A gyrokinetic study of electromagnetic effects on particle and toroidal momentum transport

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J. Candy, A.G. Peeters and B.D. Scott

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Motivations

- High β operation without central neutral beam fuelling and external momentum input envisaged for a reactor
- Present high β (hybrid) scenarios usually achieved with strong auxiliary NBI heating
- Density peaking and toroidal rotation velocity are important elements to have good plasma performance and fusion energy production
- Behaviour of particle and toroidal momentum transport with increasing beta only marginally studied so far from the theoretical standpoint
- Present approach is based on a local gyrokinetic model, and combines analytical derivations aiming at identifying the main physical mechanisms with numerical simulations with gyrokinetic codes (GYRO, GS2, GKW)

Outline

- Basic concepts on impact of electromagnetic effects on different transport channels
- Direct and indirect mechanisms, difference between electron particle transport and momentum transport
- Particle transport, electromagnetic contribution to particle convection
- Momentum transport, electromagnetic effects on Prandtl number and Coriolis convection
- Experimental relevance of the theoretical findings
- Concluding remarks

Direct and indirect electromagnetic effects on different transport channels

- **Inclusion of electromagnetic effects, that is Ampère's law, in the gyrokinetic description affects the dynamics of passing electrons, whose response becomes more and more non-adiabatic with increasing β_e**

- **In addition to the magnetic flutter, connected with perpendicular fluctuations of the magnetic field, the non-adiabaticity of passing electrons impacts also the ExB transport in two different ways**

- **A direct one, by producing additional contributions to the ExB flux merely of electromagnetic type (that is which are zero in the electrostatic limit)**
- **And an indirect one, by modifying the turbulence and then by affecting those contributions to the transport which are already present in the electrostatic limit, and which are affected by the electromagnetic fluctuations**

- **Electron and momentum transport provide two interesting examples of how these two different ways of affecting the fluxes occur**

Analytical expression of electromagnetic fluxes

- **Gyrokinetic equation for wave number ky**

$$\left[\omega_{\mathrm{r,k}} + i\left(\gamma_{\mathrm{k}} + \nu_{\mathrm{k}}\right) - k_{||}v_{||} - \omega_{\mathrm{d,k}}/Z_\sigma\right] h_{\mathrm{k},\sigma} =$$

$$\left\{\omega_{\mathrm{r,k}} + i\gamma_{\mathrm{k}} - \frac{\omega_{\mathrm{D,k}}}{Z_\sigma}\left[\frac{R}{L_{\mathrm{n},\sigma}} + \left(\frac{E}{T_\sigma} - \frac{3}{2}\right)\frac{R}{L_{\mathrm{T},\sigma}}\right]\right\}\frac{Z_\sigma e}{T_\sigma} F_{\mathrm{M}} J_{0,\sigma} U_{\mathrm{k}}$$

with the generalized e.m. potential $U_{\mathrm{k}} \equiv \phi_{\mathrm{k}} - (v_{||}/c)A_{||,\mathrm{k}}$

- **Ampere's law delivers relation between parallel potential fluctuations and electrostatic potential fluctuations** $\hat{A}_{||,k} = \frac{c}{c_s}\hat{\Omega}_k\hat{\phi}_k$

- **Radial fluxes can be expressed in the general form (comprises both ExB & M. Fl.)**

$$\Gamma_{\sigma\alpha} = Re\sum_{\mathrm{k}}\langle\int d^3v\ v^{\alpha}_{(\parallel)} J_{0,\sigma} h^*_{\mathrm{k}}\left(ik_{\mathrm{y}}\rho_{\mathrm{s}}c_{\mathrm{s}}\hat{U}_{\mathrm{k}}\right)\rangle_{\mathrm{FS}}$$

Analytical expression of electromagnetic electron flux

- **Including relationship between $\hat{\phi}_k$ and $\hat{A}_{||,k}$, an expression where all electromagnetic effects can be identified is obtained**

ES ExB

EM ExB & M. Fl.

$$\Gamma_{\sigma\alpha} = \sum_{\mathrm{k}} \langle k_{\mathrm{y}} \rho_{\mathrm{s}} c_{\mathrm{s}} \int d^3 v \, v^{\alpha}_{(\|)} \left[1 - 2\hat{v}_{||} \hat{\Omega}_{\mathrm{r,k}} + \hat{v}_{||}^2 \left(\hat{\Omega}^2_{\mathrm{r,k}} + \hat{\Omega}^2_{\mathrm{i,k}} \right) \right] F_0 J^2_{0,\mathrm{e}} |\hat{\phi}_{\mathrm{k}}|^2 \times$$

$$\frac{(\hat{\gamma}_{\mathrm{k}} + \hat{\nu}_{\mathrm{k}}) \, k_{\mathrm{y}} \rho_{\mathrm{s}} \left[R/L_{\mathrm{n}} + (E/T_{\mathrm{e}} - 3/2) \, R/L_{\mathrm{Te}} \right] - \left[\hat{\gamma}_{\mathrm{k}} \left(\hat{k}_{||} \hat{v}_{||} + \hat{\omega}_{\mathrm{d,k}} \right) - \hat{\omega}_{\mathrm{r,k}} \hat{\nu}_{\mathrm{k}} \right]}{\left(\hat{\omega}_{\mathrm{r,k}} + \hat{k}_{||} \hat{v}_{||} + \hat{\omega}_{\mathrm{d,k}} \right)^2 + (\hat{\gamma}_{\mathrm{k}} + \hat{\nu}_{\mathrm{k}})^2} \rangle_{\mathrm{FS}}$$

- **Compared to the electrostatic ExB transport, additional EM contributions scale as $\hat{v}_{||}$ & $\hat{v}_{||}^2$, that is as $\sqrt{m_{\mathrm{D}}/m_{\sigma}}$ & $m_{\mathrm{D}}/m_{\sigma}$, whereas $\hat{\Omega}_{\mathrm{k}}$ scales with β_{e}**
- **Direct EM components of the flux are small (often negligible) compared to the ES ExB component, unless $m_{\mathrm{D}}/m_{\sigma}$ is large, which is the case for $\sigma = e$**
- **When $\sigma = i$ (deuterons or impurities) dominant EM effects occur in an indirect form, affecting the instability and therefore affecting the ExB ES component**

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Analytical expression of electromagnetic passing electron flux

- **Previous generic expression is made specific for the flux of passing electrons**

$$\Gamma_{\rm pass} = (1-f_{\rm t})\sum_{\rm k}\langle k_{\rm y}\rho_{\rm s}c_{\rm s}n_0 \sum_{\varsigma=\pm 1}\frac{1}{\sqrt{\pi}}\int_0^\infty d\epsilon\sqrt{\epsilon}\exp(-\epsilon)\times$$

$$\times\left[1-2\varsigma\sqrt{2\epsilon\hat{\mu}}\hat{\Omega}_{\rm r,k}+4\epsilon\hat{\mu}\left(\hat{\Omega}^2_{\rm r,k}+\hat{\Omega}^2_{\rm i,k}\right)\right]\times$$

$$\times\frac{\left(\hat{\gamma}_{\rm k}+\hat{\nu}_{\rm k}\right)k_{\rm y}\rho_{\rm s}\left[R/L_{\rm n}+(\epsilon-3/2)\,R/L_{\rm T}\right]-\left[\hat{\gamma}_{\rm k}\left(\varsigma\sqrt{2\epsilon\hat{\mu}}/q+\hat{\omega}_{\rm d,k}\right)-\hat{\omega}_{\rm r,k}\hat{\nu}_{\rm k}\right]}{\left(\hat{\omega}_{\rm r,k}+\varsigma\sqrt{2\epsilon\hat{\mu}}/q+\hat{\omega}_{\rm d,k}\right)^2+\left(\hat{\gamma}_{\rm k}+\hat{\nu}_{\rm k}\right)^2}|\hat{\phi}_{\rm k}|^2\rangle_{\rm FS}$$

$$\hat{\mu} = m_{\rm D}/m_{\rm e}$$

[Hein PoP 2010]

Passing electron flux in the electrostatic limit, adiabatic response

[Hein PoP 2010]

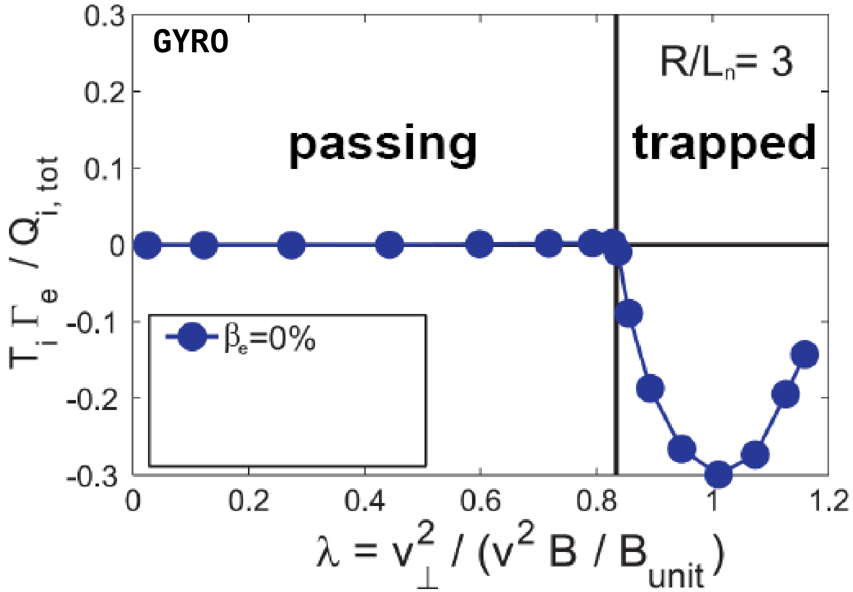
- **High parallel mobility for electrons, $\mu_e = m_D/m_e$**

Electrons: μ_e = 3600 such that terms ~ μ_e >> all other terms (order 1)

$$\Gamma_e \propto \sum_k \sum_{\varsigma=\pm 1} \langle J_0^2(k_\perp \rho_e) |\hat{\phi}_k|^2 \times \frac{(\hat{\gamma}_k + \hat{\nu}_k) k_y \rho_s [R/L_{n,e} + (\epsilon - 3/2) R/L_{T,e}] + Z_e [\hat{\gamma}_k (\varsigma\sqrt{2\epsilon\mu_e}/q + \hat{\omega}_{d,k}) - \hat{\omega}_{r,k}\hat{\nu}_k]}{(\hat{\omega}_{r,k} + \varsigma\sqrt{2\epsilon\mu_e}/q + \hat{\omega}_{d,k})^2 + (\hat{\gamma}_k + \hat{\nu}_k)^2} \rangle_{FS}$$

- **Linear electron flux from GYRO [Candy, JCP 186 (2003)] for ITG mid-radius case in s-α geometry**

Electrostatic limit:
passing electrons adiabatic, flux at the null as a consequence of large μ



Electromagnetic, non-adiabatic response, outward convection of passing electrons

[Hein PoP 2010]

$$\Gamma_e \propto \sum_k \sum_{\varsigma=\pm1} \left\langle \left[1 - 2\varsigma\sqrt{2\epsilon\mu_e}\,\hat{\Omega}_{\mathrm{r},k} + 4\epsilon\mu_e\left(\hat{\Omega}_{\mathrm{r},k}^2+\hat{\Omega}_{\mathrm{i},k}^2\right)\right] J_0^2(k_\perp\rho_e)\,|\hat{\phi}_k|^2 \times \frac{(\hat{\gamma}_k+\hat{\nu}_k)\,k_{\mathrm{y}}\rho_{\mathrm{s}}\left[R/L_{\mathrm{n,e}}+(\epsilon-3/2)\,R/L_{\mathrm{T,e}}\right] + Z_e\left[\hat{\gamma}_k\left(\varsigma\sqrt{2\epsilon\mu_e}/q+\hat{\omega}_{d,k}\right)-\hat{\omega}_{r,k}\hat{\nu}_k\right]}{\left(\hat{\omega}_{r,k}+\varsigma\sqrt{2\epsilon\mu_e}/q+\hat{\omega}_{d,k}\right)^2+(\hat{\gamma}_k+\hat{\nu}_k)^2}\right\rangle_{FS}$$

- Electromagnetic passing electron flux finite because μ_e-terms of same order in numerator and denominator due to finite β_e

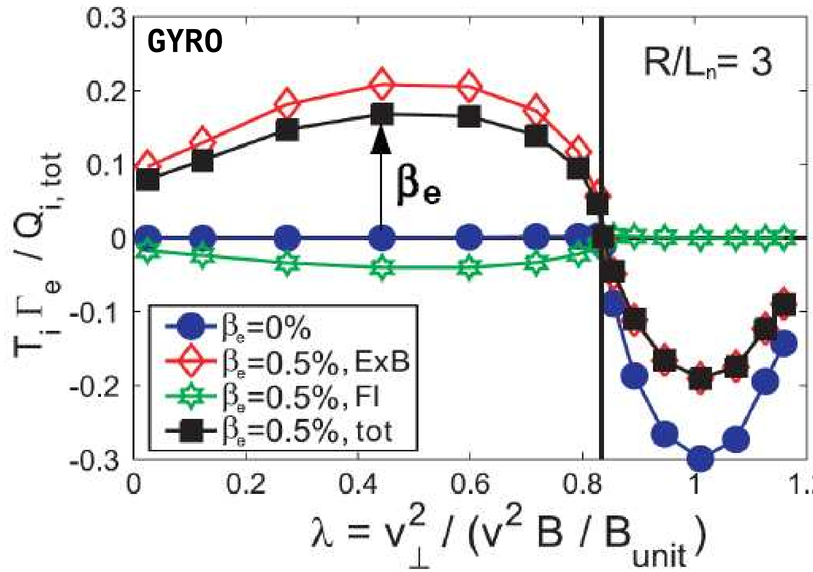
Main contribution:

$-2\varsigma\sqrt{2\epsilon\mu_e}\,\hat{\Omega}_{\mathrm{r},k}$ **multiplied by**

$\varsigma\sqrt{2\epsilon\mu_e}/q$ **term, since μ_e >> 1**

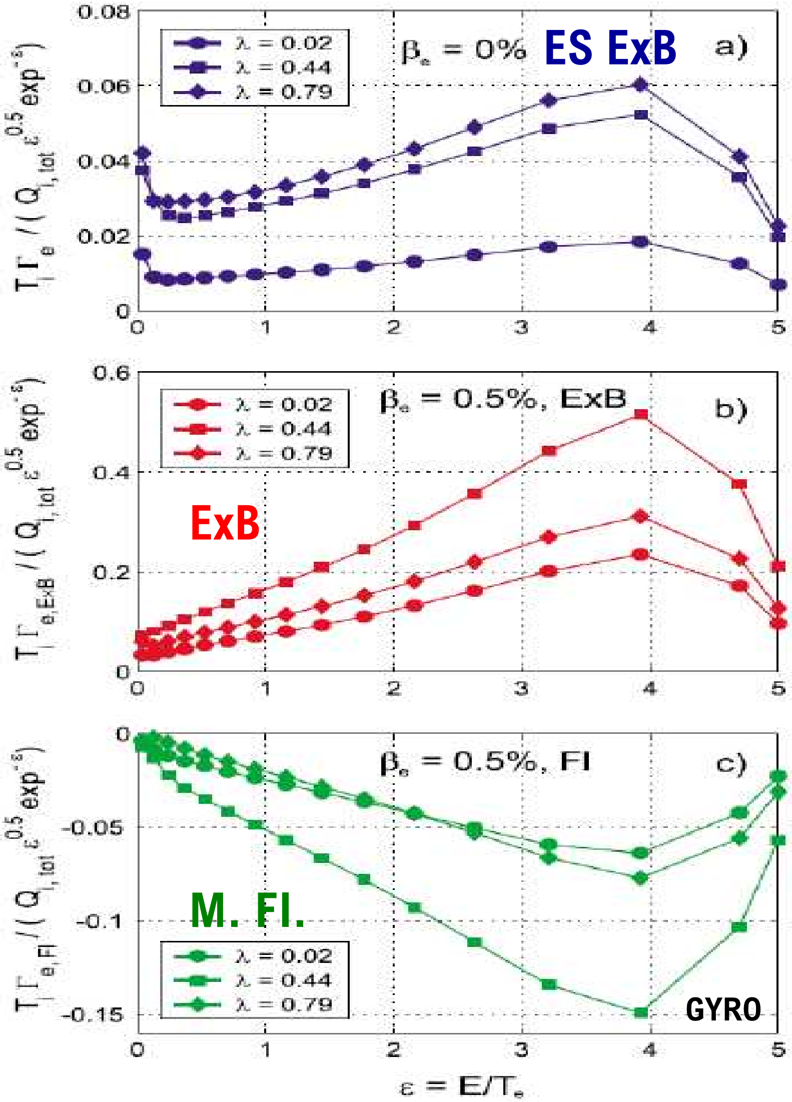
=> electromagnetic passing particle flux is of convective type

For typical ITG case:
ExB outwards, Flutter slightly inwards



Electromagnetic, passing electrons transported over the full energy range

- **ITG case**
- **Electrostatic, small passing electron flux**
- **Electromagnetic, passing electron flux becomes large at all energies**
- **Both ExB**
- **and Magnetic Flutter**



[Hein PoP 2010]

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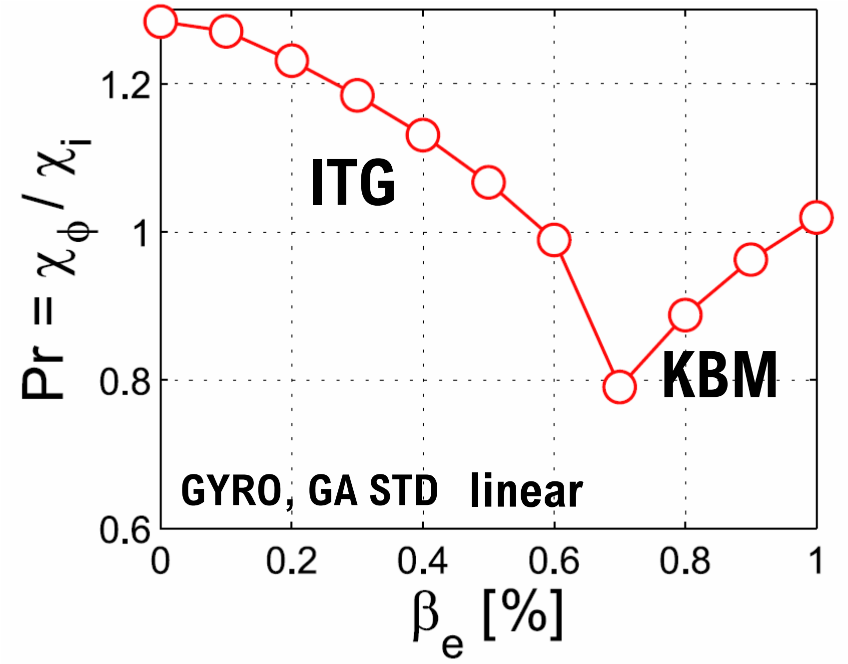
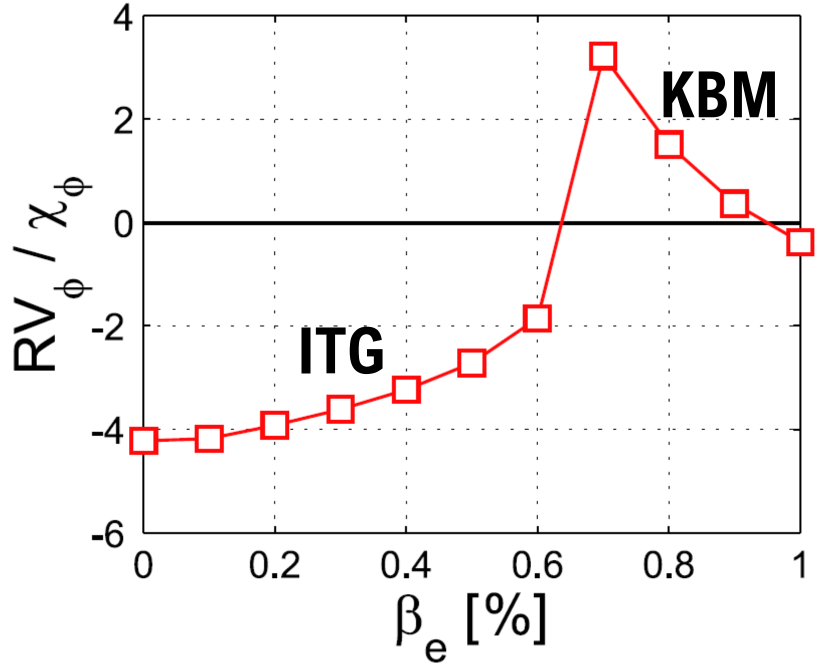
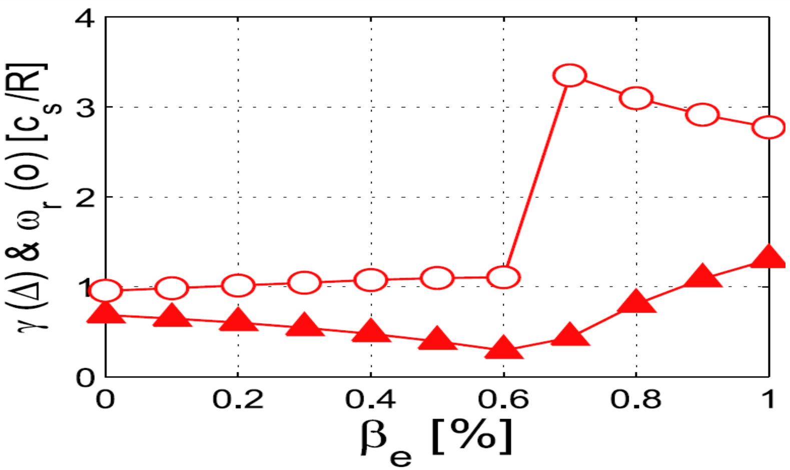
Toroidal momentum transport, Pr number and Coriolis pinch change as a function of β







$$\Gamma_\phi = \frac{n v_{\mathrm{th}}}{R}\left[\chi_\phi \hat{u}' + R V_\phi \hat{u}_\parallel\right]$$

- **In the presence of ITG modes, an increase of β reduces in size both the Pr and the Coriolis pinch numbers**

Toroidal momentum transport, radial flux of ion parallel velocity

- **Back to gyrokinetic eq. (electrostatic, parallel dynamics and Coriolis highlighted)**

$$\left[-i\hat{\omega} + i\,2\langle\hat{\omega}_{\rm D}\rangle\bar{k}_{||}\hat{v}_{||} + i\left(\langle\hat{\omega}_{\rm d,0}\rangle + 2\langle\hat{\omega}_{\rm D}\rangle\hat{v}_{||}\hat{u}\right)\right]\tilde{h}_{\rm i} = -i\left(\hat{\omega} - \hat{\omega}_*\right)F_{\rm M}J_0\hat{\phi}$$

$$2\langle\hat{\omega}_{\rm D}\rangle\bar{k}_{||} = 2\langle\hat{\omega}_{\rm D}\rangle\left(\xi\sqrt{\langle\hat{k}_{||}^2\rangle} + \langle\hat{k}_{||}\rangle\right) \qquad \hat{\omega}_* = k_{\rm y}\rho_{\rm s}\left[\frac{R}{L_{\rm n}} + \left(\epsilon - \frac{3}{2}\right)\frac{R}{L_{\rm T}} + \hat{v}_{||}\hat{u}'\right]$$

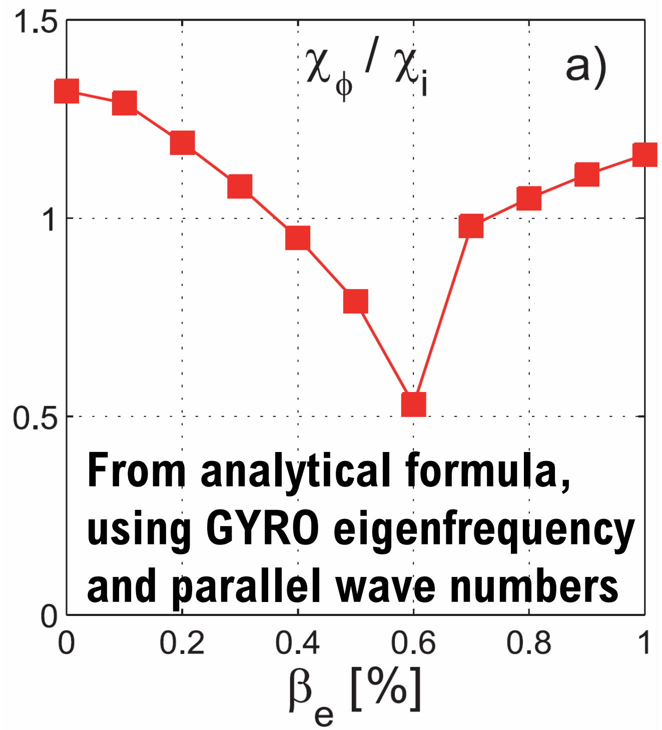
- **Includes both finite width $\langle\hat{k}_{||}^2\rangle$ and distortion $\langle\hat{k}_{||}\rangle$ of the eigenfunction along the field line ($\xi = \pm 1$)**

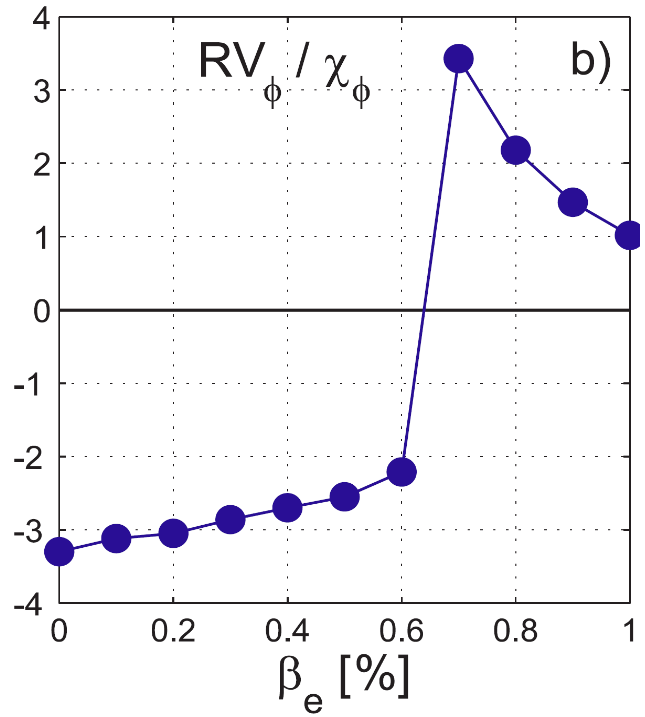
$$\{\Gamma_\phi, Q\} = -\frac{1}{2}\sum_{\xi=\pm}\Im\left\{k_{\rm y}\rho_{\rm s}c_{\rm s}^2|\hat{\phi}|^2\int d^3v\,\left\{c_{\rm s}\hat{v}_{||}, c_{\rm s}^2\epsilon\right\}\times\right.$$

$$\left.\times\;\frac{\left(\hat{\omega} - \hat{\omega}_*\right)F_{\rm M}J_0}{\hat{\omega} - \hat{\omega}_{\rm d,0} - 2\langle\hat{\omega}_{\rm D}\rangle\hat{v}_{||}\left(\xi\sqrt{\langle\hat{k}_{||}^2\rangle} + \langle\hat{k}_{||}\rangle + \hat{u}\right)}\right\}$$

Electrostatic ExB formula recovers the numerical fully electromagnetic results

- The analytical electrostatic formula for ExB transport recovers rather precisely the numerical fully electromagnetic results



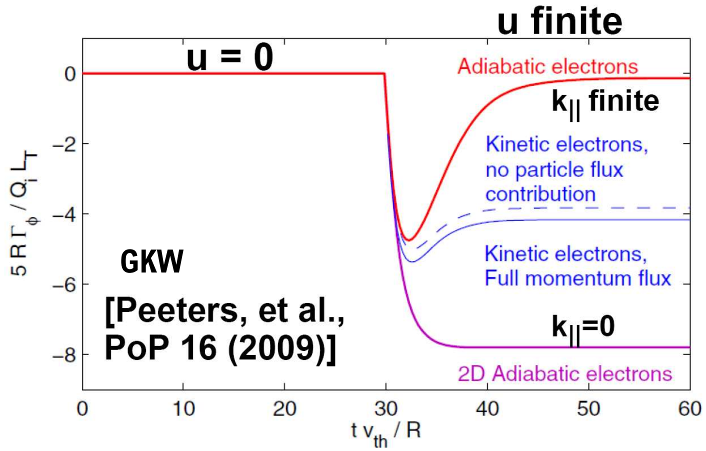


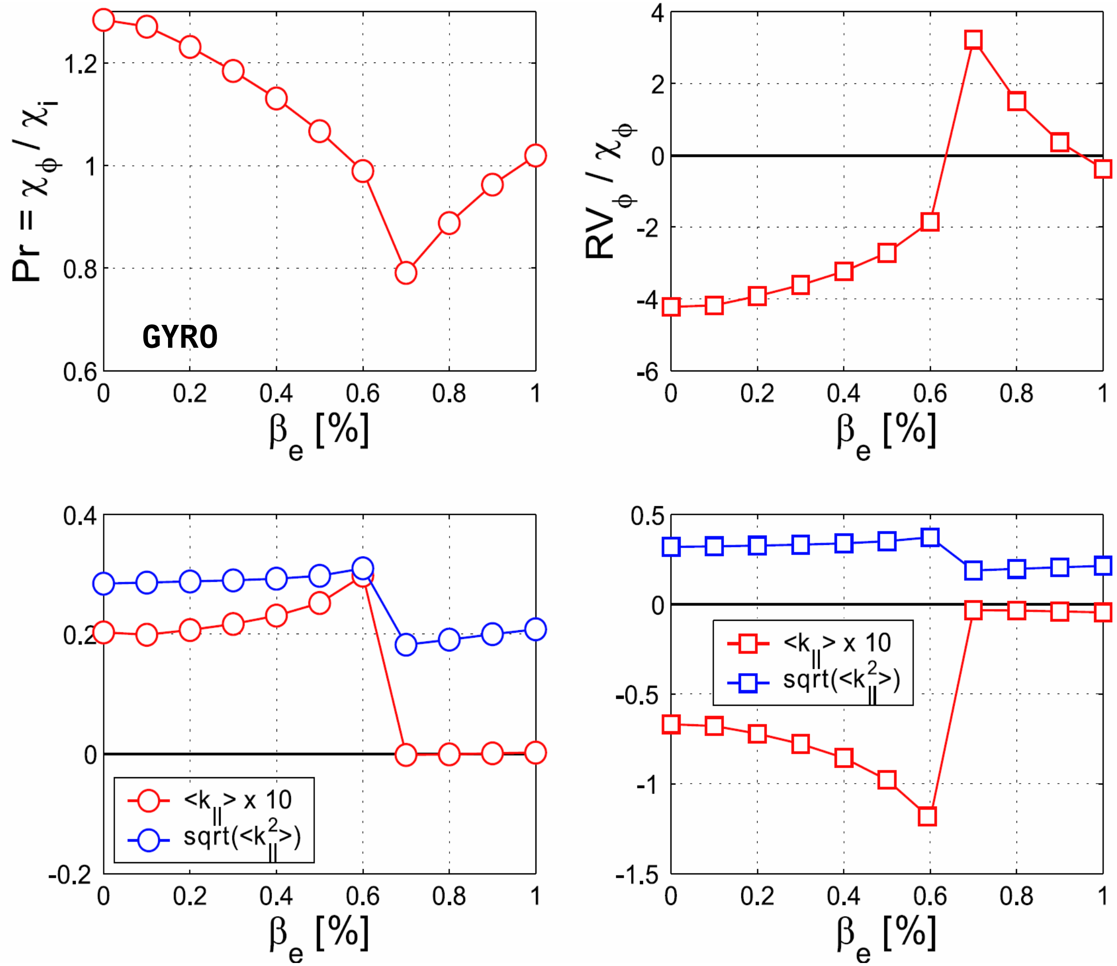
- Electromagnetic effects on Prandtl and Coriolis pinch numbers occur mainly through an indirect way (β affects the mode, and the "electrostatic" ExB transport changes, other effects are small)

Compensation effect [Peeters PoP 09] causes β dependence of Coriolis pinch

- **By affecting passing electron dynamics, electromagnetic fluctuations modify the mode eigenfunction, and change the parallel wave number $\langle k_{||} \rangle$**
- **In ITG, β dependence of $\langle k_{||} \rangle$ determines β dependence of Pr and pinch numbers**
- **Electromagnetic effects encapsulated in**

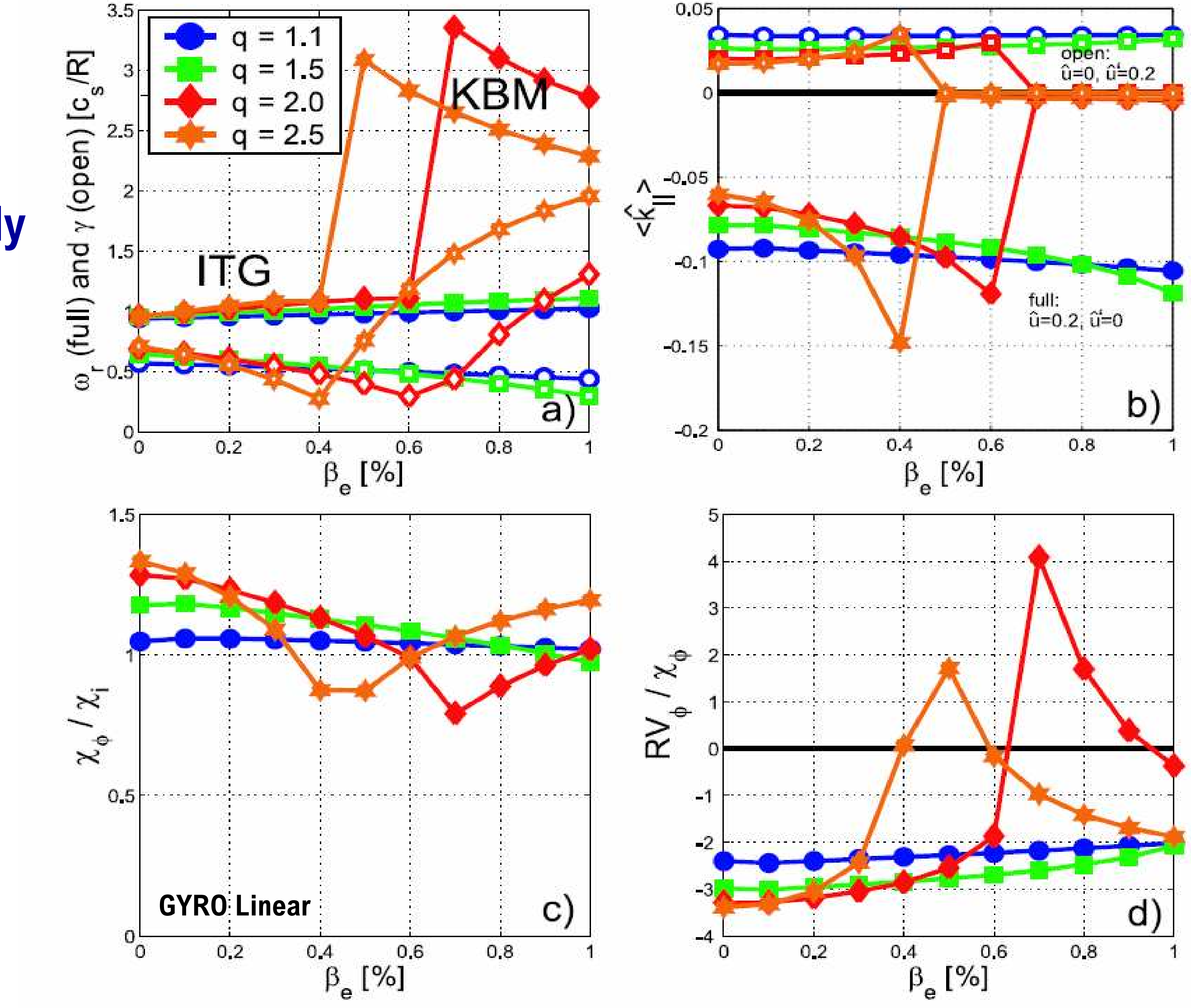
$$\langle k_{||} \rangle = \langle k_{||} \rangle(\beta)$$





Proximity to kinetic ballooning mode threshold plays essential role

- **Safety factor scan reveals that β dependence of Pr & Pinch numbers strongly depends on q**

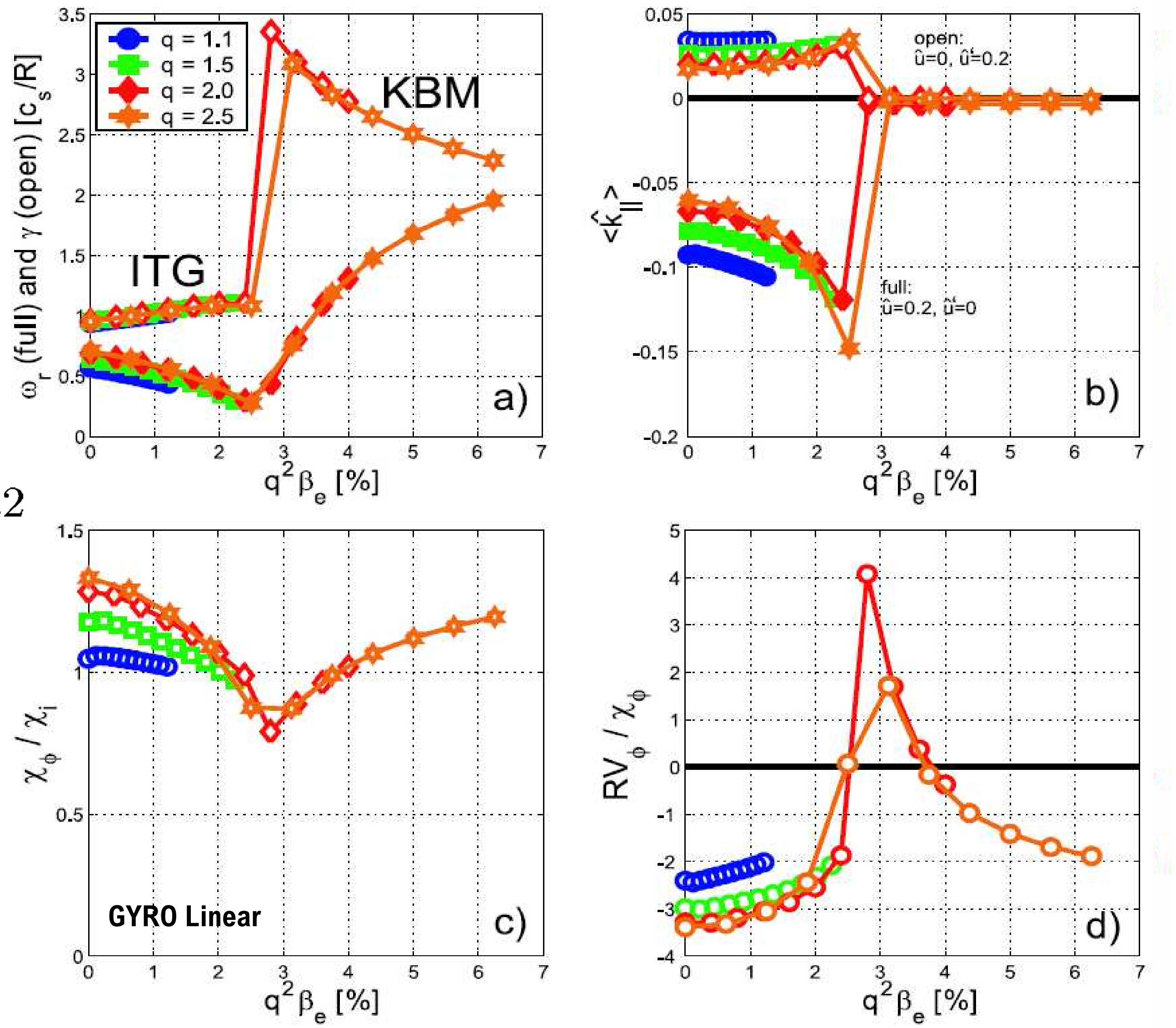


Proximity to kinetic ballooning mode threshold plays essential role

- **Safety factor scan reveals that β dependence of Pr & Pinch numbers strongly depends on q**

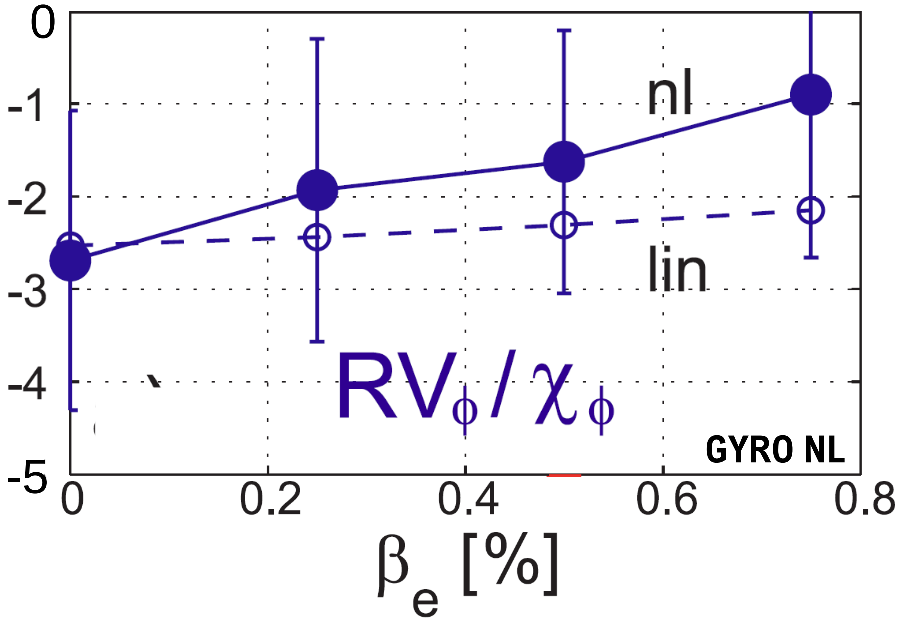
- **Results plotted against relevant parameter $\beta_e q^2$ describing strength of e.m. effects ($A_{||}$)**

- **Proximity to KBM threshold plays critical role in determining the strength of the β dependence**



Initial nonlinear simulations confirm linear results, reduction of Coriolis pinch number with increasing β

- **First nonlinear results confirm dependence found in linear calculations**



- **Requires further investigations in various parameter regimes**
- **Nonlinear momentum flux strongly fluctuating and bursting requires long time averages (NL e.m. investigations of momentum transport very expensive)**

observed also in other NL flux tube simulations [Waltz PoP 07, Peeters PoP 09]

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Reduction of R/Ln with increasing β in absence of central fuelling

- With typical H-mode plasma parameters around mid-radius, look for dependence of local R/Ln as a function of beta

- Effect of beam fuelling source included by considering that

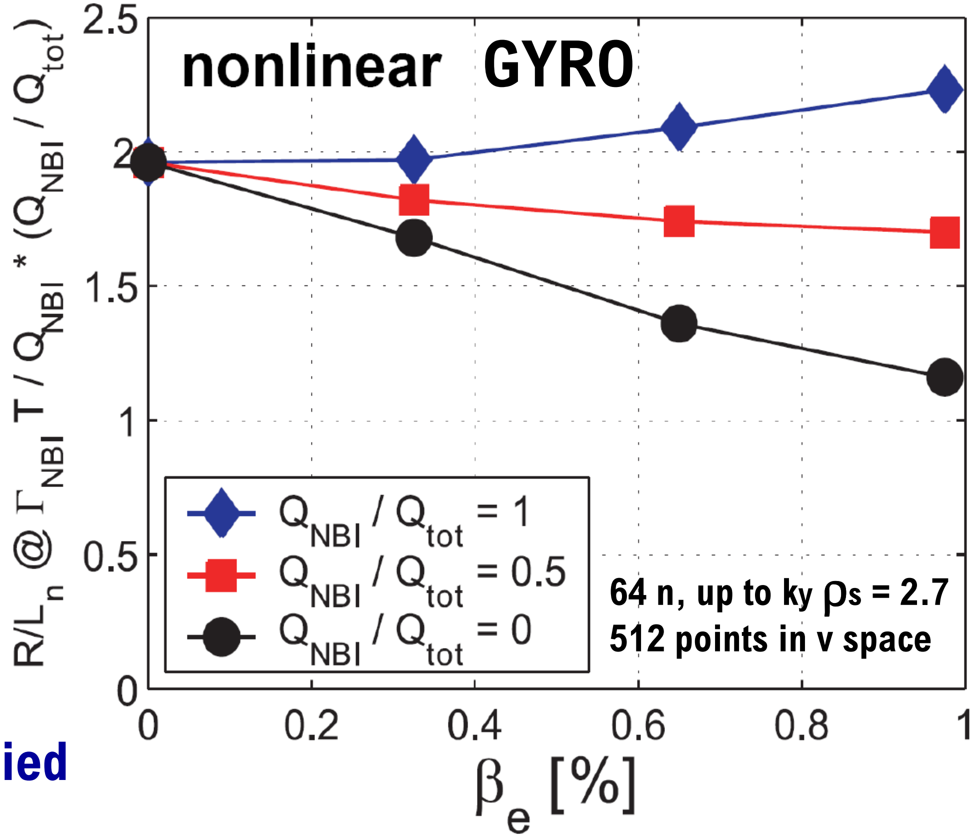
$$\Gamma_{\rm NBI} T/Q_{\rm NBI} \simeq T/E_{\rm NBI}$$

which implies

$$\Gamma_e T/Q_{\rm tot} \simeq (T/E_{\rm NBI})\,(Q_{\rm NBI}/Q_{\rm tot})$$

- At constant density, $T \propto \beta$

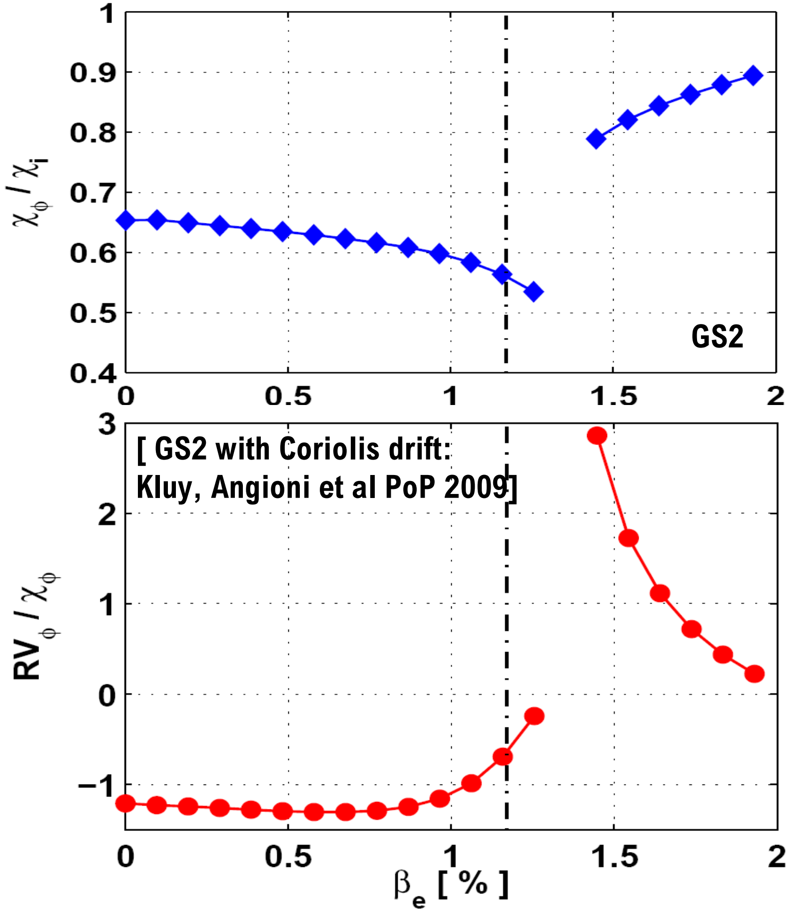
- Typical AUG H-mode mid-radius parameters, and NBI parameters applied

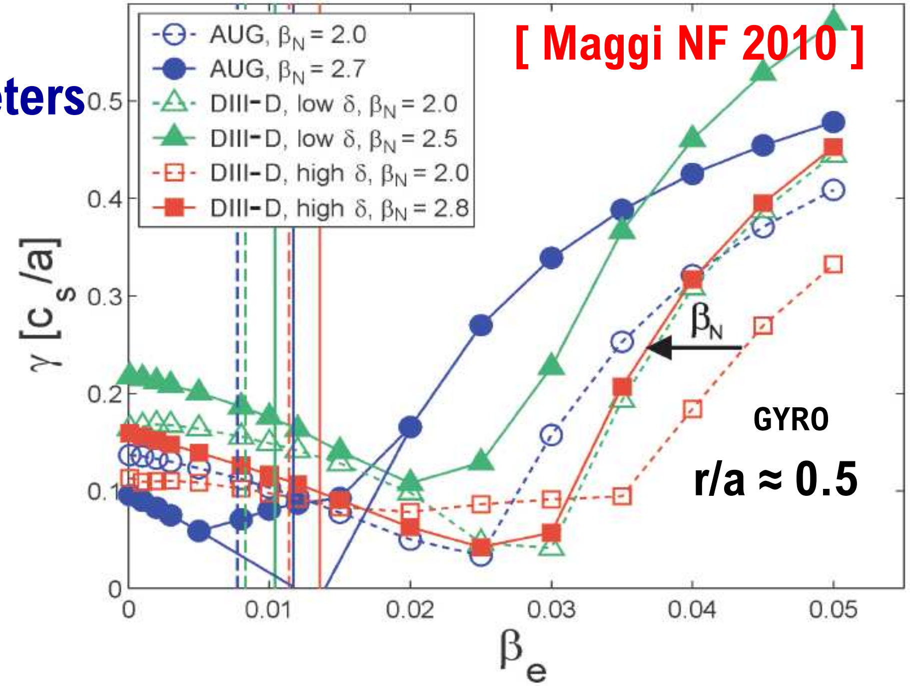


- High β predicted to lead to a significant reduction of density peaking in the absence of central fuelling (keeping other parameters constant)

High β plasmas can be close to KBM threshold, where β effects on tor. mom. transport can be significant

- **Analysis of hybrid scenarios shows that experimentally achieved plasma parameters can sit close to the KBM threshold**





- **There, reduction of both Pr and Coriolis pinch numbers become significant, but have opposite (compensating) effects on the toroidal velocity profile when an external torque is present**
- **Collisions weaken dependence at low β**

Conclusions

- A concurrent study of electromagnetic effects on particle and toroidal momentum transport highlights different ways by which $A_{||}$ fluctuations can affect transport
- Electron particle flux: additional electromagnetic contributions occur, dominant (direct) effect is the convection of passing electrons, outward in ITG turbulence
- Toroidal momentum flux: main effect (indirect) due to modification of "electro-static" ExB flux, produced by the dependence on β of the av. parallel wave number
- In both cases, the physical mechanisms can be ultimately re-conducted to the non-adiabatic response of passing electrons

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- In both cases, the physical mechanisms can be ultimately re-conducted to the non-adiabatic response of passing electrons
- In the absence of central NBI fuelling, in ITG turbulence, a reduction of density peaking with increasing β is predicted
- With ITG modes, both Pr and Coriolis pinch numbers decrease in size with increasing β, strongly in proximity of KBM threshold (at high β, predicted reduction of density peaking concurrently contributes to reduce Coriolis pinch)
- This topic would deserve some consideration from the experimental side (particle and momentum transport at high β in the absence of NBI fuelling and torque)