Reduced turbulent transport in toroidal configurations through shaping

H.E. Mynick[a], N. Pomphrey[a] & P.Xanthopoulos[b]

[a]Princeton Plasma Physics Lab
[b]IPP/MPG, Greifswald

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A.Boozer, M.Chance, L-P Ku, M.Zarnstorff

-Introduction:

-Present day stellarator designs seek to optimize neoclassical (nc) transport, while also mitigating turbulent transport (usually the dominant channel in such designs) has not been addressed up to now.

-However, with the advent of 2 powerful numerical tools, viz,
-gyrokinetic (gk) codes valid for 3D nonlinear simulations such as GENE[1,2], and
-stellarator optimization codes such as STELLOPT[3],
also optimizing for turbulent transport becomes a realistic possibility.

-Using these tools, we have demonstrated[4] that stellarators with appreciably reduced transport can be evolved from stellarators without this optimization. The same method can also be applied to axisymmetric devices[5], raising the prospect of new classes of stellarators and tokamaks with greatly improved overall confinement.

[1] F. Jenko, W. Dorland, et al., *Phys. Plasmas* **7**, 1904 (2000).
[2]P.Xanthopoulos, W.Cooper et al., *Phys.Plasmas* **16**, 082303 (2009).
[3] A. Reiman, G. Fu, S. Hirshman, L. Ku, et al, *Plasma Phys. Control. Fusion* **41** B273 (1999).
[4]H. Mynick, N. Pomphrey, P. Xanthopoulos, *Phys. Rev. Letters* **105**, 095004 (2010).
[5]H. Mynick, N. Pomphrey, P. Xanthopoulos, (to appear in *Phys. Plasmas*, 2011).

-Optimization, with STELLOPT:

-Attempts to minimize cost function $\chi^2(\mathbf{z}) = \sum_i \chi_i^2 = \sum_i w_i^2 \hat{\chi}_i^2$ (1)

In “shape space” $\mathbf{z} \equiv \{z_{j=1,..N_z}\}$

where for fixed-boundary equilibria, **z** specializes to the set of Fourier harmonics specifying the boundary.

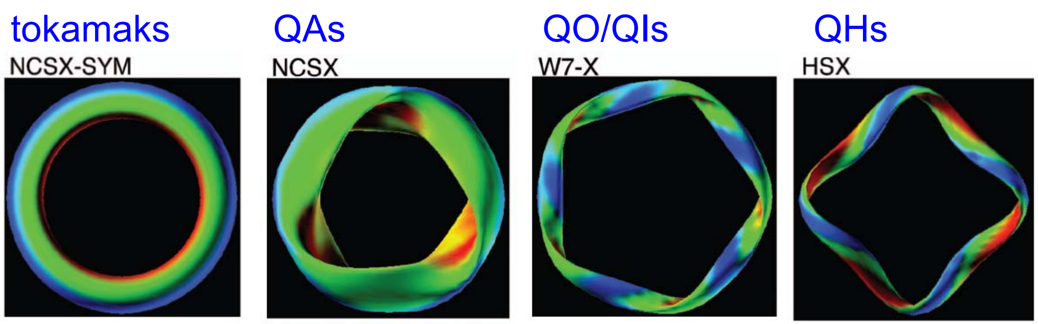
To these we now add χ^2_{txp} = turbulent transport measure.

-For a figure of merit, can take χ_{txp} = Q = radial heat flux.

-Using the averaged heat flux Q_{gk} from GENE nonlinear runs is too computationally expensive: Many 100s of equilibria examined in a typical STELLOPT run, & takes ~ 100 cpu-days for a GENE run for a single flux tube.

To overcome this, use a "proxy function" Q_{prox} to stand in for Q_{gk}.

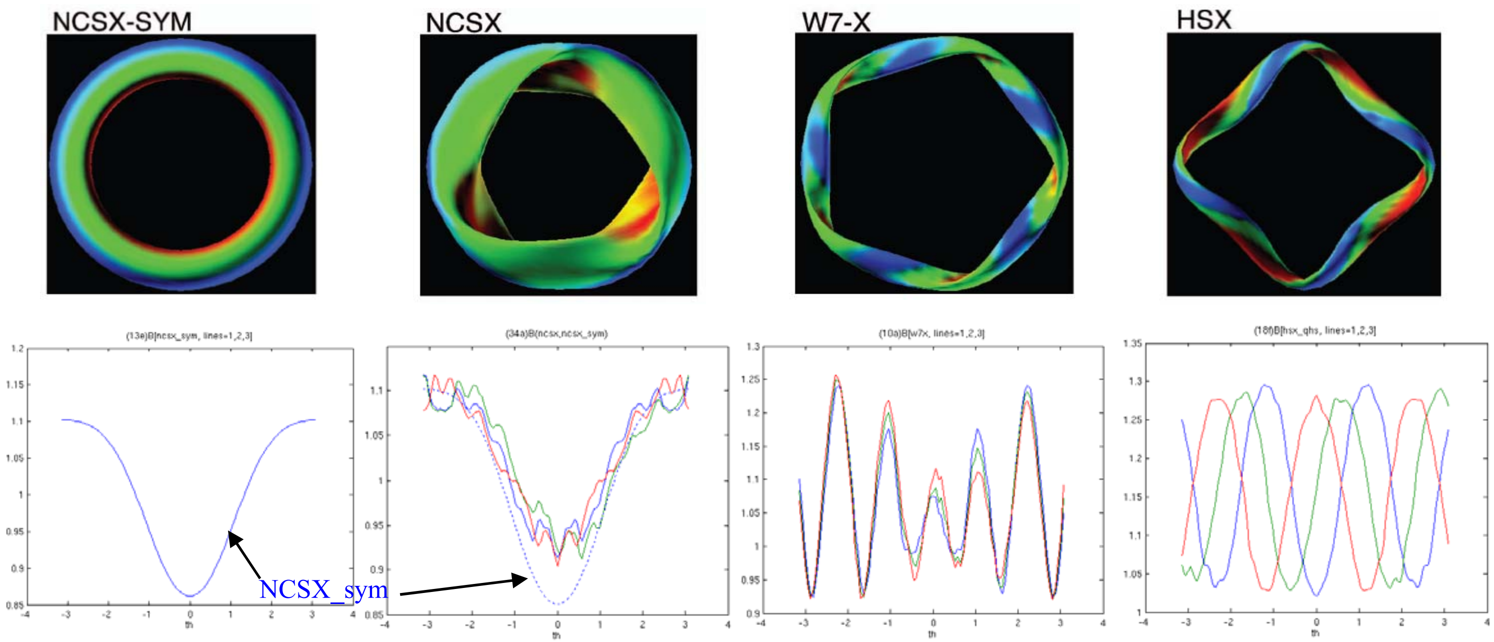
-Q_{prox} developed from analytic theory & from of GENE studies in a range of toroidal configurations[6]:



[6] H. Mynick, P.Xanthopoulos, A.Boozer, *Phys.Plasmas* **16**, 110702 (2009).

-NC-transport-optimized configurations: [5]

	tokamaks	QAs	QO/QIs	QHs
$\varepsilon_h/\varepsilon_t$:	0	<<1	~1	>>1



$$B(\theta \,|\, \alpha = cst)$$

[6] H. Mynick, P.Xanthopoulos, A.Boozer, *Phys.Plasmas* **16**, 110702 (2009).

-Some lessons from these GENE studies:

-Spatial form of heat flux $Q_i(\theta)$ along a field line resembles turbulent $<\varphi>(\theta)$, with approximate scaling $Q_i(\theta) \sim <\varphi^2>(\theta)$, suggesting quasilinear treatment appropriate.

- $<\varphi>$ resembles the most unstable linear modes φ_k, localizing in the wells of effective potential V of the linear mode equation.

-The form of $V(\theta)$ is dominated by that of radial curvature $\kappa_1(\theta) \equiv \mathbf{e}_r \cdot \boldsymbol{\kappa} \approx a\partial_r B/B$, from the drift term in the ITG dispersion eqn, for typical parameters. The deepest wells of V are thus where κ_1 is worst (most negative), occurring on the outboard side around the device corners (bean cross-section).

-Large local shear $s_l(\theta) \equiv \partial_\theta(g^{ry}/g^{xx})$ tends to suppress $<\varphi>(\theta)$, $Q_i(\theta)$, diminishing the radial extent of the modes, playing a role similar to flow shear.

-The relatively simple relationship between GENE outputs $<\varphi>$, Q_i and inputs like κ_1, s_l suggests an optimization may be done, minimizing a semi-analytic proxy $Q_{prox}(\kappa_1, s_l)$ for the transport, which can be quickly computed.

-Proxy function $\chi_{txp}{}^2$:

-Take $\chi_{txp} = Q_{prox}$, a proxy model for the radial ion heat flux.
For 1st try, use simple ITG turbulent model, from the quasilinear (ql) expression

$$Q_i = -\chi n_0 T_i', \chi = \sum_{\mathbf{k}} D_k,\ D_k \equiv (\omega_{*s} L_n)^2 \langle |\frac{e\phi_{\mathbf{k}}}{T_s}|^2 \rangle \gamma_k / |\omega|^2 \simeq \gamma_k / k_r^2, \quad (2)$$

with $\omega_{*s} \equiv -(ck_y T_s/eB)\kappa_n,\ \kappa_n \equiv L_n^{-1} \equiv -\partial_r \ln n_0$.

-GENE studies identify radial curvature $\kappa_1 \equiv \mathbf{e}_r \cdot \boldsymbol{\kappa} \approx a\partial_r B/B$, local shear $s_l \equiv \partial_\theta(g^{ry}/g^{rr})$ as key geometric quantities affecting turbulence.

-Use simplified expression for γ_k from ITG dispersion eqn,

$$\gamma_{\mathbf{k}} \simeq (\omega_{*i}/\kappa_n)|\tau\kappa_1(\kappa_p - \kappa_{cr})|^{1/2} H(\kappa_p - \kappa_{cr}) H(-\kappa_1). \quad (3)$$

-Model k_r^{-2} on the intuition that effect of local shear s_l is similar to that of flow shear, diminishing radial extent of mode from mesoscale ($k_r^{-2} \sim L_p\rho_i$) to microscale ($k_r^{-2} \sim \rho_i^2$) when ExB shearing freq $\omega_E \equiv \partial_r(cE_r/BR)$ goes from << to >> inverse fluctuation correlation time τ_E^{-1} in absence of ExB flow[7]

$$k_r^{-2}(\omega_E, s_l) \simeq \rho_i^2 + \rho_i L_p/[1 + (\tau_E\omega_E)^2 + \langle(\tau_s s_l)^2\rangle_{\Delta z}], \quad \text{with} \quad (4)$$

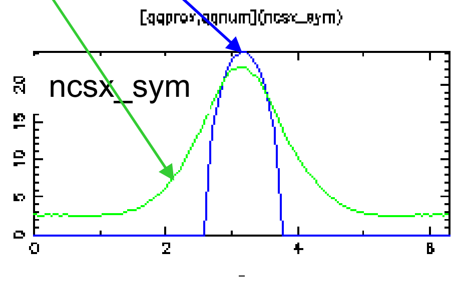
$\tau_s \equiv$ empirical constant, $\langle..\rangle_{\Delta z} \equiv$ mode-amplitude weighted avg along field line.

$\chi \rightarrow$ Bohm diffusion coefficient $\chi_B \equiv c_B \rho_i v_i$ [for $(\tau_E\omega_E)^2$ and $\langle(\tau_s s_l)^2\rangle \ll 1$], and

$\chi \rightarrow$ gyroBohm coefficient $\chi_{gB} \equiv (\rho_i/L_p)\chi_B =$ [for $(\tau_E\omega_E)^2$ or $\langle(\tau_s s_l)^2\rangle \gg 1$].

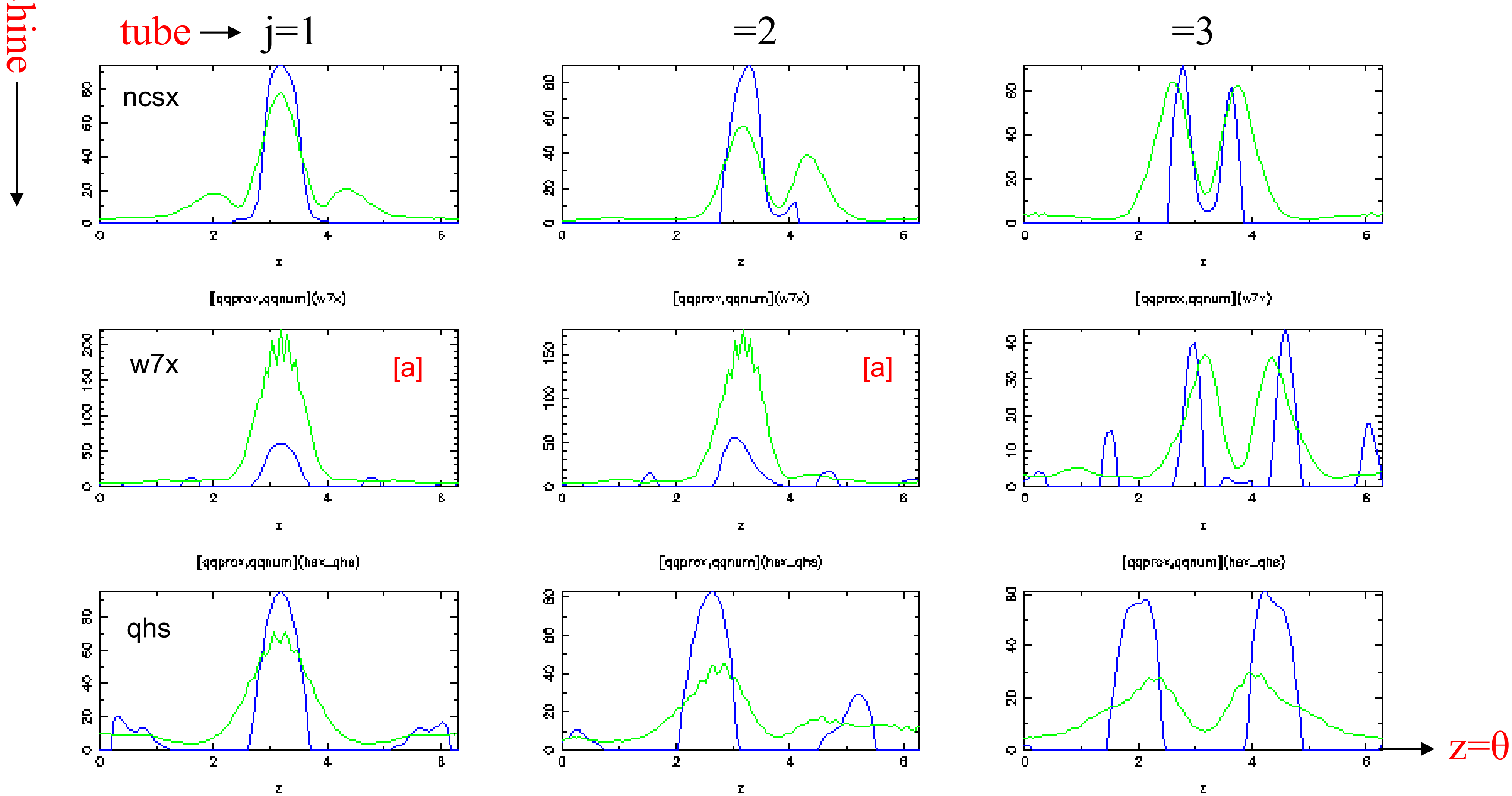
[7] J. Garcia, K. Yamazaki, J. Dies, J. Izquierdo, *PPCF* **48**, p15-27, (2006).

- Q_{gk} vs Q_{prox} comparison: With parameters optimized (*via* simulated annealing).



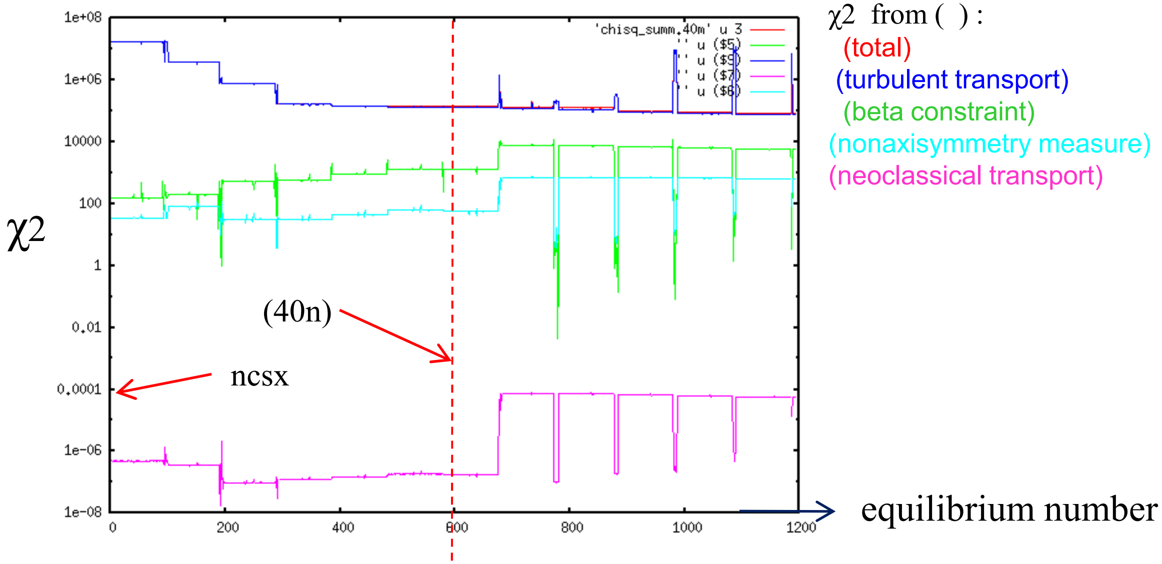
X $=[\kappa_{cr}, \tau_s, \Delta z, c_B]=[5.335E\text{-}02, 1.1267, 0.2073, 0.9596]$

Q_{prox} captures much of the behavior of Q_{gk}

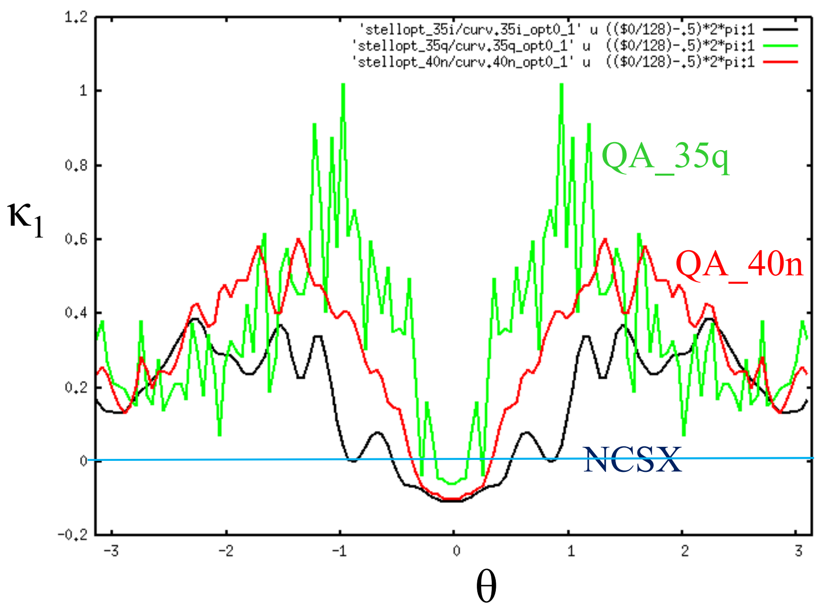


[a]Main deficiency of proxy: underestimates Q for W7X, tubes 1,2.

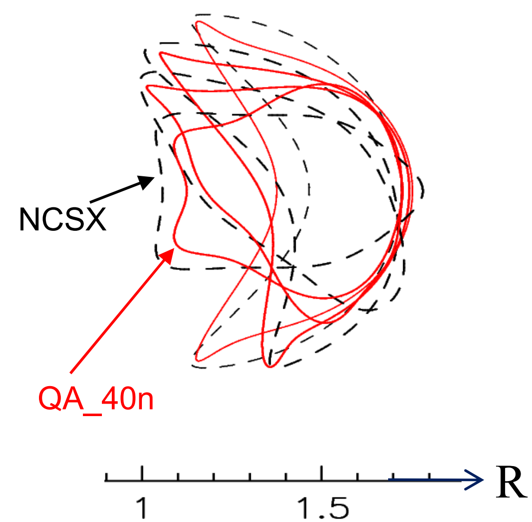
-STELLOPT runs-1: Start with NCSX (LI383) configuration, β=4.2%.



-New configurations improve χ by boosting $\kappa_1 \approx a\partial_r B/B$ in region of worst curvature .

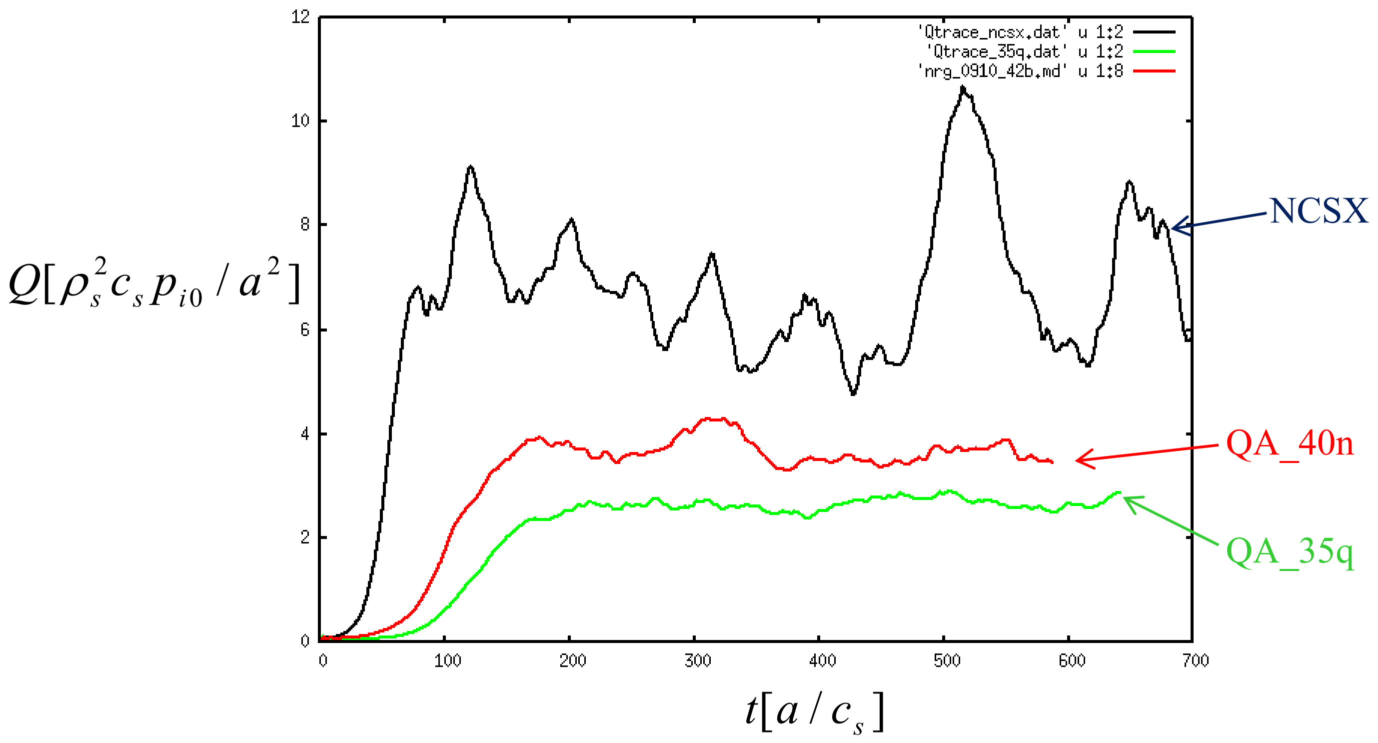


Compare poloidal cross-sections with those of NCSX:



-GENE confirms transport reduced, by factor ~ 2-2.5.

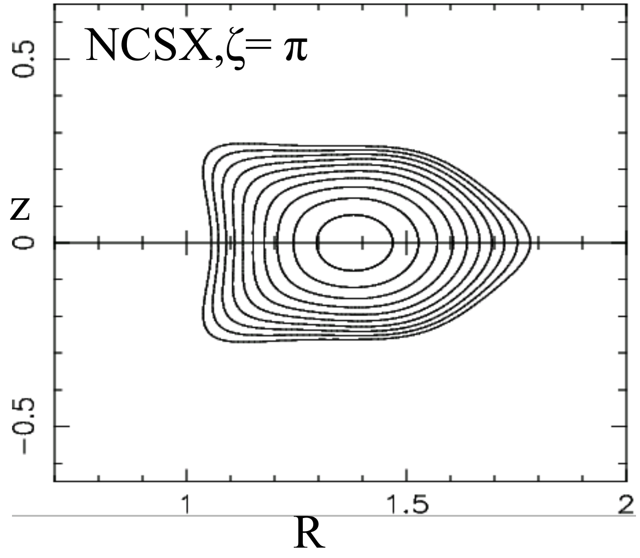
Compare $Q_{gk}(t)$ for QA_35q, QA_40n vs NCSX:

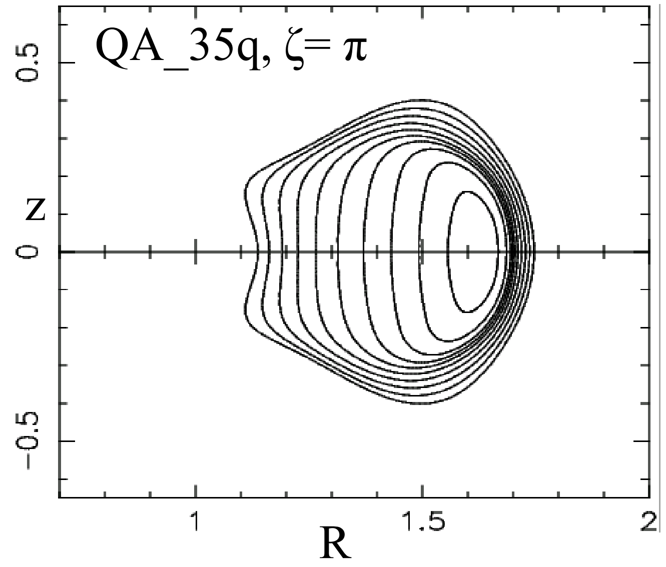


-How does STELLOPT deform NCSX to boost κ_1 ?

-A stellarator with poloidal symmetry number ℓ has rotational transform $\iota[(r/a)] \sim (r/a)^{2|\ell-2|}$ In 1965, Taylor[8] noted that applying a vertical field B_v to a stellarator with ℓ >2 would cause larger shift $\Delta \sim B_v/\iota$ for inner than for outer flux surfaces, resulting in magnetic 'well', *V"*<0.]

-Configurations 35q, 40n conform to this picture, NCSX does not:





[8] J.B. Taylor, *Phys. Fluids* **8** 1203 (1965).

-Applications:

(1)Evolve from NCSX, with ITG turbulence. (Just discussed)

(2)Evolve from other interesting toroidal configurations:
Other QAs, QHs, QOs, tokamaks.

(3)Apply to turbulence from other transport channels (e.g., ETG, TEM, ballooning)

-In the following, we discuss results from our explorations so far in these directions.

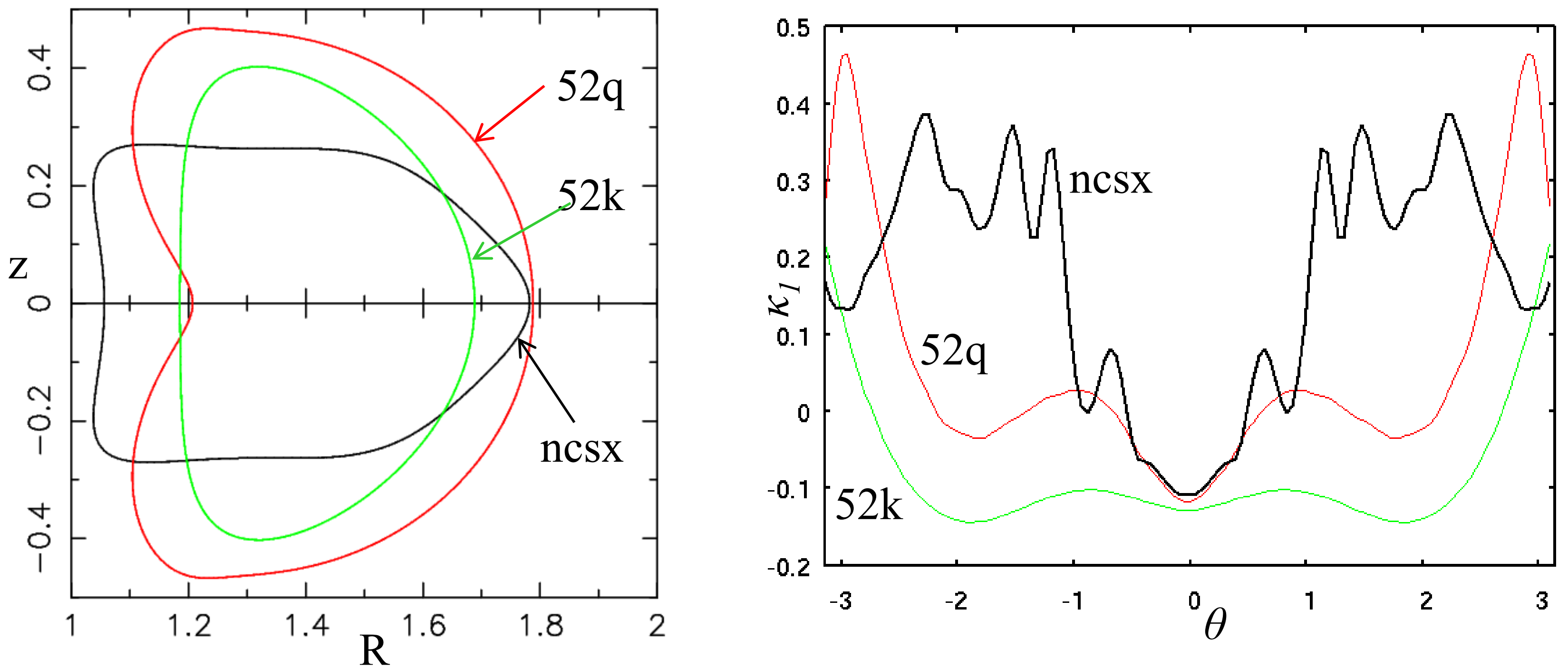
-(2): Evolving tokamaks:

-Start from TOK_52k: Shape close to axisymmetrized NCSX, tokamak-like ι(r) profile.

-Can consider 2 types of perturbations:

(a) *n=0* only, (b) *n≠0* only [Not covered here – work just beginning]

(2a) *n=0* only: Evolved config TOK_52q, possesses inboard indentation, which again displaces inner surfaces outward more than outer ones..
(This also known[9] to stabilize interchange/ballooning-type modes.)

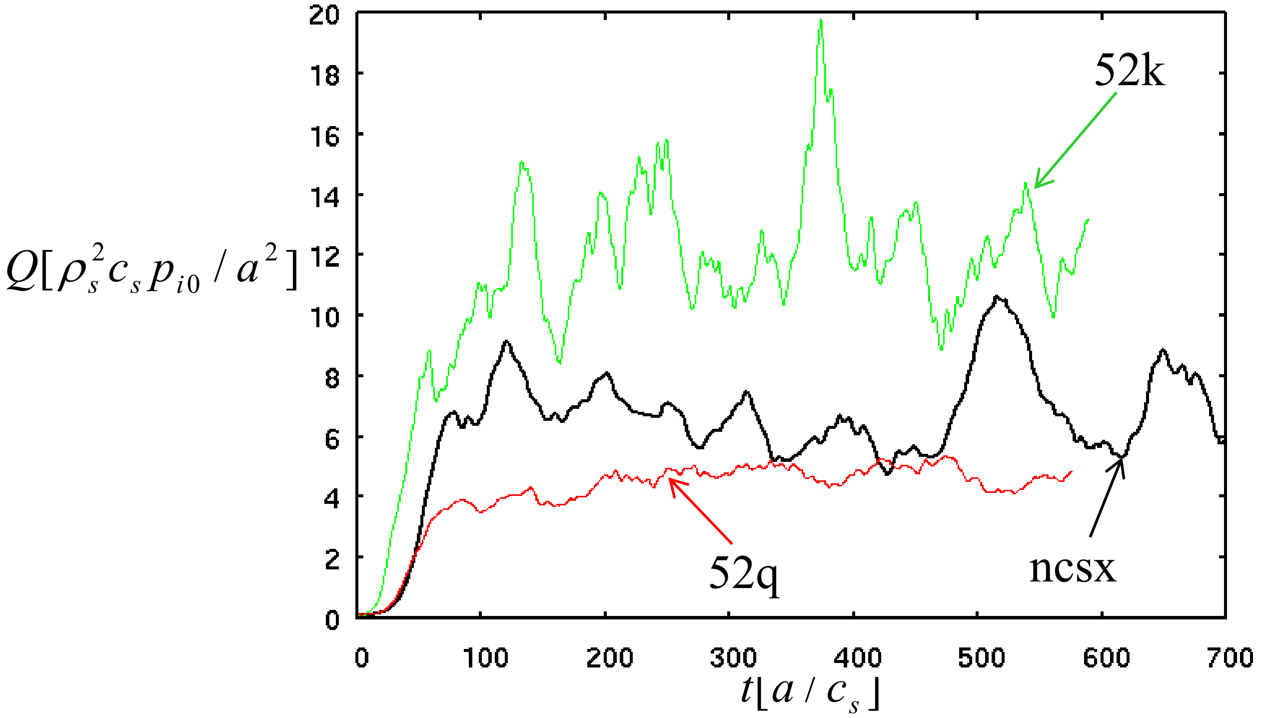


[9] M.S. Chance, S.C. Jardin, T.H. Stix, *Phys. Rev. Letters* **51** 1963 (1983).

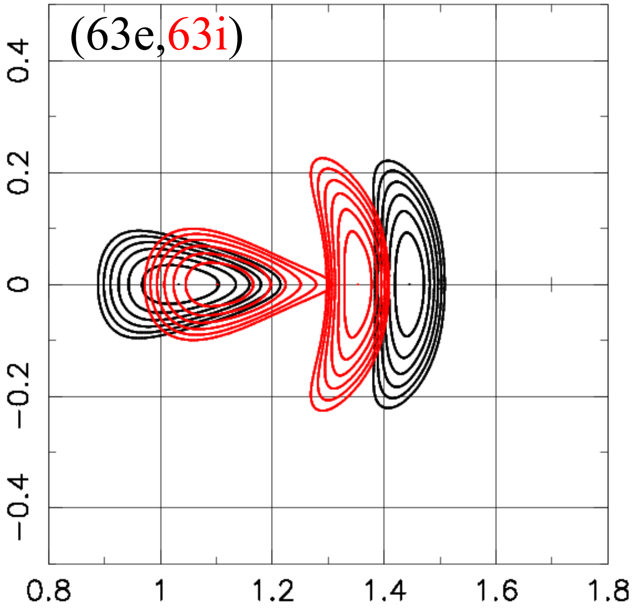
-GENE corroboration:

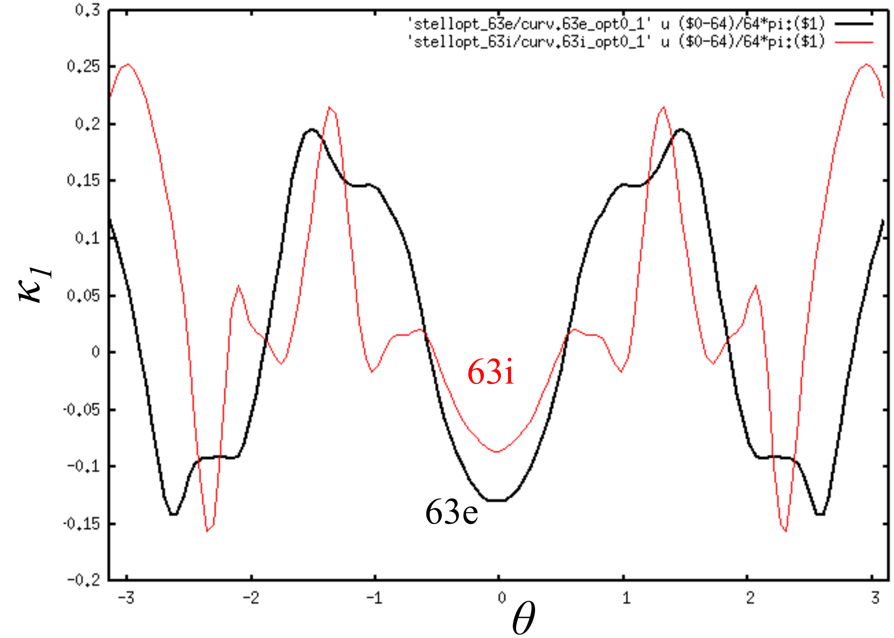
-Compare $Q_{gk}(t)$ for NCSX, TOK_52k, TOK_52q:

$Q_{gk}(t)$ for TOK_52k reduced by factor 3 for TOK_52q:

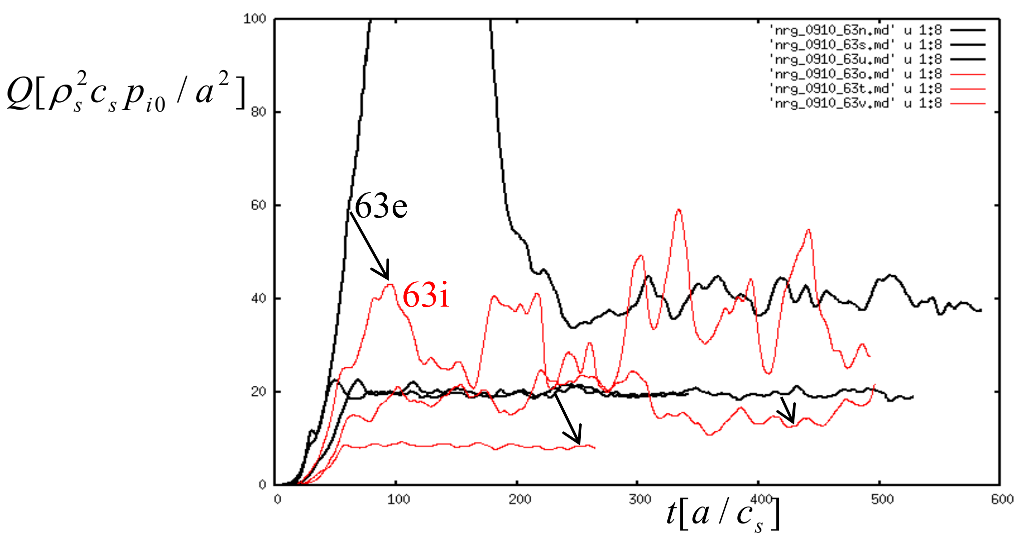


-(2c): Evolving QHs :HSX=QH_63e -> QH_63i:





-GENE corroboration:



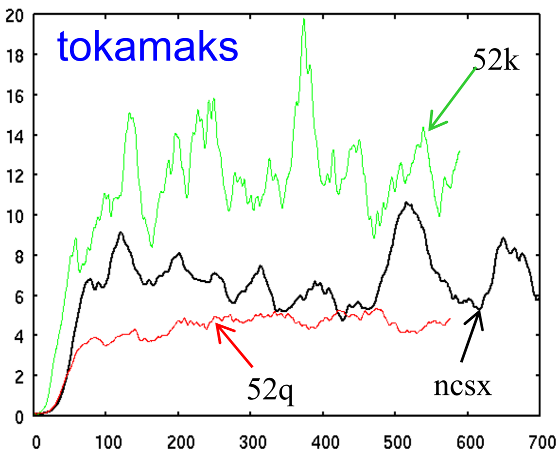
→ Spatially-avged reduction in Q_{gk} achieved ≈ 23%.

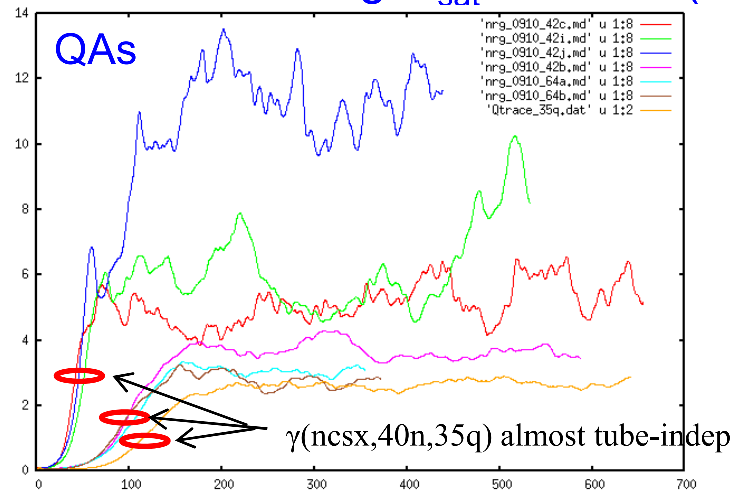
-(2d): Evolving QOs. (in progress):

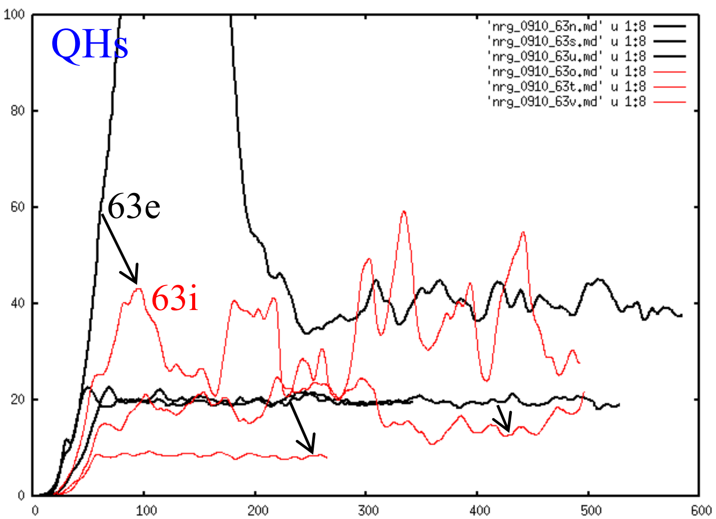
Mixing-length dependence Q~γ works better for QAs, tokamaks, QHs than for QOs.

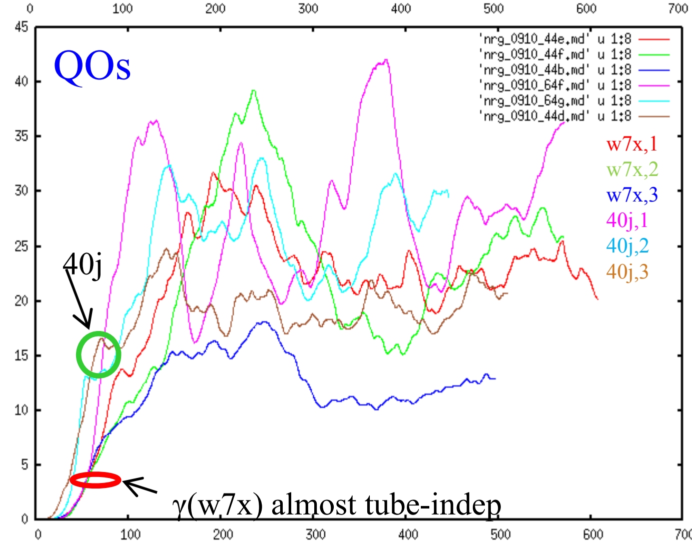
-Some possible reasons:

- -Difference in aspect ratios?
- -W7X designed to have little surface shift with change in parameters such as β.
- -Nonlinear physics appears more essential in understanding Q_{sat} for QOs. (WHY?)



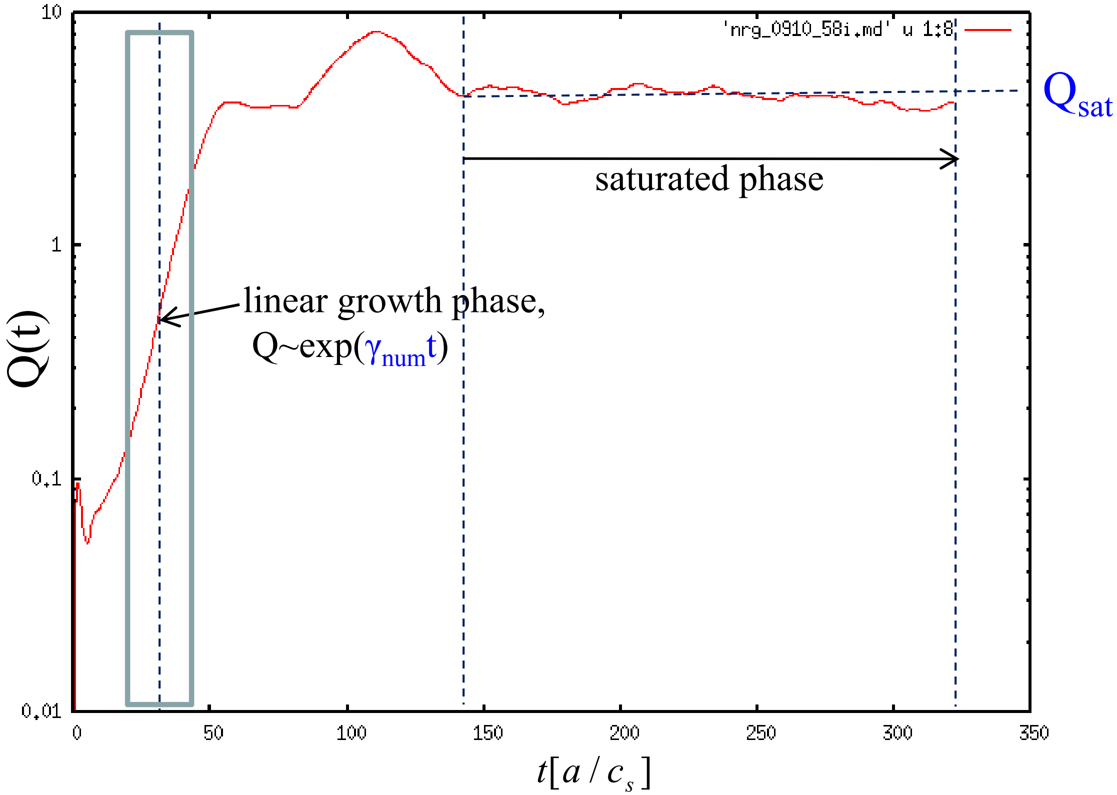






-(2e): Analyze Q(t) traces:

Infer numerical growth rates γ_{num}, saturation flux Q_{sat} from Q(t) traces,
& compare with analytic expressions $Q_{prox} \sim \gamma_{anl}$:

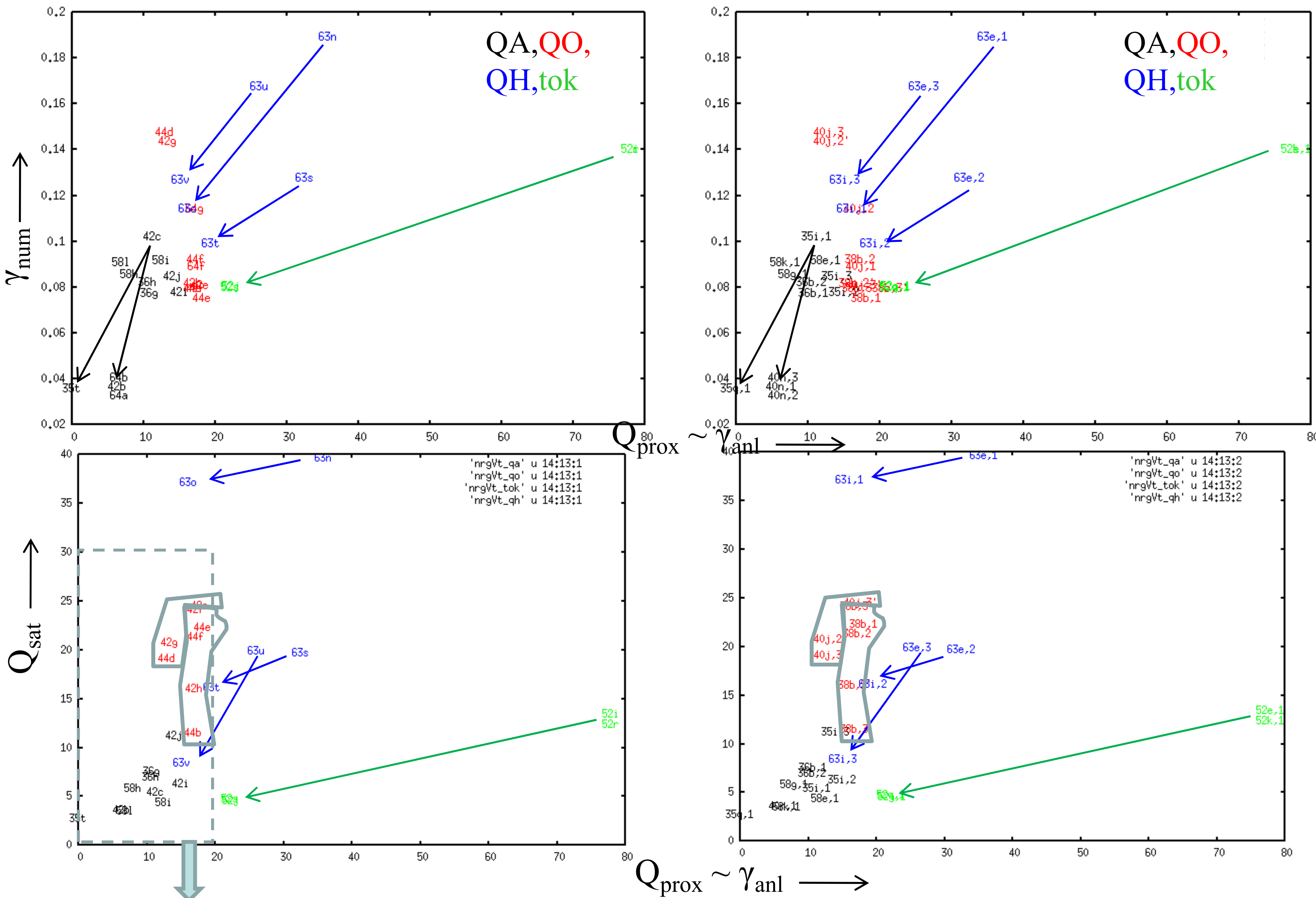


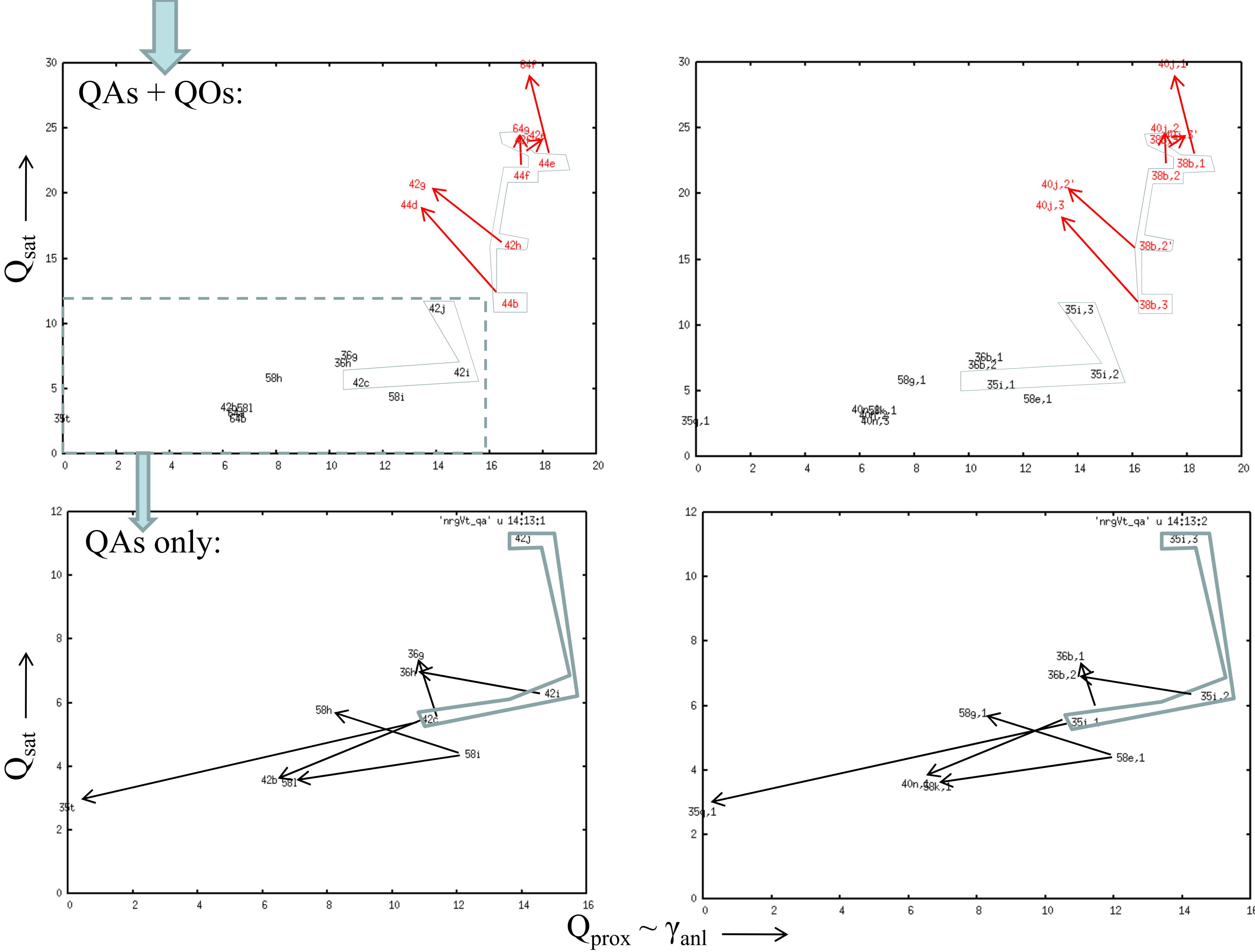
-With perfect theory, all device points on plots of Q_{prox} vs Q_{sat}, or γ_{anl} vs γ_{num} would lie on 45 degree lines.

-But to be sufficient, Q_{prox} need only have positive slope along course of evolution:

-(2e),cont: Correlate analytic results $Q_{prox} \sim \gamma_{anl}$ with numerical (gk) ones Q_{sat}, γ_{num} :

(Arrows indicate evolution dir'n, polygons group tubes from same device):





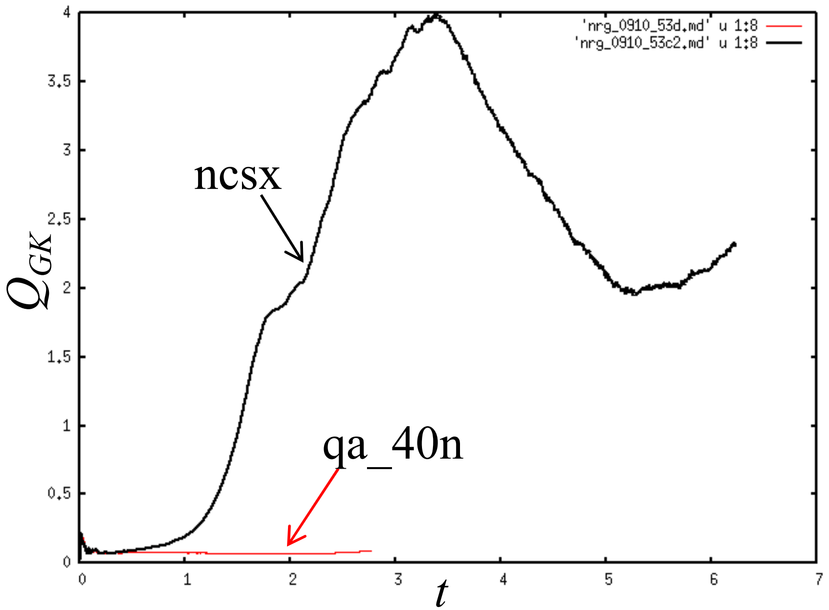
QAs + QOs:
QAs only:
Q_sat
Q_prox ~ γ_anl
'nrgVt_qa' u 14:13:1
'nrgVt_qa' u 14:13:2
64f
64g
42e
44e
44f
42g
44d
42h
44b
42j
36g
36h
58h
42c
42i
58i
42b
58l
64b
35t
40j,1
40j,2
38b,1
38b,2
40j,2'
40j,3
38b,2'
38b,3
35i,3
36b,1
36b,2
58g,1
35i,1
35i,2
58e,1
35q,1
40n,1
38k,1

-(3): Turbulence from ETG modes:

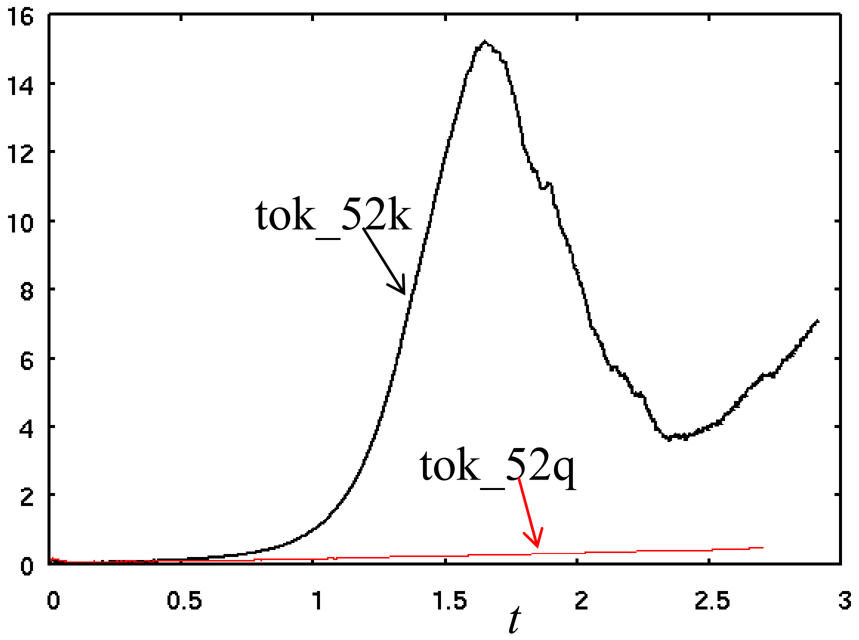
-Since ETG and ITG modes are closely related, might hope that designs which diminish ITG transport might also diminish ETG transport.

-GENE corroboration of this: using designs evolved for ITG transport (preliminary runs) :

For QA stellarators:



For tokamaks:



-The boosted κ_1 for the evolved configurations may also be expected to have improved stability to ballooning modes.[9]

-Summary:

-Using the gk code GENE and optimization code STELLOPT, we have demonstrated that stellarator and tokamak designs can be evolved which have turbulent transport levels appreciably below the starting configurations without this optimization.

-From analytic theory and GENE studies, a proxy function $Q_{prox}(\kappa_1, s_1)$ has been developed close enough to the gk prediction Q_{gk} to guide STELLOPT to configurations with diminished turbulent transport.

-Applied to the NCSX QA stellarator & optimizing for ITG transport, STELLOPT has evolved stellarator designs (QA_35q,QA_40n), with transport a factor of 2-2.5 below that of NCSX.

-Applied to the TOK_52k design (closely related to the NCSX design), STELLOPT has evolved a tokamak (TOK_52q) with transport a factor of 3 below that of TOK_52k.

-The designs optimized for ITG transport have been found to also have reduced ETG transport, due to the close relationship between these instabilities.

-The evolved configurations have improved avg curvature $<\kappa_1>$, hence should also have improved stability to ballooning/interchange-type MHD modes.

-The deformations by which STELLOPT "boosts" κ_1 appear to be those which shift inner surfaces further outward in *R* than outer ones, causing field lines to move more rapidly through the outboard bad-curvature region, resulting in configurations like those found in previous stellarator [8] and tokamak[9] work to have better MHD stability.

-Plans include applying the method to other interesting starting configurations, such as QO and QH stellarators, and extending Q_{prox} to focus on other key geometric quantities such as local shear, & to include other transport channels.

END