Effects of Particle Deposition Profile on L -> H Transition and Hysteresis Dynamics

M.A. Malkov (1), P.H. Diamond(1,2), W.W. Xiao(2)

(1)CMTFO and CASS, UCSD; USA

(2)WCI Center for Fusion Theory, NFRI; Korea

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Outline

- L-- H transitions and hysteresis in 1D, phase coexistence
- Continuous media 1D model

  - Transition point selection in a steady state:
    - regularization
    - time dependence, functional approach

  - Scan of parameter space

  - Pressure curvature (second derivative) effects

- ❑ Internal fueling
- ❑ Time dependent fueling
- 0-D, ODE-model, time dependent case

  - Fixed point analysis, bifurcations

  - Hysteresis, transition control

- Conclusions

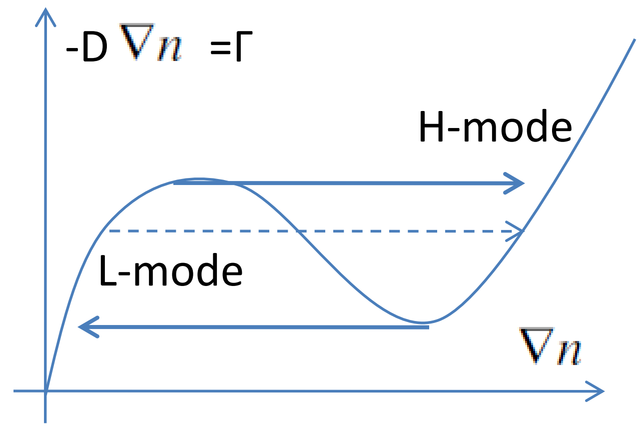
Simple two-field model of L-H transition

(Hinton, Staebler '93)

Particle transport with fueling S

$$\frac{\partial n}{\partial t} - \frac{\partial}{\partial x}\left[D_0 + \frac{D_1}{1+\alpha V_E'^2}\right]\frac{\partial n}{\partial x} = S(x)$$

Neoclassical diffusion   Turbulent part



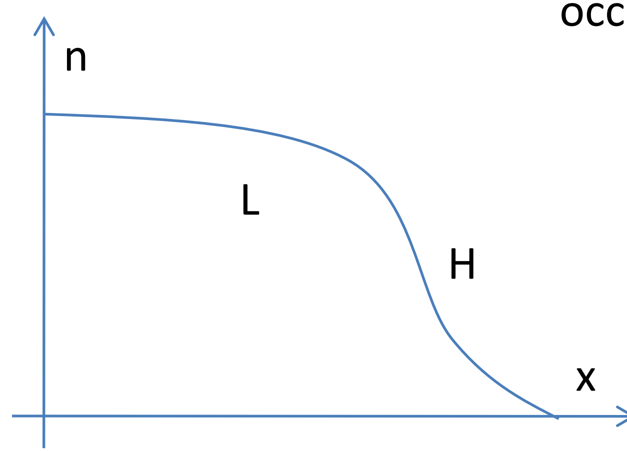
Heat transport , source H

$$\frac{3}{2}\frac{\partial p}{\partial t} - \frac{\partial}{\partial x}\left[\chi_0 + \frac{\chi_1}{1+\alpha V_E'^2}\right]\frac{\partial p}{\partial x} = H(x)$$

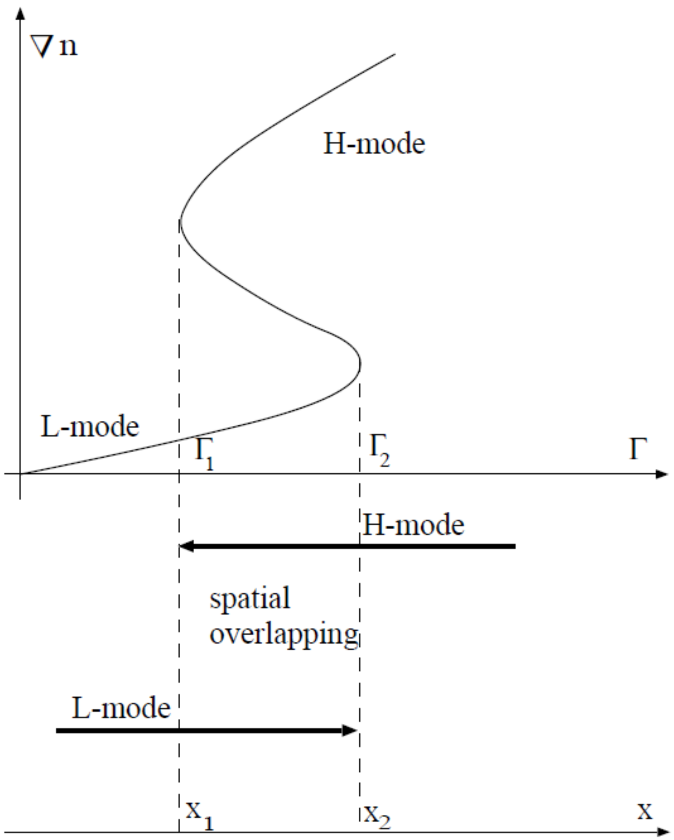
Suppression: $\mathbf{E}\times\mathbf{B}$ flow shear

$$V_E' \simeq \frac{c}{eB}\frac{\partial}{\partial x}n^{-1}(x)\,\frac{\partial}{\partial x}p(x)$$

Pedestal parameters depend on where exactly transition occurs

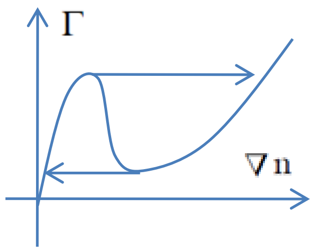


Phase coexistence dilemma

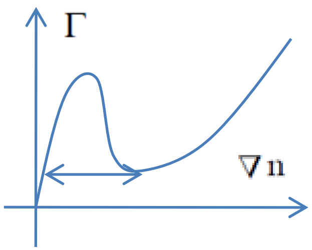


The problem can be considered as
Propagation of one phase into another

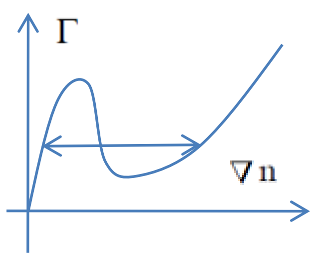
selection of transition point



Stay at a stable branch
As long as it exists
(works in 0D)



Minimum gradient jump
Criterion
Hinton and Staebler '93



Maxwell (equal area)
Rule: works in 1D, 1
field (n) model
Lebedev and Diamond
'95

Phase-coexistence conditions in a steady state

$$D_0 g_1 + \frac{D_1 g_1}{1+\alpha V_E'^2} = \int_0^x S(x)dx \equiv \Gamma_s(x) \qquad g_1 = -\frac{\partial n}{\partial x},$$

$$\chi_0 g_2 + \frac{\chi_1 g_2}{1+\alpha V_E'^2} = \int_0^x H(x)dx \equiv Q_s(x) \qquad g_2 = -\frac{\partial p}{\partial x}.$$

Algebraic problem for one of the gradients:

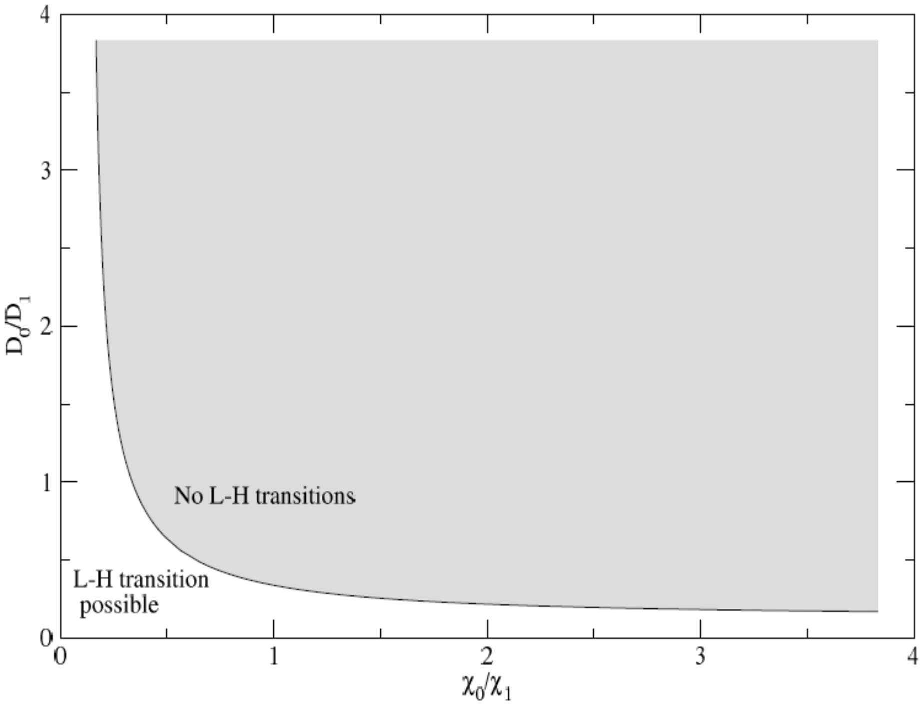
$$g + \frac{\lambda g}{1+g^4\left(1+\Theta g\right)^{-2}} = \hat{\Gamma}$$

$$\hat{\Gamma} = K D_0^{-1}\sqrt{Q_s \Gamma_s(x) D_1/\chi_1}; \qquad \lambda = D_1/D_0$$

$$\Theta = K^{-1}\sqrt{\chi_1 D_1/Q_s\Gamma_s(x)}\left(\chi_0/\chi_1 - D_0/D_1\right) \qquad K = \alpha^{1/4}\sqrt{c/eB}n^{-1}.$$

➢ model predicts a close link between fueling depth and the width of the enhanced confinement region

Phase-coexistence criteria and the depth of the bifurcation in a steady state



Neoclassical to turbulent transport
Ratio parameter space

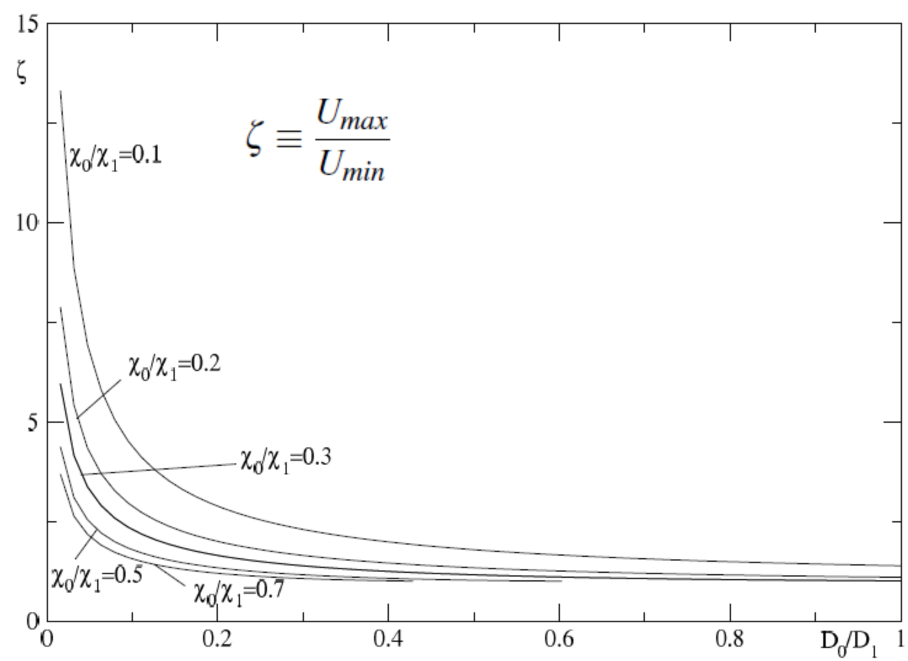
Phase coexistence range

$$U_{min} < K^2 \frac{\Gamma_s Q_s}{D_0 \chi_0} < U_{max}$$

$$K^2 = \sqrt{\alpha} c / eBn^2$$

Bifurcation depth defined as

$$\zeta \equiv \frac{U_{max}}{U_{min}}$$



Hyperdiffusion Regularization

$$\Phi'(g) - \Gamma_s(x) - \varepsilon^2 \frac{\partial^2 g}{\partial x^2} = 0 \longrightarrow \text{Maxwell rule}$$

Time dependent regularization

$$\chi_{0,1} \gg D_{0,1} \qquad \frac{\partial g}{\partial t} = \frac{\partial^2}{\partial x^2} \frac{\delta \Lambda}{\delta g} \qquad \Lambda = \int_0^a \left[\Phi(g) - \Gamma_s g\right] dx$$

$$\frac{d\Lambda}{dt} = -\int_0^a \left\{ \frac{\partial}{\partial x} \left[\Phi'(g) - \Gamma_s\right] \right\}^2 dx \leq 0$$

Minimum Λ $\longrightarrow$ Maxwell rule

- Maxwell rule governs forward and back transitions
- hysteresis is absent

Curvature of the pressure profile

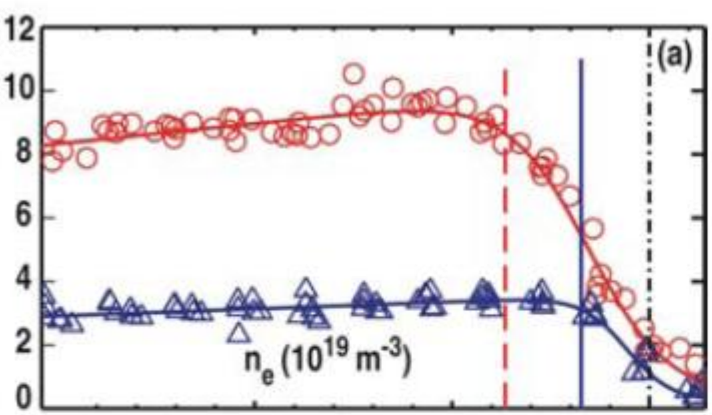
$$\frac{\partial V_E}{\partial x} \simeq -\frac{c}{eBn^2}\frac{\partial n}{\partial x}\frac{\partial p}{\partial x} + \frac{c}{eBn}\frac{\partial^2 p}{\partial x^2}$$

Second derivative resolves transition location

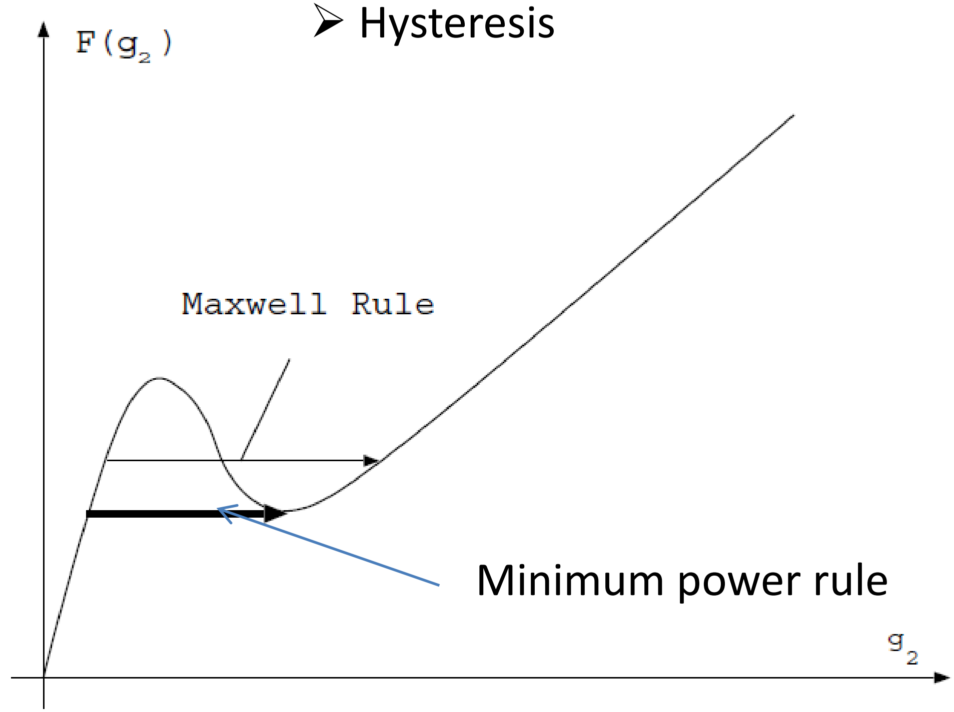
$$g_2 + \frac{(\beta - 1)\, g_2}{1 + \left(\frac{\sigma g_2^2}{1+\kappa g_2} + \mu \frac{dg_2}{dx}\right)^2} = q(x)$$

Differential equation for the pressure gradient
Instead of algebraic one

- Likely scenario: Maxwell forward
- Minimum power back transition
- Hysteresis



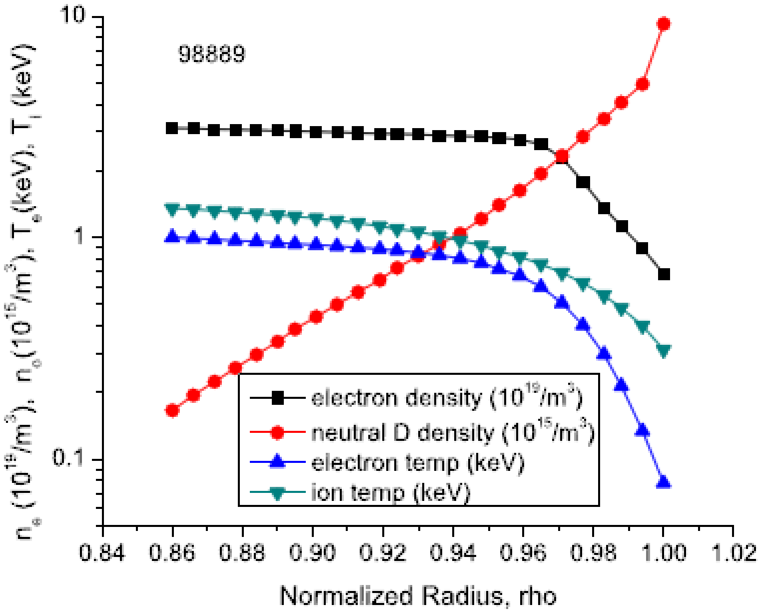
R.J. Groebner et al '09



Notations

$$q = Q_s / \chi_0, \quad \beta = 1 + \chi_1 / \chi_0;$$

$$\sigma = \sqrt{\alpha}\frac{c}{eBn^2}\frac{\Gamma_s \chi_1}{Q_s D_1}; \quad \kappa = \frac{D_0\chi_1 - \chi_0 D_1}{D_1 Q_s}; \quad \mu = \sqrt{\alpha}\frac{c}{eBn} \equiv nK^2$$



- Second derivative of pressure is clearly important, as opposed to that of the density profile
- Model correctly captures the roles of density and pressure profiles in suppressing the fluxes

Stacey and Groebner '09

Internal deposition at a finite depth within the separatrix (SMBI)

Phase transition

$$F(g) \equiv g + \frac{(\beta - 1) g}{1 + v^2 g^4 (1 + kg)^{-2}} = 1$$

$$g = \frac{\chi_0}{Q_s} g_2 \quad v = \frac{\sqrt{\alpha} c}{eBn^2} \frac{\Gamma_s \chi_1}{\chi_0^2 D_1} Q_s \quad k = \frac{D_0 \chi_1}{D_1 \chi_0} - 1$$

$kg < 1$ -reasonable approximation on the ground of a better pronounced density LH transition compared to the pressure (temperature) transition

Phase transition threshold

$$\frac{\sqrt{\alpha} c}{eBn^2} \frac{\Gamma_s Q_s}{\chi_0 D_1} > \frac{16}{3^{3/2}} \sqrt{\frac{\chi_0}{\chi_1}}$$

Heating power may be reduces at the fueling expense

- However, density build up requires more deposition and heating

Barrier propagation for time dependent deposition

$$\frac{\partial g}{\partial t} = \frac{\partial^2}{\partial x^2}\left[F(g) - \Gamma(x,t)\right]$$

F-nonlinear particle flux, Γ-internal deposition (integrated in x)

Let the barrier at x=b(t)

$$g(x,t) = \begin{cases} g^+, & x \geq b(t) \\ g^-, & x < b(t) \end{cases}$$

$$\frac{\partial g}{\partial t} - \dot{b}\frac{\partial g}{\partial \xi} = \frac{\partial^2}{\partial \xi^2}\left[f - \Gamma(x+b)\right] \qquad \xi = x - b(t).$$

Barrier propagates according to

$$\dot{b} = \frac{a\left(f_b - \Gamma_b\right) - b\left(f_a - \Gamma_a\right)}{\Delta g b\left(a - b\right)}$$

Inward barrier propagation promoted by

- under-fueling at the wall
- over-fueling at the barrier

Time dependent 0-D model

$$\frac{d\mathscr{E}}{d\tau} = \left(\mathscr{N} - a_1\mathscr{E} - a_2 d^2 \mathscr{N}^4 - a_3 V_{ZF}^2\right)\mathscr{E}$$

DW generation by temperature Gradient N, NL saturation and by By mean and zonal flows

$$\frac{dV_{ZF}}{d\tau} = \left(\frac{b_1\mathscr{E}}{1 + b_2 d^2 \mathscr{N}^4} - b_3\right) V_{ZF}$$

ZF generation by Reynolds stress Suppression by mean flow and Damping by collisions

$$\frac{d\mathscr{N}}{d\tau} = -(c_1\mathscr{E} + c_2)\mathscr{N} + q(\tau)$$

Temperature gradient maintained by Heat source q and relaxed by turbulent and Neoclassical transport

Mean flow

$$V = d\mathscr{N}^2$$

(Kim and Diamond '03)

- System demonstrated interesting behavior including dithering
- System has too many parameters
- Difficult to classify dynamics

Reduction to a five-parameter system

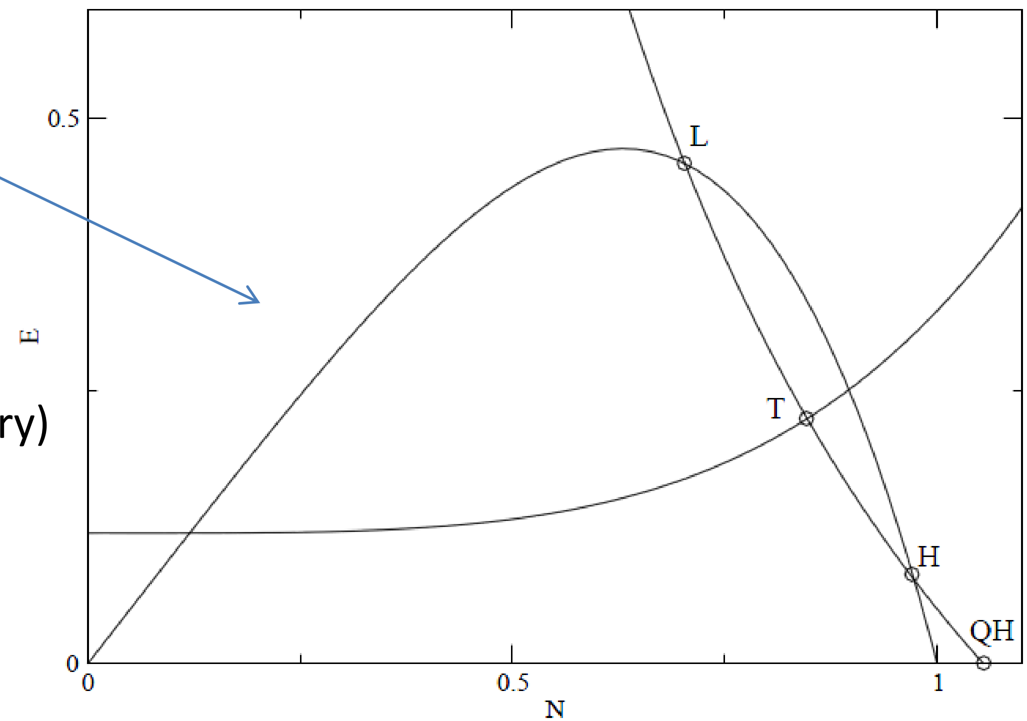
$$\frac{dE}{dt} = \left(N - N^4 - E - U\right)E$$

$$\frac{dU}{dt} = \vartheta\left(\frac{E}{1+\zeta N^4} - \eta\right)U$$

$$\frac{dN}{dt} = q\left(t\right) - \left(\rho + \sigma E\right)N$$

- Dynamics is limited to a set of 2D manifolds around fixed points (Center manifolds)

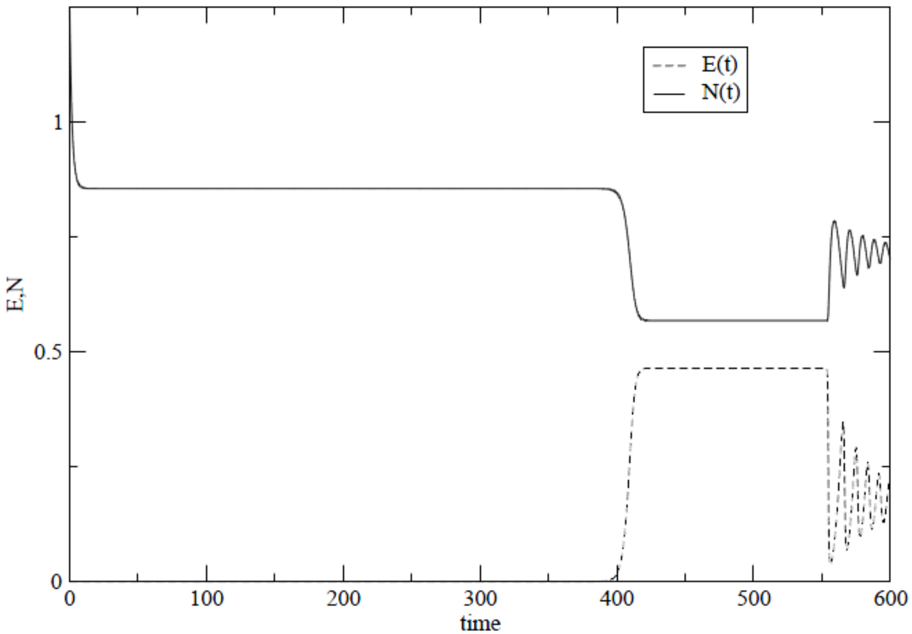
U=0 plane (no ZF) projection
Of the fixed points



q increases

- L-mode
- T-mode (transient oscillatory)
- H-mode
- QH- quiescent H mode

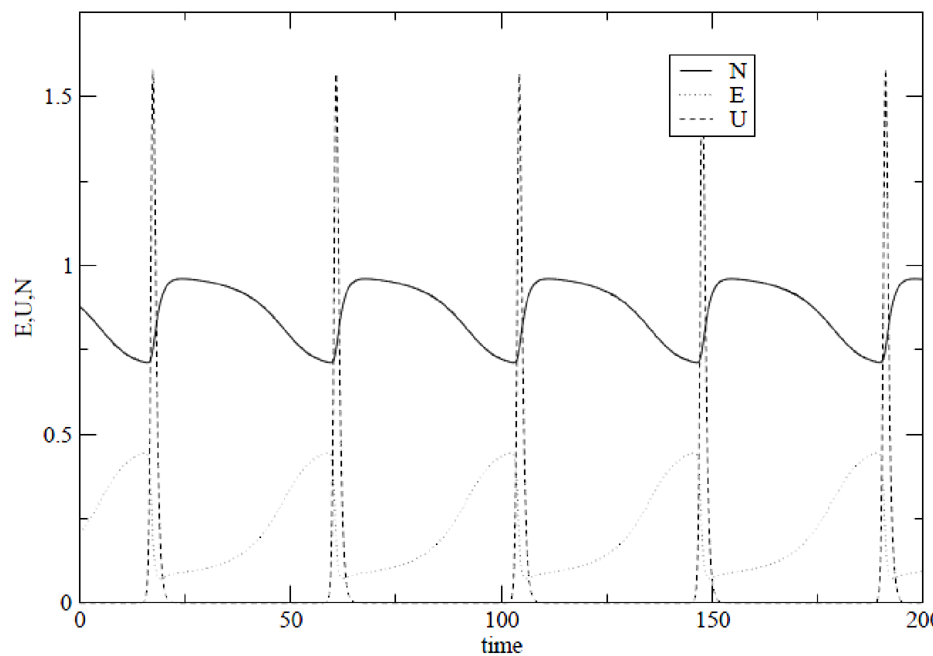
Meta-stable states and transitions



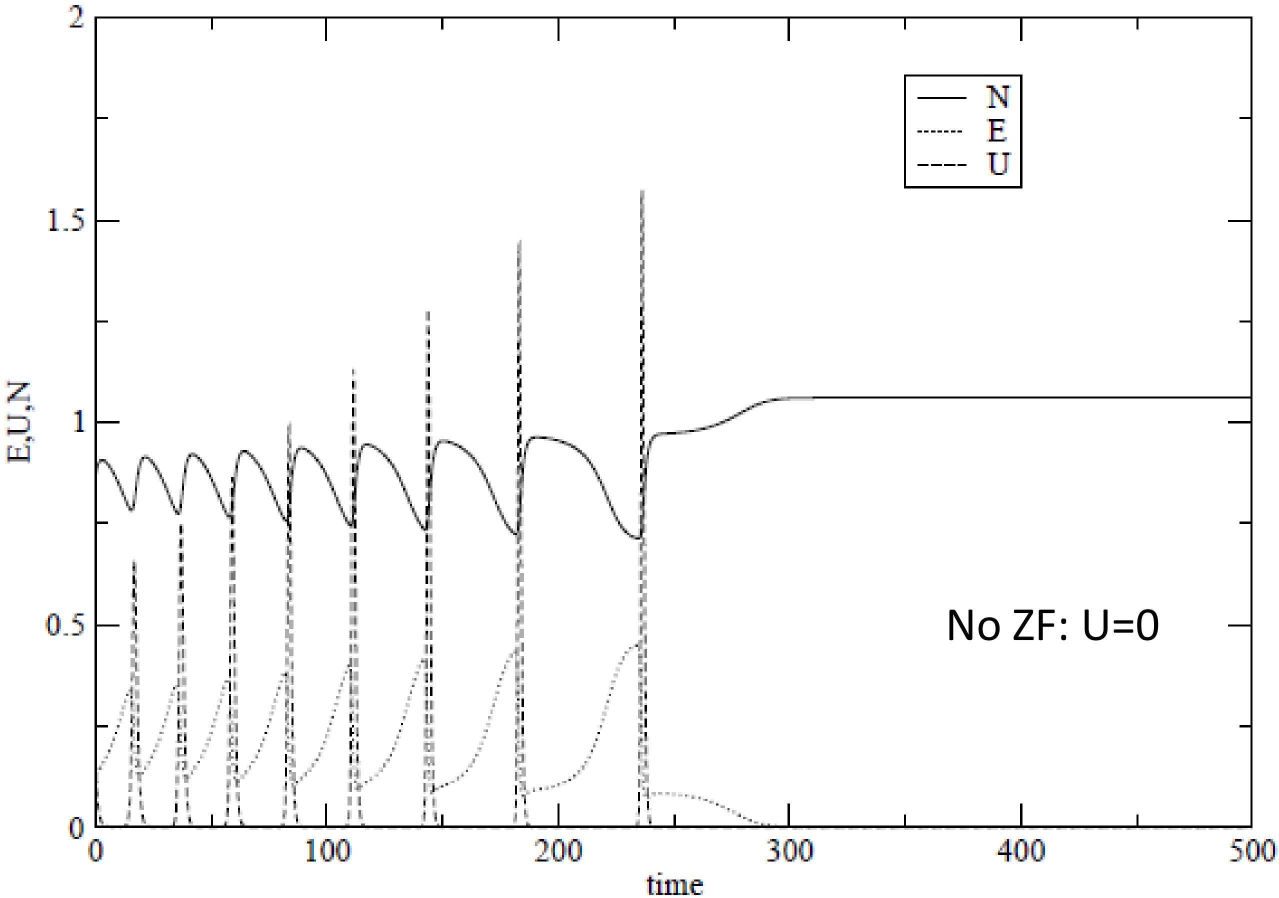
Unstable sequence, q=0.47

Unstable H→ unstable L →stable T

Hopf bifurcation:
Stable T-mode →
Stable limit cycle

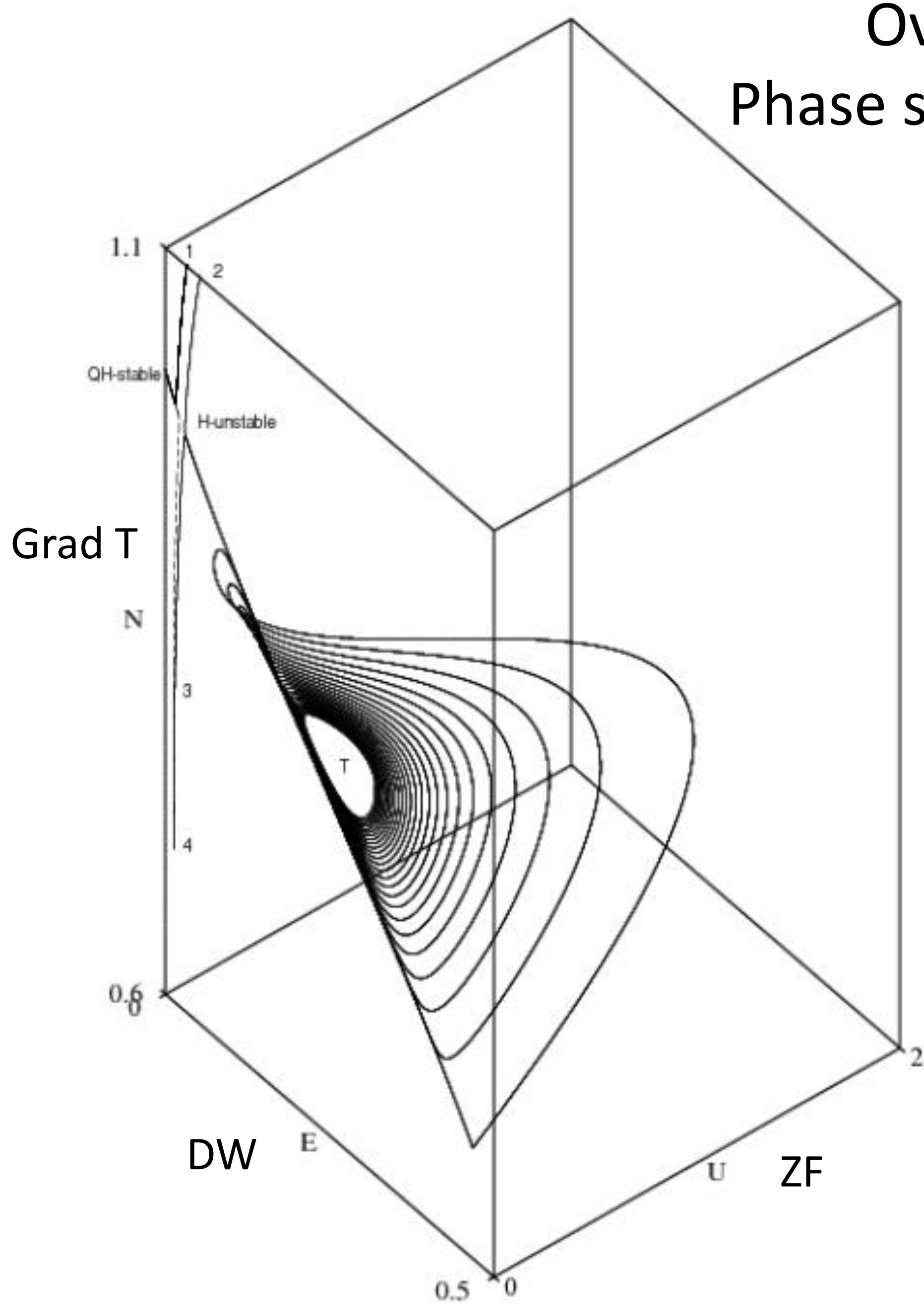


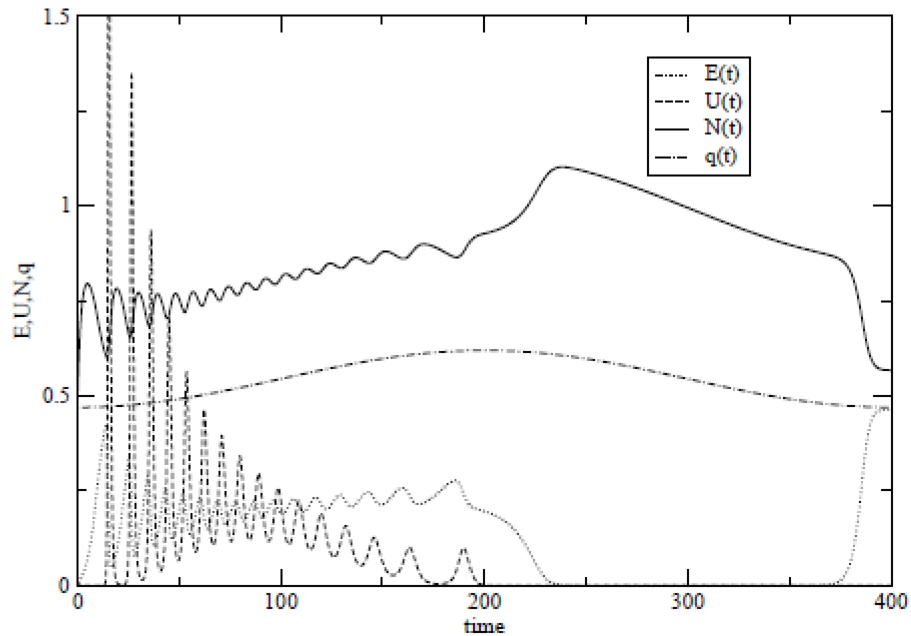
T→QH transition



At q=0.582 limit cycle disappears
System transits to the QH mode (now the only stable fixed point)

Overview of the Phase space of the system

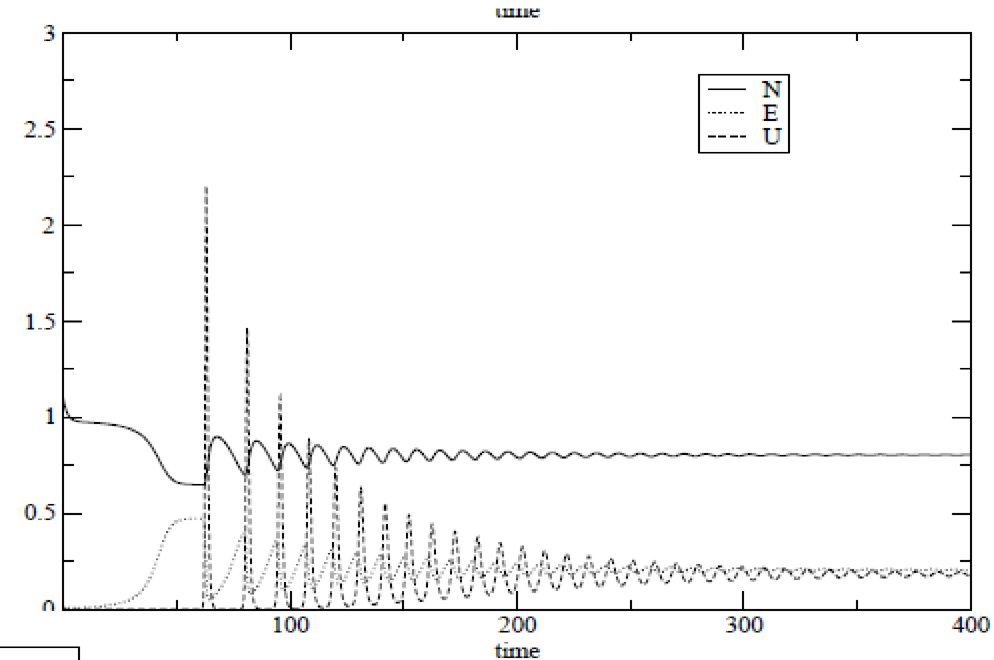


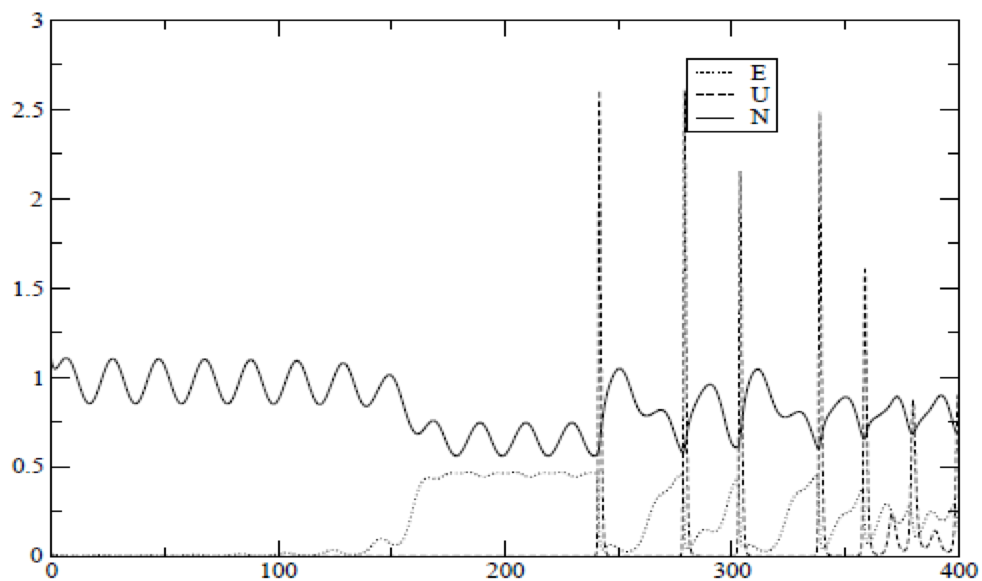


- Asymmetry of forward and back transition
- Forward transition from L to QH transition requires finite ZF (temporarily)
- Back transition may occur on ZF=0 plane
- Weak hysteresis, ~10% in heating rate q, not robust, sensitive to initial conditions

Stabilization of meta-stable H-mode fixed point by heat source modulation

Underpowered (unstable) QH mode
Undergoes transition to stable oscillatory T-mode





Applying modulated (sinusoidal) q(t) with the same average
Significantly delays transition

Conclusions

- role of phase coexistence in defining hysteresis in ***local*** two-field 1D model is classified
  - Maxwell rule governs the onset of the forward and back transitions which formally precludes *hysteresis*
  - the model predicts a close link between fueling depth and the width of the enhanced confinement region
- retaining pressure profile curvature (*some non-locality)*
  - backward transition occurs at the end point of the co-existence interval
  - ‘half-S-curve’ ***hysteresis***
  - Softening L-H transition requirements
  - the pedestal partially decouples from the fueling depth and broadens
- internal fueling further lowers power requirements for L-H transition

  -- core density build-up may somewhat increase the transition threshold
- studies of dynamical model indicate that hysteresis exists, but the basin of attraction for the H-mode shrinks rapidly with decreasing power
  - ZF generation is necessary in L→H transition but can be avoided in H→L transition
  - Power needed to maintain QH-mode can be lowered by modulating the heat source