Sheared flow dynamics in edge turbulence*

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Introduction

• Core confinement in tokamaks is sensitive to edge profile gradients.
• The edge profiles are set by turbulence, so it is important to understand the mechanisms of turbulence saturation and how these influence the edge gradients.
• For typical edge profiles, there is no scale separation between background and turbulence, $k_L L_n \sim 1$.
• Edge turbulence studies require a “global” code (with edge and SOL profiles free to evolve), not a “local” or flux-tube code. Here, we use the 2D SOLT (SOL turbulence) code.
• We are studying the roles of
  – pressure profile modification
  – Reynolds stress and sheared flows
  – radial variation in geometry and physics (open and closed field lines, drift wave, curvature driven and sheath regions)
Outline

• Quasi-linear calculations for Kelvin-Helmholtz (KH) and interchange modes
• Overview of SOLT turbulence simulations
• Comparison of quasi-linear theory with simulations
• Conclusions
Quasi-linear model

- Interchange and Kelvin-Helmholtz (KH) stability is given by

\[(\omega - k_y v_y)(\Phi'' - k_y^2)\Phi + k_y v_y' \Phi + \frac{k_y^2 n_0'}{n_0(\omega - k_y v_y)} \Phi = 0\]

- BCs: \(\Phi \to 0\) as \(x \to \pm \infty\)
- Inputs: \(k_y\) and profiles \(n_0(x)\) and \(v_y(x)\) \(\Rightarrow\) scalelengths \(L_n\) and \(L_v\)
- Compute the Reynolds’ stress (RS)-generated \(dv_y(x)/dt\).
  \(\Rightarrow\) the sign of \(dv_y(x)/dt\) depends on \(k_y L_n\) and \(L_v/L_n\).
- Let \(L_n = 1\) so that length scales are normalized to \(L_n\).
Constant velocity shear \((L_v \gg L_n)\)

\[ k_y L_n = 0.6 \]

RS tries to increase the \(v_y\) gradient at the mode center \(\Rightarrow\) ZF instability

Interchange growth rate \(\gamma\) vs \(k_y L_n\)

Shear stabilizes the interchange mode and is most effective at high \(k_y\).

Constant \(v_y\) shear \(\Rightarrow\) no KH mode.
Localized $v_y$ shear ($L_v \sim L_n$)

Note dependence of growth rate on $k_y L_n$:

(a) small $k_y L_n$ \quad \Rightarrow \quad \gamma \rightarrow 0$ for all instability drives
   (flows are not important)

(b) intermediate $k_y L_n$ \quad \Rightarrow \quad flows are destabilizing (KH mode)

(c) large $k_y L_n$ \quad \Rightarrow \quad flows stabilize interchange mode

The amp = 0.5 case is largely interchange driven.
Effect of the RS depends on $k_y L_n$

$\text{amp} \ll 1 \text{ for both cases } \Rightarrow \text{pure\ interchange\ instability\ drive}$

$k_y L_n >> 1 \Rightarrow \text{RS tries to increase the gradient in } v_y \text{ at the mode center, enhancing the flow that was present.}$

$k_y L_n \leq 1 \Rightarrow \text{The RS opposes the existing flow.}$
Quasi-linear picture

From these and other cases, varying the shapes of the profiles and the scale lengths, we obtain the following picture of how curvature-driven interchange modes interact with sheared flows:

- **Pure KH modes** act to reduce the \( v_y \) shear driving them (at least locally, although this may increase the shear elsewhere).

- Shear in \( v_y \) stabilizes the interchange mode and is most effective on the high \( k_y \) modes.

- The Reynolds Stress (RS) from pure interchange modes acts to enhance the imposed flows if the flows are large-scale with respect to the mode’s radial variation, while the RS opposes the seed flows if those flows are small scale.
Quasi-linear picture - 2

- An important implication of this work is that long wavelength modes (with respect to the pressure gradient) cannot be stabilized by self-generated sheared flows; instead, they must be stabilized by pressure profile modification (wave-breaking, plateau formation). This agrees with the SOLT simulations described subsequently.

- Finally note that these results suggest that the inverse cascade in $k_y$ may be responsible for this behavior:
  - Large $k_y$, associated with small-scale radial mode structure, feeds the (larger-scale) flow.
  - Small $k_y$, associated with large-scale radial mode structure, saps the (smaller-scale) flow.

- This picture is in qualitative agreement with SOLT simulations of turbulence saturation as a function of radial gradients
For comparison with the SOLT turbulence simulations, summarize the results of the quasi-linear calculations for curvature-driven interchange modes in the following table:

<table>
<thead>
<tr>
<th></th>
<th>$L_v \leq L_p$</th>
<th>$L_v \gg L_p$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$k_y L_p \gg 1$</td>
<td>RS+</td>
<td>RS+</td>
</tr>
<tr>
<td>$k_y L_p \leq 1$</td>
<td>RS−</td>
<td>RS+</td>
</tr>
</tbody>
</table>

where $\text{RS}^+$ ($\text{RS}^-$) $\Rightarrow$ the Reynolds Stress (RS) acts to increase (decrease) the flow shear at the mode center.

RS+ is the usual ZF instability found in flux-tube (local) simulations. RS− requires a global turbulence code (e.g. SOLT).
SOLT turbulence studies

We have done a study of the saturation of the turbulence as a function of the instability drive $\gamma_0 \propto dP/dx$ ($P = n \, T$)

Definitions:

$\alpha_{dw} = \frac{2\rho_s m_i c_s}{L_{||e} v_{ei0} m_e}$

$\beta = \frac{2\rho_s}{R}$  $\alpha_{sh} = \frac{2\rho_s}{L_{||s}}$

$\gamma_0 \equiv \left(\frac{\beta}{n} \frac{dP}{dx}\right)^{1/2}$
Large gradient case

run32 Quasi-steady state:

Below: shaded area indicates FWHM of particle flux $\Gamma = \langle n v_x \rangle$

$p$ (blue), $v_y$ (black), $\bar{dp}_y/dt$ (green)

$\Rightarrow$ RS- regime near max $\Gamma$

$\Rightarrow$ flows are not strong enough to stabilize interchange modes)

$\langle v'_y \rangle_{y,t}/\langle \gamma_{mhd} \rangle_{y,t} \approx 0.6$

$\langle k_y L_p \rangle_{y,t} \approx 1.2$
Small gradient case

run33

Right: note different initial transient and fluctuation behavior than in run32

Below: shaded area indicates FWHM of particle flux \( \Gamma = \langle n v_x \rangle \)

\( p \) (blue), \( v_y \) (black), \( dp_y/dt \) (green)

\( \Gamma(t) \)

\( 0.000 \quad 0.001 \quad 0.002 \quad 0.003 \quad 0.004 \)

\( 0 \quad 200 \quad 400 \quad 600 \quad 800 \quad 1000 \)

\( \Gamma(t) \)

\[ \Rightarrow \text{RS+ regime near max } \Gamma \]

\[ \Rightarrow \text{sheared flows help stabilize interchange modes} \]

\[ \langle v'_y \rangle_{y,t} / \langle \gamma_{\text{mhd}} \rangle_{y,t} \approx 2.4 \]

\[ \langle k_y L_p \rangle_{y,t} \approx 7.5 \]
Regime diagram for SOLT study

Plot of particle flux $\Gamma$ vs interchange growth rate $\gamma_0$ (see p. 11)

Saturation mechanisms
1 $\Rightarrow$ flow + profile mod
2 $\Rightarrow$ profile mod only
3 $\Rightarrow$ flow only (frozen profiles)

- $y$- and $t$-averaged flux
- evaluated at radial location of max flux $\Gamma$

blue curve $\Rightarrow$ stabilizing effect of sheared flow decreases rapidly as grad-P increases and $k_y L_p$ decreases (agrees with the quasi-linear model)
Conclusions: emerging picture

- Here we have studied curvature-driven interchange turbulence in the edge plasma, saturated by a combination of
  - sheared poloidal flows
  - radial profile flattening
- The dominant saturation mechanism depends on the radial gradients.
- The experimental geometry (changing topology of magnetic field lines) sets the natural radial scale $L_x$ of the edge turbulence.
- The radial mode structure of the turbulence, and profiles, determine the $k_y$ spectrum, and $k_y L_x$ determines the evolution of the sheared flows.
- The relative ordering of $k_y$, $L_p$, and $L_v$ (pressure and sheared velocity scale lengths) influences the sign of the Reynolds Stress (RS) and whether it causes the sheared flows to grow (RS+) or damp (RS-)
  - this may be related to the inverse cascade
  - see pp. 8 – 10 for details
Conclusions (cont.)

- The **RS regime influences the saturation mechanism** for the interchange turbulence
  - RS+ ⇒ shear flow stabilization
  - RS- ⇒ profile flattening

- When the mode significantly overlaps with the DW region inside the edge or the sheath-connected region in the far SOL, **additional physics** enters:
  - DW ⇒ directionality, poloidal flows
  - sheath ⇒ sink for particles, heat and momentum;
    Bohm sheath-potential-driven flows

- Preliminary **comparisons with SOLT turbulence simulations** are encouraging but more work is needed to obtain optimal simulations for this comparison.