 0.18$ for C-mod parameters

- Future plan: Role of outflux of entropy, heat in intrinsic rotation drive?